This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

+ *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.

+ *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.

+ *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.

+ *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at `http://books.google.com/`

DAVIE

PRACTICAL

ARITHME

UNIT
Increased by the Scale of Ten
10000 1000 100 10
1
.1 .01 .00
Increased by varying Scales
Divided by varying
1
1
Unit of Measure of all Numbers

New York:

duc T 118.87.315

2044 097 001 036

Harvard College
Library

VE RI
TAS

By Exchange

PRACTICAL

ARITHMETIC:

EMBRACING

THE SCIENCE AND APPLICATIONS OF NUMBERS.

BY

CHARLES DAVIES, LL.D.,

PROFESSOR OF HIGHER MATHEMATICS IN COLUMBIA COLLEGE, AND
AUTHOR OF DIFFERENTIAL CALCULUS, ANALYTICAL GEOMETRY,
DESCRIPTIVE GEOMETRY, ELEMENTS OF SURVEYING,
ALGEBRAS AND ARITHMETICS.

A. S. BARNES & COMPANY,
NEW YORK, AND CHICAGO,

Educ T 118.87.315

3 2044 097 001 036

Harvard College Library

VE RI TAS

By Exchange

PRACTICAL

ARITHMETIC:

EMBRACING

THE SCIENCE AND APPLICATIONS OF NUMBERS.

BY

CHARLES DAVIES, LL.D.,

PROFESSOR OF HIGHER MATHEMATICS IN COLUMBIA COLLEGE, AND
AUTHOR OF DIFFERENTIAL CALCULUS, ANALYTICAL GEOMETRY,
DESCRIPTIVE GEOMETRY, ELEMENTS OF SURVEYING,
ALGEBRA AND ARITHMETIC.

[illegible]

Educ T 118.87.315
✓

HARVARD COLLEGE LIBRARY
BY EXCHANGE
JAN 8 1937

A NEW SERIES OF MATHEMATICS,

By CHARLES DAVIES, LL.D.,

AUTHOR OF THE WEST POINT COURSE OF MATHEMATICS.

The following named volumes are entirely new works, written within the past ten years, to conform to all modern improvement, and take the place of the author's older series.

NO CONFLICT OF EDITIONS

is possible, if patrons will be particular to order the book they want by its exact title. Whenever any change is made so radical as to be likely to cause confusion in classes,

THE NAME OF THE BOOK IS CHANGED.

Teachers using any work by DAVIES not here-in-after enumerated, are not availing themselves of the advantages offered by

THE NEW SERIES.

☞ *Primary, Intellectual, and Practical Arithmetics constitute the Series proper. Other volumes are optional.*

DAVIES' PRIMARY ARITHMETIC.
The elementary combinations, by object lessons.

DAVIES' INTELLECTUAL ARITHMETIC.
Referring all processes to the Unit for analysis.

DAVIES' ELEMENTS OF WRITTEN ARITH.
Prominently practical, with few rules and explanations.

DAVIES' PRACTICAL ARITHMETIC.
Complete theory and practice. Substitute for this volume.

DAVIES' UNIVERSITY ARITHMETIC.
A purely scientific presentation for advanced classes.

ALSO,

DAVIES' NEW ELEMENTARY ALGEBRA.
A connecting link between Arithmetic and Algebra.

AND A FULL

COURSE OF HIGHER MATHEMATICS.

Copyright, 1876, by CHARLES DAVIES.

PRACTICAL

PREFACE.

ARITHMETIC embraces the science of numbers, together with all the rules which are employed in applying the principles of the science to practical purposes. It is the foundation of the exact and mixed sciences, and the first subject, in a well-arranged course of instruction, to which the reasoning powers of the mind are directed. Because of its great uses and applications, it has become the guide and daily companion of the mechanic and man of business. In the present work, a few general principles are laid down, to which all the operations in numbers may be referred:

1st. The unit 1 is regarded as the base of every number, and the consideration of it is the first step in the analysis of every question relating to numbers.

2d. Every number is treated as a collection of units, or as made up of sets of such collections; each collection having its own base, which is either 1, or some number derived from 1.

3d. The series of numbers expressing the relation between the different units of a number, is called the SCALE; and the employment of this term enables us to generalize the laws which regulate the formation of numbers.

4th. By employing the term "*fractional unit*," the same principles are made applicable to fractional numbers; for all fractions are but collections of fractional units, these units having a known relation to 1.

5th. The presentation of the fractional units to the minds of young pupils, by means of a diagram, as exhibited in the Primary and Intellectual Arithmetics, has greatly simplified the operations in fractions; and they may now be placed before Denominate Numbers, where, in a purely scientific arrangement, they properly belong.

In the preparation of the work, two objects have been kept constantly in view:

1st. To make it Educational; and,

2d. To make it Practical.

To attain these ends, the following plan has been adopted:

1. To introduce every new idea to the mind of the pupil by a simple question, and then to express that idea in general terms under the form of a *definition.*

2. When a sufficient number of ideas are thus fixed in the mind, they are combined to form the basis of an analysis; so that all the principles are developed by analysis in their proper order.

3. The work has been divided into sections, each containing a number of connected principles; and these sections constitute a series of dependent propositions that make up the entire system of principles and rules which the work develops.

Great pains have been taken to make the work PRACTICAL in its general character, by explaining and illustrating the various applications of Arithmetic in the transactions of business, and by connecting, as closely as possible, every principle or rule, with all the applications which belong to it.

I have great pleasure in acknowledging my obligations to many teachers who have favored me with valuable suggestions in regard to definitions, rules, and methods of illustration. Their generous appreciation of my labors has been both an encouragement and a reward.

A Key to this volume has been prepared for the use of *Teachers only.*

CONTENTS.

FIRST FIVE RULES.

PAGE

Definitions 9
Notation and Numeration 10
Addition of Simple Numbers 22
Applications in Addition 30
Subtraction of Simple Numbers 33
Applications in Subtraction 37
Multiplication of Simple Numbers 41
Composite Numbers—Factors 47
Contractions in Multiplication 48
Applications 51
Division of Simple Numbers 54
Proof of Multiplication 62
Contractions in Division 63
Applications in the preceding Rules 67

PROPERTIES OF NUMBERS.

Exact Divisors 72
Composite and Prime Numbers 72
Cancellation 74
Least Common Multiple 77
Greatest Common Divisor 79

COMMON FRACTIONS.

Definitions 82
Properties of Fractions 85
Different kinds of Fractions 86
Six Fundamental Propositions 87
Reduction of Fractions 90
Addition of Fractions 98
Subtraction of Fractions 100
Multiplication of Fractions 102
Division of Fractions 107
Complex Fractions 111
Miscellaneous Examples 112

DECIMAL FRACTIONS.

PAGE
Definition of Decimal Fractions.... 114
Decimal Numeration.... 114
Annexing and Prefixing Ciphers.... 118
Addition of Decimals.... 119
Subtraction of Decimals.... 122
Multiplication of Decimals.... 123
Division of Decimals.... 125
Applications in the Four Rules.... 128

UNITED STATES CURRENCY.

United States Currency.... 130
Reduction of Currency.... 131
Addition of United States Currency.... 132
Subtraction of United States Currency.... 135
Multiplication of United States Currency.... 136
Division of United States Currency.... 138
Applications in the Four Rules.... 139

DENOMINATE NUMBERS.

Definitions.... 147
Different kinds of Units.... 147
Abstract Numbers.... 147
United States Currency.... 148
English Currency.... 149
Reduction Descending.... 150
Reduction Ascending.... 150
Long Measure.... 151
Surveyors' Measure.... 153
Cloth Measure.... 154
Square Measure.... 155
Cubic Measure, or Measure of Volume.... 157
Liquid Measure.... 159
Dry Measure.... 160
Avoirdupois Weight.... 161
Troy Weight.... 163
Apothecaries' Weight.... 164
Measure of Time.... 165
Circular Measure, or Motion.... 168
Miscellaneous Tables.... 169
Miscellaneous Examples.... 170

PAGE
DENOMINATE FRACTIONS 172
Addition of Compound Numbers........ 178
Addition of Fractions........ 182
Subtraction of Compound Numbers........ 183
Time between Dates........ 185
Subtraction of Fractions........ 187
Multiplication of Compound Numbers........ 188
Division of Compound Numbers........ 190
Applications........ 193
Longitude and Time........ 195

DUODECIMALS........ 198
Addition and Subtraction........ 199

RATIO AND PROPORTION.

Ratio defined—Simple and Compound........ 202
Simple Proportion........ 205
Compound Proportion........ 207
Rule of Three........ 208
Double Rule of Three........ 213

APPLICATIONS.

Partnership........ 216
Percentage........ 219
Commission........ 223
Profit and Loss........ 227
Insurance........ 231
Interest........ 233
Partial Payments........ 239
Problems in Interest........ 242
Compound Interest........ 243
Stocks and Brokerage........ 246
Discount........ 248
Banking........ 250
Exchange........ 253
Equation of Payments........ 257
Assessing Taxes........ 262
Custom-house Business........ 264
Currency........ 266
Analysis........ 267

PAGE
Alligation Medial.................................... 275
Alligation Alternate.................................... 276

INVOLUTION.

Definition of, &c.................................... 280

EVOLUTION.

Definition of, &c.................................... 281
Extraction of the Square Root.................................... 281
Applications in Square Root.................................... 286
Extraction of the Cube Root.................................... 289
Applications in Cube Root.................................... 293

ARITHMETICAL PROGRESSION.

Definition of, &c.................................... 294
Different Cases.................................... 295

GEOMETRICAL PROGRESSION.

Definition of, &c.................................... 298
Cases.................................... 299

MENSURATION.

To find the area of a Triangle.................................... 302
To find the area of a Square, Rectangle, &c.................................... 303
To find the area of a Trapezoid.................................... 303
To find the circumference and diameter of a Circle.................................... 304
To find the area of a Circle.................................... 304
To find the surface of a Sphere.................................... 305
To find the contents of a Sphere.................................... 305
To find the convex surface of a Prism.................................... 306
To find the contents of a Prism.................................... 306
To find the convex surface of a Cylinder.................................... 307
To find the contents of a Cylinder.................................... 307
To find the contents of a Pyramid.................................... 308
To find the contents of a Cone.................................... 309

GAUGING.

Rules for Gauging.................................... 310

Promiscuous Examples.................................... 311

Metric System of Weights and Measures.................................... 312

ARITHMETIC.

DEFINITIONS.

1. A UNIT is a single thing, or one.

2. A NUMBER is a unit, or a collection of units.

3. An ABSTRACT NUMBER is one whose unit is not named; as, one, two, three, etc.

4. A DENOMINATE NUMBER is one whose unit is named; as, one foot, two yards, three pounds, &c. Such numbers are also called, *Concrete* numbers.

5. A SIMPLE NUMBER is a single collection of like units, whether abstract or denominate.

6. ARITHMETIC is the Science of Numbers. It is also the Art of applying numbers to practical purposes.

7. A PROPOSITION is something to be done, or demonstrated.

8. An ANALYSIS is an examination of the separate parts of a proposition; and of their connection with each other.

9. AN OPERATION is the doing of something with numbers.

10. A RULE is the direction for performing an operation.

11. An ANSWER is the result of a correct operation.

---

1. What is a unit?—2. What is a number?—3. What is an abstract number?—4. What is a denominate number? What other name has it?—5. What is a simple number?—6. What is Arithmetic.—7. What is a proposition?—8. What is an analysis?—9. What is an operation?—10. What is a rule?—11. What is an answer?

OPERATIONS OF ARITHMETIC.

13. There are, in Arithmetic, five fundamental operations: Notation and Numeration, Addition, Subtraction, Multiplication, and Division.

EXPRESSING NUMBERS.

14. There are three methods of expressing numbers:

1. By words, or common language, spoken or written.
2. By capital letters; called, the Roman method.
3. By figures; called, the Arabic method.

EXPRESSING NUMBERS BY WORDS.

15. A single thing, is called	*One.*
One and one more,	*Two.*
Two and one more,	*Three.*
Three and one more,	*Four.*
Four and one more,	*Five.*
Five and one more,	*Six.*
Six and one more,	*Seven.*
Seven and one more,	*Eight.*
Eight and one more,	*Nine.*
Nine and one more,	*Ten.*
&c.,	&c.

Each of the words, *one, two, three, four, five, six,* &c., expresses a number, and denotes how many units are taken.

NOTATION AND NUMERATION.

16. NOTATION is the method of expressing numbers, either by letters or figures.

NUMERATION is the art of reading, correctly, any number expressed by letters or figures.

There are two methods of Notation: the one by Letters, the other by Figures. The method by letters is called, the *Roman Notation;* the method by figures is called, the *Arabic Notation.*

Roman Notation.

17. The Roman notation employs seven capital letters. They express the following values:

I	V	X	L	C	D	M
One,	five,	ten,	fifty,	one hundred,	five hundred,	one thousand.

All other numbers are expressed by combining these letters, according to the following principles:

1. Every time a letter is repeated, the number which it denotes is repeated.

2. If a letter denoting a less number be written on the right of one denoting a greater, the number expressed will be denoted by the sum of the numbers.

3. If a letter denoting a less number be written on the left of one denoting a greater, the number expressed will be the difference of the numbers.

4. A dash (—), placed over a letter, increases the number for which it stands, a thousand times.

Roman Table.

I	One.
II	Two.
III	Three.
IV	Four.
V	Five.
VI	Six.
VII	Seven.
VIII	Eight.
IX	Nine.
X	Ten.
XX	Twenty.
XXX	Thirty.
XL	Forty.
L	Fifty.
LX	Sixty.
LXX	Seventy.
LXXX	Eighty.
XC	Ninety.
C	One hundred.
CC	Two hundred.
CCC	Three hundred.
CCCC	Four hundred.
D	Five hundred.
DC	Six hundred.
DCC	Seven hundred.
DCCC	Eight hundred.
DCCCC	Nine hundred.
M	One thousand.
MD	Fifteen hundred.
MM	Two thousand.
$\bar{V}$	Five thousand.
$\bar{X}$	Ten thousand.

EXAMPLES IN ROMAN NOTATION.

Express the following numbers by letters:

1. Fifteen.
2. Nineteen.
3. Twenty-nine.
4. Thirty-five.
5. Forty-seven.
6. Ninety-nine.
7. One hundred and sixty.
8. Four hundred and forty-one.
9. Five hundred and sixty-nine.
10. One thousand one hundred and six.
11. Two thousand and twenty-five.
12. Six hundred and ninety-nine.
13. One thousand nine hundred and twenty-five.
14. Two thousand six hundred and eighty.
15. Four thousand nine hundred and sixty-five.
16. Two thousand seven hundred and ninety-one.
17. One thousand nine hundred and sixteen.
18. Two thousand six hundred and forty-one.
19. One thousand eight hundred and sixty-two.
20. Twenty thousand five hundred and twelve.

---

13. How many fundamental operations are there in Arithmetic? Name them.

14. How many methods are there of expressing numbers? What are they?

15. What is a single thing called? One and one more? Six and one more? Eight and one more?

16. What is Notation? What is Numeration? How many methods of notation are there? What is the Roman method? What, the Arabic?

17. How many letters does the Roman notation employ? Which are they? What value does each represent? What is the effect of repeating a letter? What is the number, when a letter denoting a less number is placed on the right of one denoting a greater? What is the number, when a letter denoting a less number is placed on the left of one denoting a greater? What is the effect of placing *a dash over a letter?*

Arabic Notation.

18. Arabic Notation is the method of expressing numbers by figures. Ten figures are used. They are,

0	1	2	3	4	5	6	7	8	9
Naught,	one,	two,	three,	four,	five,	six,	seven,	eight,	nine.

These figures are the *Alphabet of the Arabic Notation.*

The 0 is called, naught, cipher, or zero. It denotes *no number.* Thus, if there are no apples in a basket, we write, the number of apples in the basket is 0. The other nine figures are called, *Significant Figures,* or *Digits.*

Orders of Units.

19. Nine is the highest number which can be expressed by a single figure. To express ten, we write 0 on the right of 1;

Thus, ... 10;

which is read, *ten.*

This 10 is equal to ten of the units expressed by 1. It is but a *single* ten, and is a unit, the value of which is *ten times* as great as the unit one. It is called, a *unit of the second order.*

20. When two figures are written by the side of each other, the one on the right is in the place of *units,* and the other in the place of *tens,* or of *units of the second order.* Each unit of the second order is equal to ten units of the first order. When units simply are named, *units of the first order* are always meant.

---

18. What is Arabic Notation? How many figures are used? Name the figures. What do they form? How many things does 1 express? How many things does 5 express? How many units in 3? In 7? In 9? In 8? In 0? What are the figures, with one exception, called? Which are the significant figures?

19. What is the highest number that can be expressed by a single figure? How do we express ten? To how many units 1 is ten equal? May we consider it a single unit? Of what order?

Units of the second order are written thus:

One ten, or	10.
Two tens, or twenty,	20.
Three tens, or thirty,	30.
Four tens, or forty,	40.
Five tens, or fifty,	50.
Six tens, or sixty,	60.
Seven tens, or seventy,	70.
Eight tens, or eighty,	80.
Nine tens, or ninety,	90.

The intermediate numbers between 10 and 20, between 20 and 30, &c., may be expressed by considering their tens and units. For example, the number 12 is made up of one ten and two units. It is written by setting 1 in the place of tens, and 2 in the place of units;

Thus,	12.
Eighteen, has 1 ten and 8 units,	18.
Twenty-five, has 2 tens and 5 units,	25.
Thirty-seven, has 3 tens and 7 units,	37.
Fifty-four, has 5 tens and 4 units,	54.
Ninety-nine, has 9 tens and 9 units,	99.

Hence, any number greater than nine, and less than one hundred, may be expressed by two figures.

21. In order to express *ten units of the second order*, or *one hundred*, we form a new combination,

thus, ... 100,

by writing two ciphers on the right of 1. This number is read, *one hundred*.

---

20. When two figures are written by the side of each other, what is the place on the right called? The place on the left? When units simply are named, what units are meant? How many units of the second order in 20? In 30? In 40? In 50? In 60? In 70? In 80? In 90? Of what is the number 12 made up? Also, 18, 25, 37, 54, 99? What numbers may be expressed by two figures?

This one hundred expresses 10 *units of the second order*, or 100 *units of the first order.* The one hundred is but *a single hundred,* and is a unit of the *third order.*

We can now express any number less than one thousand.

For example, in the number three hundred and seventy-five, there are 5 units, 7 tens, and 3 hundreds. Write, therefore, 5 units of the first order, 7 of the second order, and 3 of the third; and read from the left, *three hundred and seventy-five.*

hundreds.	tens.	units.
3	7	5

In the number eight hundred and ninety-nine, there are 9 units of the first order, 9 of the second, and 8 of the third; it is read, *eight hundred and ninety-nine.*

hundreds.	tens.	units.
8	9	9

In the number four hundred and six, there are 6 units of the first order, 0 of the second, and 4 of the third.

hundreds.	tens.	units.
4	0	6

Hence, we have the following law of the units:

The right-hand figure always expresses units of the first order; the second, units of the second order; and the third, units of the third order.

22. To express *ten* units of the third order, or one thousand, we form a new combination by writing three ciphers on the right of 1;

Thus, ... 1000.

This is but *one single thousand,* and is a unit of the *fourth order.*

---

21. How do you write one hundred? To how many units of the second order is it equal? To how many of the first order? May it be considered a single unit? Of what order is it? How many units of the third order in 200? In 300? In 400? In 500? In 600? Of what is the number 375 composed? The number 899? The number 406? What numbers may be expressed by three figures? What order of units will each figure express?

Thus, we may form as many orders of units as we please:

A unit of the first order is expressed by	1.
A unit of the second order, by 1 and 0; thus, . .	10.
A unit of the third order, by 1 and two 0's, . .	100.
A unit of the fourth order, by 1 and three 0's, . . .	1000.
A unit of the fifth order, by 1 and four 0's, . . .	10000.

And so on, for units of higher orders.

23. Therefore,

1st. *The same figure expresses different units, according to the place it occupies:*

2d. *Units of the first order occupy the place on the right; units of the second order, the second place; units of the third order, the third place; and the unit of any figure is determined by the number of its place:*

3d. *Ten units of the first order make one of the second; ten of the second, one of the third; ten of the third, one of the fourth; and so on, for the higher orders:*

4th. *When figures are written by the side of each other, ten units in any place make one unit of the place next to the left.*

---

22. To what are ten units of the third order equal? How do you write it? How is a unit of the first order written? How do you write a unit of the second order? One of the third? One of the fourth? One of the fifth?

23. On what does the unit of a figure depend? What is the unit of the first place on the right? What is the unit of the second place? What is the unit of the third place? Of the fourth? Of the fifth? Sixth? How many units of the first order make one of the second? How many of the second one of the third? How many of the third one of the fourth? &c. When figures are written by the side of each other, how many units of any place make one unit of the place next to the left? To how many units is 1 hundred equal? To how many tens? To how many tens is 1 thousand equal? To how many hundreds? To how many units of the first order is one unit of the third order equal? To how many of the second order?

***Give* the rule for Notation. Give the Rule for Numeration.**

EXAMPLES IN WRITING THE ORDERS OF UNITS.

1. Write 3 tens.

2. Write 8 units of the second order.

3. Write 9 units of the first order.

4. Write 4 units of the first order, 5 of the second, 6 of the third, and 8 of the fourth.

5. Write 9 units of the fifth order, none of the fourth, 8 of the third, 7 of the second, and 6 of the first.

6. Write one unit of the sixth order, 5 of the fifth, 4 of the fourth, 9 of the third, 7 of the second, and 0 of the first.

7. Write 9 units of the 5th order, 0 of the 4th, 8 of the 3d, 1 of the 2d, and 3 of the 1st.

8. Write 7 units of the 6th order, 8 of the 5th, 0 of the 4th, 5 of the 3d, 7 of the 2d, and 1 of the 1st.

9. Write 9 units of the 7th order, 0 of the 6th, 2 of the 5th, 3 of the 4th, 9 of the 3d, 2 of the 2d, and 9 of the 1st.

10. Write 8 units of the 8th order, 6 of the 7th, 9 of the 6th, 8 of the 5th, 1 of the 4th, 0 of the 3d, 2 of the 2d, and 8 of the 1st.

11. Write 14 units of the 12th order, with 9 of the 10th, 6 of the 8th, 7 of the 6th, 6 of the 5th, 5 of the 3d, and 3 of the 1st.

12. Write 13 units of the 13th order, 8 of the 12th, 7 of the 9th, 6 of the 8th, 9 of the 7th, 7 of the 6th, 3 of the 4th, and 9 of the first.

13. Write 9 units of the 18th order, 7 of the 16th, 4 of the 15th, 8 of the 12th, 3 of the 11th, 2 of the 10th, 1 of the 9th, 0 of the 8th, 6 of the 7th, 2 of the 3d, and 1 of the 1st.

14. Write 8 units of the 8th order, 6 of the 7th, 9 of the 6th, 8 of the 5th, 1 of the 4th, 0 of the 3d, 2 of the 2d, and 8 of the 1st.

15. Write 1 unit of the 9th order, 6 of the 8th, 9 of the 7th, 7 of the 6th, 6 of the 5th, 5 of the 4th, 4 of the 3d, 3 of the 2d, and 2 of the 1st.

NUMERATION TABLE

6th Period. Quadrillions.			5th Period. Trillions.			4th Period. Billions.			3d Period. Millions.			2d Period. Thousands.			1st Period. Units.		
Hundreds of Quadrillions	Tens of Quadrillions	Quadrillions	Hundreds of Trillions	Tens of Trillions	Trillions	Hundreds of Billions	Tens of Billions	Billions	Hundreds of Millions	Tens of Millions	Millions	Hundreds of Thousands	Tens of Thousands	Thousands	Hundreds	Tens	Units
		.			.			.			.			.			6
		.			.			.			.			.		7	5
		.			.			.			.			.	8	7	9
		.			.			.			.			6,	0	2	3
		.			.			.			.		8	2,	3	0	1
		.			.			.			.	1	2	3,	0	8	7
		.			.			.			7,	0	0	0,	7	3	5
		.			.			.		4	3,	2	1	0,	4	6	0
		.			.			.	5	4	8,	0	0	0,	0	8	7
		.			.			6,	2	4	5,	2	8	9,	4	2	1
		.			.		7	2,	5	4	9,	1	3	6,	8	2	2
		.			.	8	9	4,	6	0	2,	0	4	3,	2	8	8
		.			7,	6	4	1,	0	0	0,	9	0	7,	4	5	6
		.		8	4,	9	1	2,	8	7	6,	4	1	9,	2	8	5
		.	9	1	2,	7	6	1,	2	5	7,	3	2	7,	8	2	6
		6,	4	0	7,	2	1	2,	9	3	6,	8	7	6,	5	4	1
	5	7,	2	8	9,	6	7	8,	5	4	1,	2	9	7,	3	1	3
9	2	0,	3	2	3,	8	4	2,	7	6	8,	3	1	9,	6	7	5

1. Numbers expressed by more than three figures, are separated into periods of three figures each, beginning at the right, and are written and read by periods, as shown in the above table.

2. Each period contains three figures, except the last, which may contain one, two, or three figures.

3. The unit of the first period is 1; the unit of the second period, 1 thousand; of the third, 1 million; of the fourth, 1 billion; and so, for periods, still to the left.

4. To quadrillions succeed quintillions, sextillions, septillions, &c.

5. The pupil should be required to commit, thoroughly, the names of the periods, so as to repeat them in their regular order *from left to right,* as well as from right to left.

Rule for Notation.

1st. *Begin at the left hand and write each period, as if it were a period of units:*

2d. *When the number in any period, except the left-hand period, is expressed by less than three figures, prefix one or two ciphers; and when a vacant period occurs, fill it with ciphers.*

Examples in Notation.

Express the following numbers in figures:

1. One hundred and five.
2. Three hundred and two.
3. Five hundred and nineteen.
4. One thousand and four.
5. Eight thousand, seven hundred and one.
6. Forty thousand, four hundred and six.
7. Fifty-eight thousand and sixty-one.
8. Ninety-nine thousand, nine hundred and ninety-nine.
9. Four hundred and six thousand and forty-nine.
10. Six hundred and forty-one thousand, seven hundred and twenty-one.
11. One million, four hundred and twenty-one thousand, six hundred and two.
12. Nine millions, six hundred and twenty-one thousand and sixteen.
13. Ninety-four millions, eight hundred and seven thousand, four hundred and nine.
14. Four billions, three hundred and six thousand, nine hundred and nine.
15. Forty-nine trillions, nine hundred and forty-nine thousand and sixty-five.
16. Nine hundred and ninety quadrillions, nine hundred and ninety-nine millions, nine hundred and ninety thousand, nine hundred and ninety-nine.
17. Four hundred and nine sextillions, two hundred and nine thousand, one hundred and six.

RULE FOR NUMERATION.

I. *Separate the number into periods of three figures each, beginning at the right hand:*

II. *Name the order of each figure, beginning at the right hand:*

III. *Then, beginning at the left hand, read each period as if it stood alone, naming its unit.*

EXAMPLES IN NUMERATION.

Let the pupil point off and read the following numbers; then write them in words:

1.	67	7.	6124076	13.	804321049
2.	125	8.	8073405	14.	90067236708
3.	6256	9.	26940123	15.	870432697082
4.	4697	10.	9602316	16.	1704291672301
5.	23697	11.	87000032	17.	3409672103604
6.	412304	12.	1987004086	18.	4970134264174

19.	8760218760541	23.	9080620359704567
20.	904326170365	24.	9806071234560078
21.	30267821040291	25.	30621890367081263
22.	907620380467026	26.	350673123051672607

NOTE.—Let each of the above examples, after being written on the blackboard, be analyzed as a class exercise; thus:

Ex. 1. How many tens in 67? How many units over?

2. In 125, how many hundreds in the hundreds place? How many tens in the tens place? How many units in the units place? How many tens in the number?

3. In 6256, how many thousands in the thousands place? How many hundreds in the hundreds place? How many tens in the tens place? How many units in the units place?

4. In 4697, how many tens are there? The 6 hundreds are equal to how many tens? To how many tens are the 4 thousands equal? To how many tens are the 4 thousands and 6 hundreds *equal?*

Examples in Notation and Numeration.

1. Write two hundred and nine.

2. Write five thousand and five.

3. Write twelve thousand and twelve.

4. Read 1040; 30706; 6606.

5. Read 2001; 35006; 4070070.

6. Write one hundred thousand, one hundred and one.

7. Read 207600042; 1000860005.

8. Read 100000100; 5000000750001.

9. Write forty-seven millions, two hundred and four thousand, eight hundred and fifty-one.

10. Write six quadrillions, forty-nine trillions, seventy-two billions, four hundred and seven thousand, eight hundred and sixty-one.

11. Write eight hundred and ninety-nine quadrillions, four hundred and sixty trillions, eight hundred and fifty billions, two hundred millions, five hundred and six thousand, four hundred and ninety-nine.

12. Write and read, fifty-nine trillions, fifty-nine billions, fifty-nine millions, fifty-nine thousand, nine hundred and fifty-nine.

13. Eleven thousand, eleven hundred and eleven.

14. Nine billions and sixty-five.

15. Write and read, three hundred and four trillions, one million, three hundred and twenty-one thousand, nine hundred and forty-one.

16. Write and read, nine trillions, six hundred and forty billions, with 7 units of the ninth order, 6 of the seventh order, 8 of the fifth, 2 of the third, 1 of the second, and 3 of the first.

17. Write and read, three hundred and five trillions, one hundred and four billions, one million, with 4 units of the fifth order, 5 of the fourth, 7 of the second, and 4 of the first.

18. Write and read, three hundred and one billions, six millions, four thousand, with 8 units of the fourteenth order, 6 of the third, and two of the second.

ADDITION.

24. 1. John has two apples, and Charles has three: how many have both?

Analysis.—They have, together, as many apples as are equal to 2 apples counted with 3 apples, which are 5 apples.

2. James had 5 marbles, and William gave him 7 more: how many had he then?
3. Mary has 6 pins, and Jane 9: how many have both?
4. How many are 5 and 3? 6 and 4?
5. How many are 4 and 9? 8 and 5?
6. How many are 3 and 7? 10 and 0? 0 and 10?
7. How many are 1 and 5 and 6? 3 and 4 and 9?

The answer to any of the above questions, is called, the *Sum* of the numbers, and the operation by which we find it, is called, *Addition.*

25. Hence, the Sum of two or more numbers, is a number which contains as many units as there are in all the numbers added.

Addition is the operation of finding the sum of two or more numbers.

Of the Signs.

26. The sign +, is called, *plus*, which signifies, more. When placed between two numbers, it denotes that they are to be added together.

The sign =, is called, the sign of equality. When placed between two numbers, it denotes that they are equal to each other. Thus, $3 + 2 = 5$, denotes that the sum of 3 and 2 is equal to 5.

---

24. What is the sum of two or more numbers? What is Addition?

26. What is the sign of Addition? What is it called? What *does it* signify? Express the sign of equality. When placed *between two numbers,* what does it show?

ADDITION TABLE.

2 + 0 = 2	3 + 0 = 3	4 + 0 = 4	5 + 0 = 5
2 + 1 = 3	3 + 1 = 4	4 + 1 = 5	5 + 1 = 6
2 + 2 = 4	3 + 2 = 5	4 + 2 = 6	5 + 2 = 7
2 + 3 = 5	3 + 3 = 6	4 + 3 = 7	5 + 3 = 8
2 + 4 = 6	3 + 4 = 7	4 + 4 = 8	5 + 4 = 9
2 + 5 = 7	3 + 5 = 8	4 + 5 = 9	5 + 5 = 10
2 + 6 = 8	3 + 6 = 9	4 + 6 = 10	5 + 6 = 11
2 + 7 = 9	3 + 7 = 10	4 + 7 = 11	5 + 7 = 12
2 + 8 = 10	3 + 8 = 11	4 + 8 = 12	5 + 8 = 13
2 + 9 = 11	3 + 9 = 12	4 + 9 = 13	5 + 9 = 14
2 + 10 = 12	3 + 10 = 13	4 + 10 = 14	5 + 10 = 15
6 + 0 = 6	7 + 0 = 7	8 + 0 = 8	9 + 0 = 9
6 + 1 = 7	7 + 1 = 8	8 + 1 = 9	9 + 1 = 10
6 + 2 = 8	7 + 2 = 9	8 + 2 = 10	9 + 2 = 11
6 + 3 = 9	7 + 3 = 10	8 + 3 = 11	9 + 3 = 12
6 + 4 = 10	7 + 4 = 11	8 + 4 = 12	9 + 4 = 13
6 + 5 = 11	7 + 5 = 12	8 + 5 = 13	9 + 5 = 14
6 + 6 = 12	7 + 6 = 13	8 + 6 = 14	9 + 6 = 15
6 + 7 = 13	7 + 7 = 14	8 + 7 = 15	9 + 7 = 16
6 + 8 = 14	7 + 8 = 15	8 + 8 = 16	9 + 8 = 17
6 + 9 = 15	7 + 9 = 16	8 + 9 = 17	9 + 9 = 18
6 + 10 = 16	7 + 10 = 17	8 + 10 = 18	9 + 10 = 19

2 + 3 = how many?
1 + 2 + 4 = how many?
2 + 3 + 5 + 1 = how many?
6 + 7 + 2 + 3 = how many?
1 + 6 + 7 + 2 + 3 = how many?
1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = how many?

27. The operation of Addition is governed by four principles, viz.:

1. A single number expresses a collection of like units.
2. Like units alone can be added together; that is, units must be added to units, tens to tens, hundreds to hundreds, &c.
3. Every number, expressed by two or more figures, is

the sum of its units, tens, hundreds, &c.; thus, 279 is the sum of 2 hundreds, 7 tens, and 9 units.

4. The sum of several numbers is equal to the sum of all their parts.

1. James has 14 cents, and John gives him 21: how many cents has he then?

ANALYSIS.—Since units must be added to units, and tens to tens, the numbers are written so that units of the same order may fall in the same column, and a line is drawn beneath them. The column of the lowest order is first added, and contains 5 units, which are written under the column. The tens are next added, and they amount to 3 tens, which are written under the tens. The sum is 3 tens and 5 units, or thirty-five.

OPERATION.
14
21
35 cents.

2. A gentleman bought a carriage for 385 dollars, a team of horses for 286 dollars, and two sets of harness for 96 dollars: what did he pay for all?

ANALYSIS.—Write the numbers so that units of the same value shall fall in the same column; then add each order of units separately.

OPERATION.
385
286
96

Sum of the units	17
Sum of the tens	25
Sum of the hundreds	5
Sum total	767

The following, however, is the method in practice:

ANALYSIS.—Write the numbers as before. The units are added together, and their sum is 17, which is 1 ten and 7 units; the units are placed under the column of units, and the 1 ten is added with the column of tens, which then amounts to 26 tens, equal to 2 hundreds and 6 tens; the 6 tens are placed under the tens, and the hundreds are added with the column of hundreds, which amounts to 7, and is therefore placed under the hundreds.

OPERATION.
385
286
96
767 dollars.

---

27. By how many principles is the operation of adding governed? *Name them.*

When a column amounts to ten, or more than ten, the unit figure is set down, and the tens' figure is added to the next column, because, 10 *units of any order make* 1 *unit of the next higher order.* This process is called, *carrying to the next column.*

28. Hence, to find the sum of two or more numbers, we have the following

RULE.

I. *Write the numbers to be added, so that units of the same order shall fall in the same column:*

II. *Add the column of units: set down the units of the sum, and carry the tens to the next column:*

III. *Add the column of tens: set down the tens, and carry the hundreds to the next column; and so on, till all the columns are added, and set down the entire sum of the last column.*

PROOF.

The PROOF of any operation in Addition, consists in showing that the result, or answer, contains as many units as there are in all the numbers added, and no more. There are two methods of proof, for beginners:

I. *Begin at the top of the units column, and add all the columns downward, carrying from one column to the other, as when they were added upward. If the two results agree, the work is supposed to be right.*

II. *Draw a line, dividing the numbers into parts. Add the parts separately, and then add the sums. If the last sum is the same as the sum first found, the work may be regarded as right.*

---

28. How do you set down numbers for addition? Where do you begin to add? If the sum of any column can be expressed by a single figure, what do you do with it? When it cannot, what do you do? When you add to the next column, what is it called? What do you set down in the last column? Of what does the proof consist, in Addition? What is the first method of proof? What is the second method of proof?

READING.

The pupil should be early taught to omit the intermediate words in the addition of a column of figures. Thus, in example 18, instead of saying, 7 and 5 are 12 and 1 are 13 and 6 are 19; he should say, twelve, thirteen, nineteen; and in the column of tens, ten, nineteen, twenty-three; and similarly for the other columns. This is called, *reading the columns.* Let the pupils be often practiced in it, both separately, and in concert in classes.

EXAMPLES.

1. A farmer has 160 sheep in one field, 20 in another, and 16 in another: how many has he in all?

2. If a gentleman travels 328 miles one day, 171 miles the next day, and 250 miles the third day, how far will he travel in all?

(3.)	(4.)	(5.)	(6.)
604	199	776	398
743	367	407	999

(7.)	(8.)	(9.)	(10.)
427	329	3034	8094
242	260	6525	1602
330	100	230	103

(11.)	(12.)	(13.)	(14.)
4096	9976	9875	67954
3271	8757	9988	98765
4722	8168	8774	37214

(15.)	(16.)	(17.)	(18.)
6412	90467	87052	432046
1091	10418	64108	210491
6741	91467	74981	809765
9028	41290	21360	542137

(19.)	(20.)	(21.)	(22.)
21467	89479	74167	9947621
80491	75416	21094	704126
67421	7647	2947	81267
4304	214	674	9241
2191	19	85	495

PROOF OF ADDITION.

Either of the following methods may be used in proving examples in Addition.

	(23.)	PROOF.
	34578	34578
	3750	
	87	
	328	4509
	17	
	327	
Sum,	39087	39087

	(24.)	PROOF.
	23456	
	78901	125813
	23456	
	78901	
	23456	181258
	78901	
Sum,	307071	307071

(25.)	(26.)	(27.)
672981043	91278976	8416785413
67126459	7654301	6915123460
39412767	876120	31810213
7891234	723456	7367985
109126	31309	654321
84172	4871	47853
72120	978	2685

28. What is the sum of 304 and 273?
29. What is the sum of 3607 and 4082?
30. What is the sum of 30704 and 471912?
31. What is the sum of 398463 and 401536?
32. If a top costs 6 cents, a knife 25 cents, a slate 12 cents: what does the whole amount to?
33. John gave 30 cents for a bunch of quills, 18 cents for an inkstand, 25 cents for a quire of paper: what did the whole cost him?

34. If 2 cows cost 143 dollars, 5 horses 621 dollars, and 2 yoke of oxen 124 dollars, what will be the cost of them all?

35. What is the sum of 8 hundreds, 4 tens, 6 units, and 6 thousands?

36. What is the sum of 3 units, 5 units, 6 tens, 3 tens, 4 hundreds, 3 hundreds, 5 thousands, and 4 thousands?

37. What is the sum of five units of the 4th order, 1 of the 3d, three of the 4th, five of the 3d, and one of the 1st?

38. What is the sum of six units of the 2d order, five of the 3d, six of the 4th, three of the 2d, four of the 3d, two of the 1st, and four of the 2d?

39. What is the sum of 3 and 6, 5 tens and 2 tens, and 3 hundreds and 6 hundreds?

40. What is the sum of 4 and 5, 5 tens, 3 hundreds and 2 hundreds?

41. Add 8635, 2194, 7421, 5063, 2196, and 1245 together.

42. Add 246034, 298765, 47321, 58653, 64218, 5376, 9821, and 340 together.

43. Add 27104, 32547, 10758, 6256, 704321, 730491, 2787316, and 2749104 together.

44. Add 1, 37, 39504, 6890312, 18757421, and 265 together.

45. What is the sum of the following numbers, viz.: seventy-five; one thousand and ninety-five; six thousand four hundred and thirty-five; two hundred and sixty-seven thousand; one thousand four hundred and fifty-five; twenty-seven millions and eighteen; two hundred and seventy millions and twenty-seven thousand?

46. What is the sum of 372856, 404932, 2704793, 9078961, 304165, 207708, 41274, 375, 271, 34, and 6?

47. What is the sum of 4073678, 4084162, 3714567, 27413121, 27049, 87419, 27413, 604, 37, and 9?

48. What is the sum of 36704321, 2947603, 999987, 76, *47213694, 21612090,* 8746, 31210496, and 3021?

49. Add together fifty-eight billions, nine hundred and eighty-two millions, four hundred and eighty-seven thousand, six hundred and fifty-four; seven hundred and forty billions, three hundred and fifty millions, five hundred and forty thousand, seven hundred and sixty; four hundred and twenty-five billions, seven hundred and three millions, four hundred and two thousand, six hundred and three; thirty-four billions, twenty millions, forty thousand and twenty; five hundred and sixty billions, eight hundred millions, seven hundred thousand and five hundred.

(50.)
READ
87406
89507
41299
47208
71625
72428
97206
41278
28907
25412
27049
28416
72204
70412
27426
62081
81697
87489
21642
24672

(51.)
READ
92674
27049
28372
37041
49741
57214
59261
41219
57267
40216
87614
92742
87046
90212
17618
40261
57274
21859
42673
51814

(52.)
READ
25043
97069
81216
75850
90417
19216
} 388811

20428
60594
72859
43706
21441
87604
71215
} 377847

18972
27042
59876
54301
87415
32018
72687
} 352311

53. By the census of 1850, the population of the ten largest cities was as follows: New York, 515547; Philadelphia, 340045; Baltimore, 169054; Boston, 136881; New Orleans, 116375; Cincinnati, 115436; Brooklyn, 96838; St. Louis, 77860; Albany, 50763; Pittsburgh, 46601: what was their entire population?

APPLICATIONS.

29. In all the applications of Arithmetic, *the numbers added together must have the same unit.*

In the question, How many head of live-stock in a field, there being 6 cows, 2 oxen, 3 steers, and 15 sheep? the unit is 1 head of live stock. And the same principle is applicable to all similar questions.

1. How many days are there in the twelve calendar months? January has 31, February 28, March 31, April 30, May 31, June 30, July 31, August 31, September 30, October 31, November 30, and December 31.

2. A merchant bought a horse for 112 dollars; after keeping him a short time, he sold him, and gained 25 dollars: how much did he receive for the horse?

3. A person sold a house and lot for 3650 dollars, and, by so doing, lost 375 dollars: what had they cost him?

4. A speculator bought a house and lot for 1964 dollars, expended 384 dollars in repairing and refitting the property, paid taxes and insurance amounting to 56 dollars, and then sold so as to gain 396 dollars: what did he get for the property?

5. What is the total weight of seven casks of merchandise; No. 1 weighing 960 pounds, No. 2, 725 pounds, No. 3, 830 pounds, No. 4, 798 pounds, No. 5, 698 pounds, No. 6, 569 pounds, No. 7, 987 pounds?

6. At the Custom House, on the first day of June, there were entered 1800 yards of linen; on the 10th, 2500 yards; on the 25th, 600 yards; on the day following, 7500 yards; and on the last three days of the month, 1325 yards each day: what was the whole amount entered during the month?

7. A farmer has his live-stock distributed in the following manner: in pasture No. 1 there are 5 horses, 14 cows, 8

---

29. What principles govern all the additions in Arithmetic? What is the unit in the question, How many head of cattle in a *pasture?*

oxen, and 6 colts; in pasture No. 2, 3 horses, 4 colts, 6 cows, 20 calves, and 12 head of young cattle; in pasture No. 3, 320 sheep, 16 calves, 2 colts, and 5 head of young cattle. How much live-stock had he of each kind, and how many head had he altogether?

8. What is the interval of time between an event which happened 125 years ago, and one that will happen 267 years hence?

9. In 1861, John was 24 years old: in what year, should he live, will he be 19 years older?

10. A merchant has real estate worth 45615 dollars, furniture worth 2862 dollars, merchandise in store worth 25659 dollars, cash in bank 9645 dollars, and cash on hand 456 dollars: what is the amount of his fortune?

11. Of a debt, George paid 475 dollars, and Samuel paid the remainder, which was 625 dollars: what was the debt?

12. A lot of ground cost 750 dollars; a house was erected thereon, whose cost was, for carpenters' work, 2000 dollars, masons' work, 765 dollars, painters' work, 265 dollars, and for other work, 327 dollars: what was the cost of the house and lot?

13. There are 60 seconds in a minute, 3600 in an hour, 86400 in a day, 604800 in a week, 2419200 in a month, and 31557600 in a common year: how many seconds in the periods of time named above?

14. Suppose a merchant to buy the following parcels of cloth: 3912 yards, 1856, 2011, 4540, 937, 6338, 3603, 1586, 2044, 2951, 4228, 1345, 1011, 6138, 960, 607, 5150, 43886, 617, 7513, 4079, 743, 612, 2519, 1238, and 2445 yards: how many yards does he buy in all?

15. What is the sum of two millions bushels of corn, five hundred and thirty-one thousand bushels, one hundred and twenty bushels, fourteen thousand bushels, thirty thousand and twenty-four bushels, five hundred and sixty bushels, and seven hundred and two bushels?

16. The mail route from Albany to New York is 144 miles, from New York to Philadelphia 90 miles, from Philadelphia

to Baltimore 98 miles, and from Baltimore to Washington City 38 miles: what is the distance from Albany to Washington?

17. A man, dying, leaves to his only daughter nine hundred and ninety-nine dollars, and to each of three sons two hundred dollars more than he left the daughter: what was each son's portion, and what the amount of the whole estate?

18. The number of acres of the public lands sold in 1834 was 4658218; in 1835, 12564478; in 1836, 25167833. The number of acres sold in 1840 was 2236889; in 1841, 1164796; in 1842, 1129217. How many acres were sold in the first three, and how many in the last three years?

19. What was the population of the British provinces in North America in 1834, the population of Lower Canada being stated at 549005; of Upper Canada, 336461; of New Brunswick, 152156; of Nova Scotia and Cape Breton, 142548; of Prince Edward's Island, 32292; of Newfoundland, 75000?

20. By the census of 1850, the number of deaf and dumb in the United States was 9803; of blind, 9794; of insane, 15610; of idiots, 15787: what was the aggregate?

21. A vessel took for her cargo, from New York to London, cotton, wheat, flour, corn, and tobacco; the cotton was valued at 16562 dollars, the wheat 5690 dollars, the flour 25645 dollars, the corn 10684 dollars, and the tobacco 35760 dollars: what was the value of the cargo?

22. By the census of 1850, the population of the District of Columbia was 51687; of the Territory of Minnesota, 6077; of New Mexico, 61547; of Oregon, 13294; of Utah, 11380: what was the population of the Territories, including the District of Columbia?

23. By the census of 1850, the population of Maine was 583169; of New Hampshire, 317976; of Vermont, 314120; of Massachusetts, 994514; of Rhode Island, 147545; and of Connecticut, 370792: what was the population of the six New England States?

24. A person, who was born in 1801, died at the age of 46 years: his son died 15 years afterward: in what year did *the son die?*

SUBTRACTION.

30. John has 5 apples, and Charles 2: how many more apples has John than Charles?

ANALYSIS.—As many more as, added to what Charles has, will make his number equal to John's: 3 added to 2, gives 5: Therefore, John has 3 apples more than Charles.

31. The DIFFERENCE between two numbers, is that number which added to the less, will give the greater.

32. SUBTRACTION is the operation of finding the difference between two numbers.

33. The MINUEND is the greater of the two numbers.

34. The SUBTRAHEND is the less of the two numbers.

35. The REMAINDER, or *Difference*, is the result of the operation.

If the numbers are equal, the remainder is 0, and either may be taken as the minuend.

Of the Signs.

36. The sign —, is called *minus*, which signifies, *less*. When placed between two numbers, the one before it is the minuend, and the one after it, the subtrahend; thus,

$$5 - 3 = 2,$$

denotes that 5 is the minuend, 3 the subtrahend, and 2 the remainder.

$8 - 4 =$ how many?
$12 - 5 =$ how many?
$17 - 8 =$ how many?
$19 - 10 =$ how many?

37. The principles which control the operations of Subtraction are,

1. That the difference, added to the less number, gives the greater.
2. That the minuend and subtrahend must have the same unit.

38. 1. James has 27 apples, and gives 14 to John: how many has he left?

Analysis.—The 27 is made up of 7 units and 2 tens; and the 14, of 4 units and 1 ten. Subtract 4 units from 7 units, and 3 units will remain; subtract 1 ten from 2 tens, and 1 ten will remain: hence, the remainder is 13.

OPERATION.

27 Minuend.
14 Subtrahend.
13 Remainder.

2. A farmer had 378 sheep, and sold 256: how many had he left?

Analysis.—We first write the number 378, and then 256 under it, so that units of the same order shall fall in the same column. We then take 6 units from the 8 units, 5 tens from 7 tens, and 2 hundreds from 3 hundreds, leaving for the remainder, 122.

OPERATION.

378
256
122

3. What is the difference between 843 and 562?

Analysis.—Begin at the units' column, and subtract 2 from 3. At the next place we meet a difficulty, for we can not subtract a greater number from a less.

OPERATION.

843
562

If we take 1 from the 8 hundreds (equal to 10 tens), and add it to the 4 tens, the minuend will become 7 hundreds, 14 tens, and 3 units. We then say, 6 tens from 14 tens leaves 8 tens; and 5 hundreds from 7 hundreds leaves 2 hundreds: hence, the remainder is 281.

7 14 3
5 6 2
2 8 1

The same result is obtained by adding, mentally, 10 to the 4 tens, and then adding 1 to 5, the next figure of the subtrahend at the left; for, adding 1 to the 5 is the same as diminishing the 8 by 1. This process of adding 10 to a figure of the minuend, and 1 to the next figure of the subtrahend, at the left, is called, *borrowing*.

$$\begin{array}{r} {\scriptstyle 10} \\ 843 \\ 562 \\ {\scriptstyle 1} \\ \hline 281 \end{array}$$

---

31. What is the difference between two numbers?—32. What is Subtraction?—33. What is the minuend?—34. What is the subtrahend?—35. What is the remainder?—36. What is the sign of Subtraction?—37. What are the principles which control the operations of Subtraction?

Hence, for Subtraction, we have the following

RULE.

I. *Write the less number under the greater, so that units of the same order shall fall in the same column:*

II. *Begin at the right hand, and subtract each figure of the subtrahend from the one directly over it, when the upper figure is greater than the lower, or equal to it:*

III. *When the upper figure is the less, add 10 to it, before subtracting, and then add 1 to the next figure of the subtrahend.*

PROOF.

Add the remainder to the subtrahend. If the work is right, the sum will be equal to the minuend.

SPELLING—READING.

39. What is the difference between 725 and 341 ?

By the common method, which is spelling, we say, 1 from 5 leaves 4; 4 from 12 leaves 8; 1 to carry to 3 are 4; 4 from 7 leaves 3.

Reading the words which express the final result, we should make the operations mentally, and say, 4, 8, 3.

OPERATION.

725
341
384

Let the pupils be practiced separately in the reading, and also in concert in classes.

EXAMPLES.

	(1.)	(2.)	(3.)	(4.)
Minuends,	874	999	8497	62843
Subtrahends,	642	367	7487	51720
Remainders,	232			

---

38. Give the rule for Subtraction.

39. Explain what is meant by spelling and reading.

(5.)	(6.)	(7.)	(8.)
972	278846	894862	27609465
631	167504	170641	17206105

(9.)	(10.)	(11.)	(12.)
32780	8592678	67942139	219067803
15678	1078953	9756783	104202196

(13.)	(14.)	(15.)	(16.)	(17.)
4760	3600	87000	67087	10000
1986	1863	1009	40000	4

(18.)	(19.)	(20.)	(21.)
30000	100000	20006400	100044400
9999	1	18609635	90094406

22. From 2637804 take 2376982.
23. From 3762162 take 826541.
24. From 78213609 take 27821890.
25. From thirty thousand and ninety-seven, take one thousand six hundred and fifty-four.
26. From one hundred millions two hundred and forty-seven thousand, take one million four hundred and nine.
27. Subtract one from one million.
28. From 804367 subtract 27905.
29. From 18623041 subtract 61294.
30. Erom 4270492 subtract 26409.
31. From 8741209 subtract 728104.
32. From 741874 subtract 689346.
33. From sixty-five billions, three millions, six hundred and twelve, take nine billions, one million, four thousand and 6.
34. From 14 billions, 127 millions, six hundred and fourteen thousand, 916, take twenty-nine millions, 416 thousand.
35. From forty trillions, 160 billions, 42 millions, 16 thou*sand, 14 hundred*, take 3 trillions, 63 billions, 41 hundred.

APPLICATIONS.

40. It should be observed, that in all the applications of Subtraction, one number can be subtracted from another, only, *when they both have the same unit.*

1. Suppose I had lent a man 1565 dollars, and he died, owing me 450 dollars: how much had he paid me?

2. A man bought a house and lot for 5650 dollars, on a mortgage of 3 years; he paid, at different times, sums amounting to 3756 dollars: how much did he still owe?

3. Suppose John were born in eighteen hundred and fifteen, and James in eighteen hundred and twenty-five: what is the difference of their ages?

4. A man was born in 1785: what was his age in 1830?

5. George Washington was born in the year 1732, and died in 1799: how old was he at the time of his death?

6. The Declaration of Independence was published, July 4th, 1776: how many years to July 4th, 1838?

7. In 1850, there were in the State of New York, 3,097,394 inhabitants, and in the State of Pennsylvania, 2,311,786 inhabitants: how many more inhabitants were there in New York than in Pennsylvania?

8. The revolutionary war began in 1775; the second war, in 1812: what time elapsed between their commencements?

9. A merchant bought a vessel for 12642 dollars, and gave in part payment a house that was worth 7585 dollars, and the rest in cash: how much did he pay in cash?

10. A person sold a farm for 15896 dollars, which had cost him 12264 dollars: how much did he gain?

11. A man dies, worth 1200 dollars; he leaves 504 to his daughter, and the remainder to his son: what was the son's portion?

12. A merchant bought a house for 6450 dollars; he paid 4625 dollars in cash, and the rest in merchandise: what was the value of the merchandise?

13. Washington died in 1799, at the age of 67: in what year was he born?

14. Henry Hudson sailed up the Hudson river in 1609. how many years since?

15. Pliny, the historian, died 17 years after the birth of Christ: how many years was that before the Declaration of Independence?

16. In one week, one steamer traversed 2065 miles, while another traversed 1986 miles: how much further did the one travel than the other?

17. In 1850, there were in New York, which is the largest city in the United States, 515,547 inhabitants, and in Philadelphia, the next largest city, 340,045: how many more inhabitants were there in New York than in Philadelphia?

18. At a certain period, there were 4338472 children in the United States, between the ages of 5 and 15; of this number, 2477667 were in schools: how many were out of schools?

19. The circulation of the blood was discovered in 1616: how many years to 1855?

20. A merchant bought 500 barrels of flour, for 3500 dollars; he sold 250 barrels, for 2000 dollars: how many barrels remained on hand, and how much must he sell them for, that he may lose nothing?

21. A merchant bought 1675 yards of cloth, for which he paid 5025 dollars; he then sold 335 yards, for 1005 dollars: how much had he left, and what did it cost him?

22. In 1850, the slaves in the United States amounted to 3204313; free colored, to 434495: what was their difference?

23. What length of time elapsed between the birth of Sir Francis Bacon, in 1561, and the birth of Benjamin Franklin, in 1706?

24. A merchant sold a vessel for 9768 dollars, and, by so doing, gained 1862 dollars: how much had the vessel cost?

25. A householder sold two houses: for the first, which cost 3500 dollars, he received 4760 dollars; for the second, which cost 3735 dollars, he received 5000 dollars: on which of the houses did he make the greater gain, and how much?

26. By the census of 1850, the number of white inhabitants in the United States amounted to 19553068; and the blacks, to 3638808: by how many did the white inhabitants exceed the black?

27. By the census of 1850, the entire population of the United States was 23191876; that of the six New England States, 2728116: by how many did the whole population exceed that of the six New England States?

28. An army of 75425 men is required by a general, who finds that he has only 49846 men: how many more men are required?

29. A boy, in working an example, used the number 2306400, which he afterward found was too large by 29875: what was the correct number?

Applications in Addition and Subtraction.

1. A merchant buys 19576 yards of cloth of one person, 27580 yards of another, and 375 of a third; he sells 1050 yards to one customer, 6974 yards to another, and 10462 yards to a third: how many yards has he left?

2. A person borrowed of his neighbor, at one time, 355 dollars, at another time, 637 dollars, and 403 dollars at another time; he then paid him, 977 dollars: how much did he owe him?

3. I have a fortune of 2543 dollars, to divide among my four sons, James, John, Henry, and Charles. I give James 504 dollars, John 600 dollars, and Henry 725: how much remains for Charles?

4. I have a yearly income of ten thousand dollars. I pay 275 dollars for rent, 220 dollars for fuel, 35 dollars to the doctor, and 3675 dollars for all my other expenses: how much have I left at the end of the year?

5. A man pays 300 dollars for 100 sheep, 95 dollars for a pair of oxen, 60 dollars for a horse, and 125 dollars for a chaise. He gives 100 bushels of wheat, worth 125 dollars; a cow, worth 25 dollars; a colt, worth 40 dollars, and pays the rest in cash: how much money does he pay?

6. A merchant owes 450120 dollars, and has property as

follows: bank stock, 350000 dollars; western lands, valued at 225100; furniture, worth 4000 dollars, and a store of goods, worth 96000: how much is he worth?

7. If I buy 489 oranges for 912 cents, and sell 125 for 186 cents, and then sell 134 for 199 cents, how many will be left, and how much will they have cost me?

8. By the census of 1850, the entire population of the United States was 23191876; the slave population, 3204313; free colored, 434495: what was the white population?

9. A man gains 367 dollars, then loses 423; a second time he gains 875, and loses 912; he then gains 1012 dollars: how much has he gained in all?

10. If I agree to pay a man 36 dollars for plowing 25 acres of land, 200 dollars for fencing it, and 150 for cultivating it, how much shall I owe him after paying 331 dollars?

11. A merchant bought 85 hogsheads of sugar for 28675 dollars, paid 1231 dollars freight, and then sold it for 1683 dollars less than it cost him: how much did he receive for it?

12. If a man's income is 3467 dollars a year, and he spends 269 dollars for clothing, 467 for house rent, 879 for provision, and 146 for traveling, how much will he have left at the end of the year?

13. Six men bought a tract of land, for 36420 dollars: the first man paid 12140; the second, 3035 less than the first; the third, 346; the fourth, 6070 more than the third; the fifth, 1821 less than the fourth: how much did the sixth man pay?

14. The coinage in the United States Mint, from its establishment in the year 1792 to 1836, was thus: gold, 22102035 dollars; silver, 46739182 dollars; copper, 740331 dollars. The amount coined, from the year 1837 to 1848, was 81436165 dollars: how much more was coined in the last-mentioned period than in the first?

MULTIPLICATION.

41. 1. What will 4 oranges cost, at 2 cents apiece?

Analysis.—1 orange costs, 2 cents;
2 oranges cost, 2 + 2 = 4 cents;
3 oranges cost, 2 + 2 + 2 = 6 cents;
4 oranges cost, 2 + 2 + 2 + 2 = 8 cents;

For the cost of 1 orange, 2 is taken *once;* for the cost of 2 oranges, it is taken *twice;* for the cost of 3, it is taken *three times;* and for the cost of 4, it is taken *four times:* Hence, one time 2 is 2; two times 2 are 4; three times 2 are 6; and four times 2 are 8.

2. What is the cost of 6 yards of ribbon, at 7 cents a yard?

Analysis.—Six yards of ribbon will cost 6 times as much as 1 yard; since 1 yard costs 7 cents, 6 yards will cost 6 times 7 cents, which are 42 cents: Therefore, 6 yards of ribbon, at 7 cents a yard, will cost 42 cents.

42. Multiplication is the operation of taking one number as many times as there are units in another.

43. The Multiplicand is the number to be taken.

44. The Multiplier is the number denoting how many times the multiplicand is to be taken.

45. The Product is the result of the operation.

46. The multiplicand and multiplier, are called Factors of the product.

47. The sign ×, is called the sign of Multiplication. When placed between two numbers, it denotes that they are to be multiplied together; thus,

7 × 5 = 35; and is read, 5 times 7 are 35.

---

41. What will 4 oranges cost, at 2 cents apiece?—42. What is Multiplication?—43. What is the multiplicand?—44. What is the multiplier?—45. What is the product?—46. What are the factors?—47. What is the sign of Multiplication? What is it called? When placed between two numbers, what does it denote?

MULTIPLICATION TABLE.

1 × 1 = 1	2 × 1 = 2	3 × 1 = 3	4 × 1 = 4
1 × 2 = 2	2 × 2 = 4	3 × 2 = 6	4 × 2 = 8
1 × 3 = 3	2 × 3 = 6	3 × 3 = 9	4 × 3 = 12
1 × 4 = 4	2 × 4 = 8	3 × 4 = 12	4 × 4 = 16
1 × 5 = 5	2 × 5 = 10	3 × 5 = 15	4 × 5 = 20
1 × 6 = 6	2 × 6 = 12	3 × 6 = 18	4 × 6 = 24
1 × 7 = 7	2 × 7 = 14	3 × 7 = 21	4 × 7 = 28
1 × 8 = 8	2 × 8 = 16	3 × 8 = 24	4 × 8 = 32
1 × 9 = 9	2 × 9 = 18	3 × 9 = 27	4 × 9 = 36
1 × 10 = 10	2 × 10 = 20	3 × 10 = 30	4 × 10 = 40
1 × 11 = 11	2 × 11 = 22	3 × 11 = 33	4 × 11 = 44
1 × 12 = 12	2 × 12 = 24	3 × 12 = 36	4 × 12 = 48
5 × 1 = 5	6 × 1 = 6	7 × 1 = 7	8 × 1 = 8
5 × 2 = 10	6 × 2 = 12	7 × 2 = 14	8 × 2 = 16
5 × 3 = 15	6 × 3 = 18	7 × 3 = 21	8 × 3 = 24
5 × 4 = 20	6 × 4 = 24	7 × 4 = 28	8 × 4 = 32
5 × 5 = 25	6 × 5 = 30	7 × 5 = 35	8 × 5 = 40
5 × 6 = 30	6 × 6 = 36	7 × 6 = 42	8 × 6 = 48
5 × 7 = 35	6 × 7 = 42	7 × 7 = 49	8 × 7 = 56
5 × 8 = 40	6 × 8 = 48	7 × 8 = 56	8 × 8 = 64
5 × 9 = 45	6 × 9 = 54	7 × 9 = 63	8 × 9 = 72
5 × 10 = 50	6 × 10 = 60	7 × 10 = 70	8 × 10 = 80
5 × 11 = 55	6 × 11 = 66	7 × 11 = 77	8 × 11 = 88
5 × 12 = 60	6 × 12 = 72	7 × 12 = 84	8 × 12 = 96
9 × 1 = 9	10 × 1 = 10	11 × 1 = 11	12 × 1 = 12
9 × 2 = 18	10 × 2 = 20	11 × 2 = 22	12 × 2 = 24
9 × 3 = 27	10 × 3 = 30	11 × 3 = 33	12 × 3 = 36
9 × 4 = 36	10 × 4 = 40	11 × 4 = 44	12 × 4 = 48
9 × 5 = 45	10 × 5 = 50	11 × 5 = 55	12 × 5 = 60
9 × 6 = 54	10 × 6 = 60	11 × 6 = 66	12 × 6 = 72
9 × 7 = 63	10 × 7 = 70	11 × 7 = 77	12 × 7 = 84
9 × 8 = 72	10 × 8 = 80	11 × 8 = 88	12 × 8 = 96
9 × 9 = 81	10 × 9 = 90	11 × 9 = 99	12 × 9 = 108
9 × 10 = 90	10 × 10 = 100	11 × 10 = 110	12 × 10 = 120
9 × 11 = 99	10 × 11 = 110	11 × 11 = 121	12 × 11 = 132
9 × 12 = 108	10 × 12 = 120	11 × 12 = 132	12 × 12 = 144

CASE I.

48. When the multiplier does not exceed 9.

1. What is the product of 236 multiplied by 4?

ANALYSIS.—Since the entire number 236 is to be taken 4 times, each order of units must be taken 4 times: hence, the product must contain 24 units, 12 tens, and 8 hundreds:

Therefore, the product is . . . .

OPERATION.

$$\begin{array}{rl} 236 & \\ \underline{4} & \\ 24 & \text{units.} \\ 12 & \text{tens.} \\ \underline{8} & \text{hundreds.} \\ 944 & \end{array}$$

In practice the operation is performed thus:

Say, 4 times 6 are 24; set down the 4, and then say, 4 times 3 are 12, and 2 to carry are 14; set down the 4, and then say, 4 times 2 are 8, and 1 to carry are 9; set down the 9, and the product is 944, as before.

OPERATION.

$$\begin{array}{r} 236 \\ \underline{4} \\ 944 \end{array}$$

Hence, we have the following

RULE.

Multiply each figure of the multiplicand by the multiplier, carrying and setting down as in Addition.

49. Since the multiplier denotes *times*, it is always an *abstract number;* and since the repetition of a number does not change its unit, the *unit of the product* will be the same as that of the *multiplicand.*

READING.

50. The operations of Multiplication may be much shortened, by pronouncing only the final results.

Thus, in the last example, instead of saying, 4 times 6 are 24; 4 times 3 are 12, and 2 are 14; 4 times 2 are 8, and 1 are 9, we pronounce only the final results, 24, 14, 9,—performing the operations mentally.

EXAMPLES.

	(1.)	(2.)	(3.)
Multiplicand,	867901	278904	678741
Multiplier,	1	2	3

(4.)	(5.)	(6.)	(7.)
47904	780635	910362	1790478
4	5	6	7

8. Multiply 320 by 1.
9. Multiply 9048 by 3.
10. Multiply 67049 by 8.
11. Multiply 270412 by 5.
12. Multiply 30416 by 7.
13. Multiply 204127 by 0.
14. Multiply 30497 by 6.
15. Multiply 69507 by 5.

16. If 104 yards of cotton sheeting are sufficient to supply 1 family for a year, how many yards would supply 9 families ?

17. A farmer had 309 sheep in each of 4 fields: how many sheep had he altogether ?

18. Mrs. Simpkins purchased 2 rolls of table linen, each containing 149 yards: how many yards did she buy ?

19. Each of 9 grocers bought 2974 pineapples: how many did they all buy ?

20. If each of 7 children receive 4073 dollars, how much do they all receive ?

21. How many sheep are there in 9 droves, if in each drove there are 598 ?

22. What will be the cost of 9 horses, at 165 dollars apiece ?

---

48. What is the rule, when the multiplier contains but one figure ?

49. What kind of a number is the multiplier ? What is the unit of the product ?

50. How may the operations of Multiplication be abridged ? Give *an example.*

51. **The product of two numbers is the same, whichever be taken as the multiplier.**

Multiply any two numbers together; as, 8 by 6.

Analysis.—Place as many 1's in a horizontal row as there are units in the multiplicand, and make as many rows as there are units in the multiplier: the product is equal to the number of 1's in one row taken as many times as there are rows; that is, to 8 × 6 = 48.

$$6\left\{\begin{array}{c}\overbrace{1\quad 1\quad 1\quad 1\quad 1\quad 1\quad 1\quad 1}^{8}\\ 1\quad 1\quad 1\quad 1\quad 1\quad 1\quad 1\quad 1\\ 1\quad 1\quad 1\quad 1\quad 1\quad 1\quad 1\quad 1\\ 1\quad 1\quad 1\quad 1\quad 1\quad 1\quad 1\quad 1\\ 1\quad 1\quad 1\quad 1\quad 1\quad 1\quad 1\quad 1\\ 1\quad 1\quad 1\quad 1\quad 1\quad 1\quad 1\quad 1\end{array}\right.$$

If we consider the number of 1's in a vertical row to be the multiplicand, and the number of rows the multiplier, the product will be equal to the number of 1's in a vertical row taken as many times as there are vertical rows; that is, 6 × 8 = 48. Hence,

The product of two numbers is the same, whichever be taken as the multiplier.

CASE II.

52. **When the multiplier contains two or more figures.**

1. Multiply 8204 by 603.

Analysis.—The multiplicand is to be taken 603 times. Taking it 3 times, we obtain 24612.

When we come to take it 6 hundred times, the *lowest order of units* will be hundreds: hence, 4, the first figure of the product, must be written in the third place.

OPERATION.

$$\begin{array}{r} 8204\\ 603\\ \hline 24612\\ 49224\\ \hline 4947012 \end{array}$$

Note.—The product obtained by multiplying by a single figure of the multiplier, is called, a *partial product.* The sum of the partial products, is the required product.

---

51. Is the product of two numbers altered by interchanging the factors?

Hence, we have the following

RULE.

I. *Write the multiplier under the multiplicand, placing units of the same order in the same column:*

II. *Beginning with the units' figure, multiply the multiplicand by each significant figure of the multiplier, and write the first figure of each partial product directly under its multiplier:*

III. *Then add the partial products, and their sum will be the required product.*

PROOF.

Write the multiplicand in the place of the multiplier, and find the product, as before. If the two products are the same, the work is supposed to be right.

EXAMPLES.

	(1.)	(2.)	(3.)	(4.)
Multiplicand,	365	37864	23293	47042
Multiplier,	84	209	74	91

(5.)	(6.)	(7.)	(8.)
46834	679084	1098731	8971432
406	126	1987	10471

9. Multiply 12345678 by 32.
10. Multiply 9378964 by 42.
11. Multiply 1345894 by 49.
12. Multiply 576784 by 64.
13. Multiply 596875 by 144.
14. Multiply 46123101 by 72.
15. Multiply 6185720 by 132.
16. Multiply 718328 by 96.
17. Multiply 679534 by 9185.
18. Multiply 86972 by 1208.
19. Multiply 1055054 by 279.
20. Multiply 538362 by 9258.
21. Multiply 50406 by 8056.
22. Multiply 523972 by 1527.
23. Multiply 760184 by 1615.
24. Multiply 105070 by 3145.

25. What is the product of the number 728056, multiplied by 50467?

26. What is the product of the number 579073, taken 604678 times?

27. What will be the result of taking the number 590587 79904 times?

28. If the number 9127089 be taken 670456 times, what number will express the result?

29. What is the product, when the number 30726 is multiplied by 97034219?

30. What is the product of the two numbers, 870623 and 91678538?

31. Multiply five thousand nine hundred and sixty-five, by six thousand and nine.

32. Multiply eight hundred and seventy thousand six hundred and fifty-one, by three hundred and seven thousand and four.

33. Multiply four hundred and sixty-two thousand six hundred and nine, by itself.

34. Multiply eight hundred and forty-nine millions six hundred and seven thousand three hundred and six, by nine hundred thousand two hundred and four.

35. Multiply 704 millions 130 thousand 496, by three thousand three hundred and one.

36. Multiply forty-nine millions forty thousand six hundred and ninety-seven, by nine millions forty thousand seven hundred and nine.

COMPOSITE NUMBERS—FACTORS.

53. A COMPOSITE NUMBER is one which may be produced by multiplying together two or more numbers.

54. A FACTOR is any one of the numbers which, multiplied together, produce a composite number.

Thus, $2 \times 3 = 6$, 2 and 3 are the factors of the composite number 6.

---

53. What is a composite number?—54. What is a factor?

Also, 12 = 6 × 2 = 3 × 2 × 2, is a composite number, and the factors are 6 and 2; but 6 is a composite number, whose factors are 3 and 2: hence, 3, 2, and 2 are factors of 12.

1. What are the factors of 8? Of 9? Of 10? Of 14?
2. What are the factors of 4? Of 28? Of 30? Of 32?

55. When the multiplier is a composite number.

1. Multiply 8 by the composite number 6, of which the factors are 3 and 2.

$$
\begin{array}{r}8\\ \underline{\;\;3}\\ 24\\ \underline{\;\;2}\\ 48\end{array}
\qquad
2\left\{\begin{array}{l}3\left\{\vphantom{\begin{array}{c}1\\1\\1\end{array}}\right.\\ 3\left\{\vphantom{\begin{array}{c}1\\1\\1\end{array}}\right.\end{array}\right.
\overbrace{\begin{array}{cccccccc}1&1&1&1&1&1&1&1\\1&1&1&1&1&1&1&1\\1&1&1&1&1&1&1&1\\1&1&1&1&1&1&1&1\\1&1&1&1&1&1&1&1\\1&1&1&1&1&1&1&1\end{array}}^{8}
\begin{array}{l}\left.\vphantom{\begin{array}{c}1\\1\end{array}}\right\}\ 8\times2=16\\ \left.\vphantom{\begin{array}{c}1\\1\end{array}}\right\}\ 8\times2=16\\ \left.\vphantom{\begin{array}{c}1\\1\end{array}}\right\}\ 8\times2=\underline{16}\\ \qquad\qquad\quad\;\, 48\end{array}
$$

Analysis.—If we write 6 horizontal lines, with 8 units in each, the product of 8 × 6 = 48, will express the number of units in all the lines.

If we divide the horizontal lines into sets of 3 each (as on the left), there will be 2 sets; the number in each set will be, 8 × 3 = 24, and since there are 2 sets, the whole number of units will be, 24 × 2 = 48.

If we divide the lines into sets of 2 each (as on the right), there will be 3 sets; the number in each set will be 8 × 2 = 16, and since there are 3 sets, the whole number of units will be 16 × 3 = 48. Hence,

When the multiplier is a composite number, multiply by the factors, in succession.

CONTRACTIONS IN MULTIPLICATION.

56. CONTRACTIONS, in Multiplication, are short methods of finding the product, when the multiplier is a composite number.

---

55. How do you multiply, when the multiplier is a composite number?

56. What are contractions, in Multiplication?

CASE I.

57. **When the factors are any numbers.**

Rule.

I. *Separate the composite number into its factors.*

II. *Multiply the multiplicand by one factor, and the product by a second factor; and so on, till all the factors have been used: the last product will be the product required.*

Examples.

1. Multiply 327 by 12.

The factors of 12 are 2 and 6; they are also 3 and 4; or they are 3, 2, and 2.

For, 2 × 6 = 12, 3 × 4 = 12, and 3 × 2 × 2 = 12.

2. Multiply 5709 by 48.
3. Multiply 342516 by 56.
4. Multiply 209402 by 72.
5. Multiply 937387 by 54.
6. Multiply 91738 by 81.
7. Multiply 3842 by 144.

CASE II.

58. **When the multiplier is 1, with any number of ciphers annexed; as, 10, 100, 1000, &c.**

Placing a cipher on the right of a number, is called, annexing it. Annexing one cipher, increases the unit of each place ten times: that is, it changes units into tens, tens into hundreds, hundreds into thousands, &c.; and, therefore, *increases the number ten times.*

Thus, the number 5 is increased *ten times* by annexing one cipher, which makes it 50. The annexing of two ciphers increases a number *one hundred times;* the annexing of three ciphers, *a thousand times,* &c.: hence, the following

Rule.

Annex to the multiplicand as many ciphers as there are in the multiplier, and the number so formed will be the required product.

---

57. How do you multiply, when the factors are any numbers?

EXAMPLES.

1. Multiply 254 by 10.
2. Multiply 648 by 100.
3. Multiply 7987 by 1000.
4. Multiply 9840 by 10000.
5. Multiply 3750 by 100.
6. Multiply 6704 by 10000.
7. Multiply 2141 by 100.
8. Multiply 872 by 100000.

CASE III.

59. When there are ciphers on the right hand of one or both of the factors.

In this case, each number may be regarded as a composite number, of which the significant figures are one factor, and 1, with the requisite number of ciphers annexed, the other factor.

1. Let it be required to multiply 3200 by 800.

OPERATION.

$$3200 = 32 \times 100; \text{ and } 800 = 8 \times 100.$$
$$\text{Then, } 3200 \times 800 = 32 \times 100 \times 8 \times 100,$$
$$= 32 \times 8 \times 100 \times 100,$$
$$= 2560000.$$

Hence, we have the following

RULE.

Omit the ciphers, and multiply the significant figures: then place as many ciphers at the right hand of the product as there are in both factors.

EXAMPLES.

(1.)	(2.)	(3.)
76400	7532000	416000
24	580	357000
1833600		148512000000

4. 4871000 × 270000.
5. 296200 × 875000.
6. 3456789 × 567090.
7. 21200 × 70.
8. 359260 × 304000.
9. 7496430 × 695000.

---

58. If you place one cipher on the right of a number, what effect has it on its value? If you place two, what effect has it? If you place three? How much will each increase it? How do you multiply by 10, 100, 1000, &c.?

59. When there are ciphers on the right hand of one or both the *factors, how do you* multiply?

APPLICATIONS IN THE PRECEDING RULES.

60. 1. What will 5 yards of cloth cost, at 7 dollars a yard?

ANALYSIS.—Five yards of cloth will cost 5 times as much as 1 yard; since 1 yard of cloth costs 7 dollars, 5 yards will cost 5 times 7 dollars, which are 35 dollars: Therefore, 5 yards of cloth, at 7 dollars a yard, will cost 35 dollars: hence,

The cost of any number of things, is equal to the price of a single thing multiplied by the number.

We have seen that the product of two numbers will be the same (that is, will contain the same number of units), whichever be taken for the multiplicand (Art. 51). Hence, in practice, we may multiply the two factors together, taking either for the multiplier, and *then assign the proper unit to the product.* We generally take the *least* number for the multiplier.

2. There are ten bags of coffee, each containing 48 pounds: how much coffee is there in all the bags?

3. There are 24 hours in a day, and 7 days in a week: how many hours in a week?

4. A merchant bought 49 hogsheads of molasses, each containing 63 gallons: how many gallons of molasses were there in the parcel?

5. If a regiment of soldiers contains 1128 men, how many men are there in an army of 106 regiments?

6. A merchant buys a piece of cloth containing 97 yards, at 3 dollars a yard: what does the piece cost him?

7. Suppose a man were to travel 32 miles a day: how far would he travel in 365 days?

8. There are 20 pieces of cloth, each containing 37 yards, and 49 other pieces, each containing 75 yards: how many yards of cloth are there in all the pieces?

9. A farmer bought a farm containing 10 fields; three of the fields contained 9 acres each; three other of the fields, 12 acres each; and the remaining 4 fields, each 15 acres:

---

60. How do you find the cost of any number of things? How may it be done in practice?

how many acres were there in the farm, and how much did the whole cost, at 18 dollars an acre?

10. In a certain city, there are 3751 houses. If each house, on an average, contains 5 persons, how many inhabitants are there in the city?

11. If 786 yards of cloth can be made in one day, how many yards can be made in 1252 days?

12. If 30009 cents are paid for one man's labor on a railroad for 1 year, how many cents would be paid to 814 men, each man receiving the same wages?

13. There are 320 rods in a mile: how many rods are there in the distance from St. Louis to New Orleans, which is 1092 miles?

14. Suppose a book to contain 470 pages, 45 lines on each page, and 50 letters in each line: how many letters in the book?

15. Supposing a crew of 250 men to have provisions for 30 days, allowing each man 20 ounces a day: how many ounces have they?

16. There are 350 rows of trees in a large orchard, 125 trees in each row, and 3000 apples on each tree: how many apples in the orchard?

17. How many soldiers are there in a body of men that is 7 deep, if each line has 758 men?

18. If a railroad car goes 27 miles an hour, how far will it run in 3 days, running 20 hours each day? How far would it run, if its rate were 37 miles an hour?

19. If 1327 barrels of flour will feed the inhabitants of a city for 1 day, how many barrels will supply them for 2 years?

20. A regiment of men contains 10 companies, each company 8 platoons, and each platoon 34 men: how many men in the regiment?

21. Two persons start from the same place, and travel in the same direction; one travels at the rate of 6 miles an hour, the other at the rate of 9 miles an hour; if they travel *8 hours* a day, how far will they be apart at the end of 17 *days? How far,* if they travel in opposite directions?

22. The Erie railroad is about 425 miles long, and cost 65 thousand dollars a mile: if 9645635 dollars had been paid, how much would remain unpaid?

23. A drover bought 106 oxen, at 35 dollars a head; it cost him 6 dollars a head to get them to market, where he sold them at 47 dollars: did he make or lose, and how much?

24. The great Illinois Central Railroad reaches from Chicago to the mouth of the Ohio river, 815 miles, and cost 23500 dollars a mile: what was its entire cost?

Bills of Parcels.

61. When a person sells goods, he generally gives with them a bill, showing the amount charged for them, and acknowledging the receipt of the money paid; such bills are called, *Bills of Parcels.*

New York, Oct. 1, 1854.

25. *James Johnson* *Bought of W. Smith.*

4 Chests of tea, of 45 pounds each, at 1 doll. a pound
3 Firkins of butter, at 17 dolls. per firkin . . . .
4 Boxes of raisins, at 3 dolls. per box . . . . .
36 Bags of coffee, at 16 dolls. each . . . .
14 Hogsheads of molasses, at 28 dolls. each . . . .

Amount, *dollars.*

Received the amount in full. W. Smith.

Hartford, Nov. 1, 1854.

26. *James Hughes* *Bought of W. Jones.*

27 Bags of coffee, at 14 dollars per bag . . . . .
18 Chests of tea, at 25 dolls. per chest . . . . .
75 Barrels of shad, at 9 dolls. per barrel . . . . .
87 Barrels of mackerel, at 8 dolls. per barrel . . .
67 Cheeses, at 2 dolls. each . . . .
59 Hogsheads of molasses, at 29 dolls. per hogshead .

Amount, *dollars.*

Received the amount in full, for W. Jones,
per *James Cross.*

---

61. What are bills of parcels?

DIVISION.

62. 1. When a number is divided into 2 equal parts, each part is called, *one-half of the number.*

What is one-half of 4 apples? What is one-half of 4? How many times is 2 contained in 4?

2. When a number is divided into 3 equal parts, each part is called, *one-third of the number.*

What is one-third of 9 apples? What is one-third of 9? How many times is 3 contained in 9?

3. When a number is divided into 4 equal parts, each part is called, *one-fourth of the number.*

What is one-fourth of 12 pears? What is one-fourth of 12? How many times is 4 contained in 12?

4. When a number is divided into 5 equal parts, each part is called, *one-fifth of the number.*

What is one-fifth of ten marbles? What is one-fifth of 10? How many times is 5 contained in 10?

5. When a number is divided into 6 equal parts, each part is called, *one-sixth of the number.*

6. If 12 apples be equally divided among 4 boys, how many will each have?

ANALYSIS.—If 12 apples be divided equally among 4 boys, each boy will have one of the four equal parts of 12 apples: one of the 4 equal parts of 12 is 3: Therefore, each boy will have 3 apples.

7. If 24 peaches are divided equally among 6 boys, how many will each have? What is one of the six equal parts of 24 peaches? How many times is 6 contained in 24?

8. How many yards of cloth, at 3 dollars a yard, can you buy for 24 dollars?

---

62. What is one-half of a number? What is one third of a number? What is one-fourth of a number? What is one-fifth of a number?

Analysis.—Since 1 yard costs 3 dollars, you can buy as many yards as 3 is contained times in 24; 3 is contained in 24, 8 times. Therefore, at 3 dollars a yard, you can buy 8 yards of cloth for 24 dollars.

9. A farmer pays 28 dollars for 7 sheep: how much is that apiece?

Analysis.—Since 7 sheep cost 28 dollars, one sheep will cost as many dollars as 7 is contained times in 28, which is 4. Therefore, each sheep will cost 4 dollars.

10. If 12 yards of muslin cost 96 cents, how much does 1 yard cost?

11. How many oranges could you buy for 72 cents, if they cost 6 cents apiece?

63. Division is the operation of dividing a number into equal parts; or, of finding how many times one number is contained in another.

64. The Dividend is the number to be divided.

65. The Divisor is the number by which we divide: it shows into how many equal parts the dividend is divided.

66. The Quotient is the result of division. It is one of the equal parts of the dividend, and if the numbers have the same unit, shows how many times the dividend contains the divisor.

67. The Remainder is what is left after the operation.

68. Exact Division is when the remainder is 0.

69. There are three signs used to denote Division:

$18 \div 4$, expresses that 18 is to be divided by 4.

$\frac{18}{4}$, expresses that 18 is to be divided by 4.

$4\,)\,18$, expresses that 18 is to be divided by 4.

---

63. What is Division?—64. What is the dividend?—65. What is the divisor?—66. What is the quotient? What does the quotient show?—67. What is the remainder?—68. What is exact division? —69. How many signs are there of Division? Write them.

CASE I.

70. When the divisor does not exceed 12.

1. Divide 86 by 2.

Analysis.—There are 8 tens and 6 units to be divided by 2. We say, 2 in 8, 4 times, which being tens, we write it in the tens place. We then say, 2 in 6, 3 times, which being units, are written in the units place. Hence, the quotient is 43.

OPERATION.

Divisor. Dividend.

2) 86

43 quotient.

2. Divide 466 by 8.

Analysis.—We first divide the 46 tens by 8, giving a quotient of 5 tens, and 6 tens over. These 6 tens are equal to 60 units, to which add the 6 in the units place. Then say, 8 in 66, 8 times and 2 over: hence, the quotient is 58, and a remainder of 2. This remainder is written after the last quotient figure, and the 8 placed under it; the quotient is read, 58 and 2 divided by 8.

OPERATION.

8) 466

58–2 rem.

$58\frac{2}{8}$ quot.

3. Let it be required to divide 30456 by 12.

Analysis.—We first say, 12 in 3, we cannot. Then, 12 in 30, 2 times and 6 over; then, 12 in 64, 5 times and 4 over; then, 12 in 45, 3 times and 9 over; then, 12 in 96, 8 times.

OPERATION.

12) 30456

2538

Note.—When the divisor exceeds 9, we always divide two figures in the dividend by it.

Rule.

I. *Write the divisor on the left of the dividend. Beginning at the left, divide each figure of the dividend by the divisor, and set each quotient figure under its dividend:*

II. *If there is a remainder after any division, annex to it the next figure of the dividend, and divide as before:*

III. *If any dividend is less than the divisor, write 0 for the quotient figure, and annex the next figure of the dividend, for a new dividend:*

IV. *If there is a remainder, after dividing the last figure, set the divisor under it, and annex the result to the quotient.*

Proof.

Multiply the entire part of the quotient by the divisor, and to the product add the remainder: if the work is right, the result will be equal to the dividend.

Examples.

(1.)

$$\begin{array}{lr} & 3\,)\,\underline{9369} \\ \textit{Ans.} & 3123 \\ & \underline{3} \\ \textit{Proof,} & 9369 \end{array}$$

(2.)

$$\begin{array}{r} 6\,)\,\underline{825467} \\ 137577\tfrac{5}{6} \\ \underline{6} \\ 825467 \end{array}$$

(3.)

$$4\,)\,\underline{73684}$$

(4.)

$$5\,)\,\underline{673420}$$

(5.)

$$7\,)\,\underline{446396}$$

(6.)

$$5\,)\,\underline{1746809}$$

7. Divide 86434 by 2.
8. Divide 416710 by 4.
9. Divide 64140 by 5.
10. Divide 278943 by 6.
11. Divide 95040522 by 6.
12. Divide 75890496 by 8.
13. Divide 6794108 by 3.
14. Divide 21090431 by 9.
15. Divide 2345678964 by 6.
16. Divide 570196382 by 7.
17. Divide 67897634 by 9.
18. Divide 75436298 by 8.
19. Divide 674189904 by 9.
20. Divide 1404967214 by 5.
21. Divide 27478041 by 7.
22. Divide 167484329 by 3.

23. If it takes 5 bushels of wheat to make a barrel of flour, how many barrels can be made from 65890 bushels?

24. How many barrels of flour, at 7 dollars a barrel, can be bought for 609463 dollars?

25. A vessel sails 8 miles an hour: in how many hours will it sail 3756 miles?

---

70. Give the rule for Division, when the divisor does not exceed 12. How do you prove Division?

26. Suppose a wheel is 7 feet in circumference: how many times would it turn, in going 15840 feet?

27. If a pace is 3 feet, how many paces will a man take in walking 6 miles, or 31680 feet?

28. When John starts, Joseph is 37594 feet ahead; Joseph goes 251 feet a minute, and John goes 260 feet a minute: in how many minutes will John overtake Joseph?

29. A county contains 207360 acres of land, lying in 9 townships of equal extent: how many acres in each township?

30. An estate, worth 2943 dollars, is to be divided equally among a father, mother, 3 daughters, and 4 sons: what is the portion of each?

31. A railroad, worth 544806 dollars, is owned, in equal shares, by 9 persons: what is the value of the share of each?

CASE II.

71. When the divisor exceeds 12.

1. Let it be required to divide 7059 by 13.

OPERATION.

$$\begin{array}{rccccccc}
 & \text{Thous.} & \text{Hunds.} & \text{Tens.} & \text{Units.} & \text{Hunds.} & \text{Tens.} & \text{Units.} \\
13\,) & 7 & 0 & 5 & 9\,(& 5 & 4 & 3 \\
 & 6 & 5 & & & & & \\
\cline{2-4}
 & & 5 & 5 & & & & \\
 & & 5 & 2 & & & & \\
\cline{3-5}
 & & & 3 & 9 & & & \\
 & & & 3 & 9 & & & \\
\cline{4-5}
 & & & & 0 & & &
\end{array}$$

Analysis.—The divisor, 13, is not contained in 7 thousands; therefore, *there are no thousands in the quotient.*

We then consider the 0 to be annexed to the 7, making 70 hundreds, and call this a *partial dividend.*

The divisor, 13, is contained in 70 hundreds, 5 hundreds times and something over. To find how much over, multiply 13 by 5 hundreds, and subtract the product, 65, from 70, and there will remain 5 hundreds, to which bring down the 5 tens, and consider the 55 tens a new *partial dividend.*

Then, 13 is contained in 55 tens, 4 tens times and something over. Multiply 13 by 4 tens, and subtract the product, 52, from 55, and to the remainder, 3 tens, bring down the 9 units, and consider the 39 units a new *partial dividend.*

Then, 13 is contained in 39, 3 times. Multiply 13 by 3, and subtract the product, 39, from 39, and we find that the remainder *is 0.*

2. Let it be required to divide 2756 by 26.

We first say, 26 in 27, once, and place 1 in the quotient. Multiplying by 1, subtracting, and bringing down the 5, we have 15 for the first partial dividend. We then say, 26 in 15, 0 times, and place the 0 in the quotient. We then bring down the 6, and find that the divisor is contained in 156, 6 times.

OPERATION.

26) 2756 (106
26
156
156

Hence, if any one of the partial dividends is less than the divisor, write 0 for the quotient figure, and bring down the next figure, forming a new partial dividend.

Rule.

I. *Write the divisor on the left of the dividend:*

II. *Note the fewest figures of the dividend, at the left, that will contain the divisor, and set the quotient figure at the right of the dividend:*

III. *Multiply the divisor by the quotient figure, subtract the product from the first partial dividend, and to the remainder annex the next figure of the dividend, forming a second partial dividend:*

IV. *Find, in the same manner, the second and succeeding figures of the quotient, till all the figures of the dividend are brought down.*

Note 1.—There are five operations in Division: 1st. To write down the numbers; 2d. Divide, or find how many times; 3d. Multiply; 4th. Subtract; 5th. Bring down, to form the partial dividend.

2. The product of a quotient figure by the divisor must never be larger than the corresponding partial dividend; if it is, the quotient figure is too large, and must be diminished.

3. When any one of the remainders is greater than the divisor, the quotient figure is too small, and must be increased.

4. The unit of any quotient figure is the same as that of the partial dividend from which it is obtained. The pupil should always name the unit of every quotient figure.

5. The unit of a remainder is the same as that of the dividend.

PROOF.

72. In Division, the divisor shows into how many equal parts the dividend is divided: the quotient is one of these parts, and the remainder is what is left.

Hence, to prove Division,

Multiply the divisor by the quotient, and to the product add the remainder. If the work is right, the sum will be the same as the dividend.

EXAMPLES.

1. If 300 be divided into 60 equal parts, what is one of these parts?

2. How many times is 54 contained in 7574?

3. If 295470 be divided into 90 equal parts, what is one of these parts?

4. How many times is 37 contained in 7210449?

5. If 62205 dollars be divided equally among a regiment consisting of 957 men, how many dollars will each have?

6. What is one of the equal parts of the number 66708, when divided by 204?

---

71. What is the rule for division, when the divisor exceeds 12?

NOTES.—1. How many operations are there in Division? Name them.

2. If a partial product is greater than the partial dividend, what does it indicate?

3. What do you do when any one of the remainders is greater than the divisor?

4. What is the unit of any figure of the quotient? When the divisor is contained in simple units, what will be the unit of the quotient figure? When it is contained in tens, what will be the unit of the quotient figure? When it is contained in hundreds? In thousands?

5. What is the unit of the remainder?

72. In Division, what does the divisor show? What the quotient? *What is the* remainder? How do you prove Division?

7. How many times is the number 43 contained in the number 12986?

8. How many times is the number 627 contained in the number 657723?

9. What is one of the equal parts of 256 barrels of flour, divided equally among 16 families?

10. How many times is the number 804 contained in the number 320796?

11. Divide 147735 by 45.
12. Divide 947387 by 54.
13. Divide 145260 by 108.
14. Divide 79165238 by 238.
15. Divide 62015735 by 78.
16. Divide 14420946 by 74.
17. Divide 295470 by 90.
18. Divide 1874774 by 162.
19. Divide 435780 by 216.
20. Divi. 119836687 by 3041.
21. Divide 203812983 by 5049.
22. Divide 20195411808 by 3012.
23. Divide 74855092410 by 949998.
24. Divide 47254149 by 4674.
25. Divide 119184669 by 38473.
26. Divide 280208122081 by 912314.
27. Divide 293839455936 by 8405.
28. Divide 4637064283 by 57606.
29. Divide 352107193214 by 210472.
30. Divide 558001172606176724 by 2708630425.
31. Divide 1714347149347 by 57143.
32. Divide 675437149567159 4 by 678957.
33. Divide 71900715708 by 37149.
34. Divide 571943007145 by 37149.
35. Divide 671493471549375 by 47143.
36. Divide 571943007645 by 37149.
37. Divide 171493715947143 by 57007.
38. Divide 121932631112635269 by 987654321.

39. In a hogshead there are 63 gallons: how many hogsheads are there in a reservoir, containing 2645750 gallons?

40. A drover wishes to divide 15600 cattle into 75 droves: how many cattle must he put in each drove?

73. Principles resulting from Division.

1. When the divisor is 1, the quotient will be equal to the dividend.

2. When the divisor is equal to the dividend, the quotient will be 1.

3. When the divisor is less than the dividend, the quotient will be greater than 1.

4. When the divisor is greater than the dividend, the quotient will be less than 1.

Proof of Multiplication.

74. In Division, the divisor and quotient are factors of the dividend. In Multiplication, the multiplicand and multiplier are factors of the product: Hence,

If the product of two numbers be divided by the multiplicand, the quotient will be the multiplier; or, if the product be divided by the multiplier, the quotient will be the multiplicand.

Examples.

3679 Multiplicand.	3679) 1203033 (327
327 Multiplier.	11037
25753	9933
7358	7358
11037	25753
1203033 Product.	25753

---

73. When the divisor is 1, what is the quotient? When the divisor is equal to the dividend, what is the quotient? When the divisor is less than the dividend, how does the quotient compare with 1? When the divisor is greater than the dividend, how does the quotient compare with 1?

74. In Multiplication, what are the factors of the product? If the product be divided by the multiplicand, what is the quotient? If it be divided by the multiplier, what is the quotient?

2. The product of two factors is 68959488; one factor, 96; what is the other?

3. The multiplier is 270000; now, if the product be 1315170000000, what will be the multiplicand?

CONTRACTIONS IN DIVISION.

75. CONTRACTIONS IN DIVISION, are short methods of finding the quotient, when the divisor is a composite number.

CASE I.

76. When the divisor is any composite number.

1. Let it be required to divide 1407 dollars equally among 21 men. Here the factors of the divisor are 7 and 3.

ANALYSIS.—Let the 1407 dollars be first divided into 7 equal piles. Each pile will contain 201 dollars. Let *each* pile be now divided into 3 equal parts. Each part will contain 67 dollars, and the number of parts will be 21: hence the following

OPERATION.

7) 1407

3) 201 1st quotient.

67 quotient sought.

RULE.

Divide the dividend by one of the factors of the divisor; then divide the quotient, thus arising, by a second factor, and so on, till every factor has been used as a divisor; the last quotient will be the answer.

EXAMPLES.

Divide the following numbers by the factors:

1. 1260 by 12 = 3 × 4.
2. 18576 by 48 = 4 × 12.
3. 9576 by 72 = 9 × 8.
4. 19296 by 96 = 12 × 8.
5. 55728 by 4 × 9 × 4 = 144.
6. 92880 by 2 × 2 × 3 × 2 × 2.
7. 57888 by 4 × 2 × 2 × 2.
8. 154368 by 3 × 2 × 2.

---

75. What are contractions, in Division? What is a composite number?

76. What is the rule, when the divisor is any composite number?

True Remainder, when the divisor is a composite number.

Let it be required to divide 755 grapes into 24 equal parts. $24 = 2 \times 3 \times 4$.

2) 755
3) 377 . . 1, 1st rem.
4) 125 . . 2, 2d rem.
31 . . 1, 3d rem.
1st rem. . . = 1
2×2 . . . = 4
1×3×2 . . = 6
True remainder, 11

ANALYSIS.—If 755 grapes be divided into 2 equal parts, there will be 377 bunches, each containing 2 grapes, and 1 grape over. The unit of 377, is 1 bunch = 2 grapes. The unit of the first remainder, is the same as that of the dividend: hence, that remainder forms a part of the true remainder.

If we divide 377 bunches into 3 equal parts, we shall have 125 piles, each containing 3 bunches, and 2 bunches over $= 2 \times 2 = 4$ grapes.

If we divide 125 piles into 4 equal parts, we shall have 31 new piles, and 1 pile over $= 3 \times 2 = 6$ grapes. Hence, to find the true remainder, we have the following

RULE.

To the first remainder add the products which arise by multiplying each of the following remainders by all the preceding divisors, except its own; their sum will be the true remainder.

EXAMPLES.

1. Let it be required to divide 43720 by $45 = 3 \times 5 \times 3$.

3) 43720
5) 14573 . 1 = 1st rem. . . . . 1 .
3) 2914 . 3 = 2d rem. . . 3 × 3 = 9
971 . 1 = 3d rem. . 1 × 5 × 3 = 15
True remainder, 25

What is the rule for finding the true remainder?

2. Divide 956789 by 56.
3. Divide 4870029 by 72.
4. Divide 674201 by 110.
5. Divide 445767 by 144.
6. Divide 1913578 by 42.
7. Divide 146187 by 105.
8. Divide 26964 by 110.
9. Divide 93696 by 231.

CASE II.

77. When the divisor is 10, 100, 1000, &c.

1. In 476 yards of cloth, how many pieces are there of 10 yards each?

Analysis.—There will be one-tenth as many pieces as there are yards; 47 tens is one-tenth of 47 hundreds: if, then, we strike off the right-hand figure, we obtain one-tenth of 476, which is 47, and 6 over.

OPERATION.

$10\,)\,\underline{47|6}$

47–6 remainder.

$47\frac{6}{10}$ quotient.

If the divisor is 100, the quotient is one-hundredth of the dividend; 4 is one-hundredth of 4 hundreds: if, then, we strike off the two right-hand figures, thus, 4|76, we obtain one-hundredth of 476, and 76 over. Hence, the following

Rule.

I. *From the right hand, cut off, by a line, as many figures as there are ciphers in the divisor:*

II. *The figures at the left will be the quotient, and those at the right, the remainder.*

Examples.

1. Divide 49763 by 10.
2. Divide 7641200 by 100.
3. Divide 496321 by 1000.
4. Divide 64978 by 10000.

CASE III.

78. When the divisor contains significant figures, with ciphers on the right of them.

---

77. What is the rule when the divisor is 1, with any number of ciphers annexed?

1. Let it be required to divide 67389 by 700.

Analysis.—We may regard the divisor as a composite number, of which the factors are 100 and 7. We first divide by 100, by striking off the 89, and then by 7, giving 96, with a remainder of 1: this remainder we multiply by the first divisor, 100, and then add 89, forming the true remainder, 189: to the quotient, 96, we annex 189 divided by 700, for the entire quotient. Hence, the following

OPERATION.

7|00) 673|89

96 . . 1 remains.

189 true remainder.

Ans. $96\frac{189}{700}$.

Rule.

I. *Cut off the ciphers by a line, and cut off the same number of figures from the right of the dividend:*

II. *Divide the remaining figures of the dividend by the remaining figures of the divisor, and find the true remainder:*

III. *To the quotient before found, annex the true remainder, and write the divisor under it, for the true quotient.*

Examples.

1. 986327 by 210000.
2. 876000 by 6000.
3. 36599503 by 400700.
4. 5714364900 by 36500.
5. 18490700 by 73000.
6. 70807149 by 31500.
7. 8749632 by 3700.
8. 65975090 by 504000.
9. 87470469 by 8510000.
10. 97621397 by 6987000.

Applications.

79. Abstractly, the object of Division, is to find how many times one number is contained in another. Practically, all questions are reduced to three classes:

1. To find the value of any equal part of a number, or of any number of things.

2. Knowing the entire cost of a number of things, and the price of a single thing, to find the number of things.

---

78. How do you divide when the divisor is any digit, with ciphers *on the right of it?*

3. Knowing the number of things, and their entire cost, to find the price of a single thing.

The three analyses on pages 54 and 55, give the following

RULES.

1. To find the value of each part.

Divide the number of things by the number of parts into which they are to be divided: the quotient will be the value of each part.

2. To find the number of things.

Divide the entire cost by the price of a single thing: the quotient will be the number of things.

3. To find the price of a single thing.

Divide the entire cost by the number of things: the quotient will be the price of a single thing.

APPLICATIONS IN THE PREVIOUS RULES.

1. Mr. Jones died, leaving an estate worth 4500 dollars, to be divided equally among 3 daughters and 2 sons: what was the share of each?

2. The owner of an estate sold 240 acres of land, and had 312 acres left: how many acres had he at first?

3. The sum of two numbers is 3475, and the smaller is 1162: what is the greater?

4. The difference between two numbers is 1475, and the greater number is 5760: what is the smaller?

5. A gentleman bought a house for two thousand twenty-five dollars, and furnished it for seven hundred and six dollars; he paid, at one time, one thousand and ten dollars, and at another time, twelve hundred and seven dollars: how much remained unpaid?

6. At a certain election, the whole number of votes cast

---

79. What is the object of Division, abstractly? Practically, to how many forms are all questions reduced? What are they? What are the rules?

for two opposing candidates, was 12672; the successful candidate received 316 majority: how many votes did each receive?

7. What number must be multiplied by 124, to produce 40796?

8. The sum of 19125 dollars is to be distributed equally among a certain number of men, each to receive 425 dollars: how many men are to receive the money?

9. A merchant has 5100 pounds of tea, and wishes to pack it in 60 chests: how much must he put in each chest?

10. The product of two numbers is 51679680, and one of the factors is 615: what is the other factor?

11. Bought 156 barrels of flour for 1092 dollars, and sold the same for 9 dollars per barrel: how much did I gain?

12. Mr. James has 14 calves, worth 4 dollars each; 40 sheep, worth 3 dollars each; he gives them all for a horse, worth 150 dollars: what does he make or lose by the bargain?

13. Mr. Wilson sells 4 tons of hay, at 12 dollars per ton, 80 bushels of wheat, at 1 dollar per bushel, and takes in payment a horse, worth 65 dollars, a wagon, worth 40 dollars, and the rest in cash: how much money did he receive?

14. If the remainder is 17, the quotient 610, and the dividend 45767, what is the divisor?

15. If the product of two numbers is 346712, and one of the factors is 76, what is the other factor?

16. If the quotient is 482, and the dividend 135442, what is the divisor?

17. There are 31173 verses in the Bible: how many verses must be read each day, that it may be read through in a common year?

18. The distance around the earth is computed to be about 25000 miles: how long would it take a man to travel that distance, supposing him to travel at the rate of 35 miles a day?

19. A person having a yearly salary of 1500 dollars, saves at the end of the year, 405 dollars: what was his average daily expenses, allowing 365 days to the year?

20. The salary of the President of the United States is 50000 dollars a year: how much can he spend daily, and save of his salary, 9850 dollars at the end of the year?

21. A speculator bought 512 barrels of flour for 3584 dollars, and sold the same for 4608 dollars: how much did he gain per barrel?

22. Mr. Place purchased 15 cows; he sold 9 of them for 35 dollars apiece, and the remainder for 32 dollars apiece, when he found that he had lost 123 dollars: how much did he pay apiece for the cows?

23. If a man's salary is 800 dollars a year, and his expenses 425 dollars, how many years will elapse before he will be worth 10000 dollars, if he is worth 2500 dollars at the present time?

24. How long can 125 men subsist on an amount of food that will last 1 man 4500 days?

25. The income of the Bishop of Durham, in England, is 292 dollars a day: how many clergymen would this support on a salary of 730 dollars per annum?

26. The diameter of the earth is 7912 miles, and the diameter of the sun, 112 times as great: what is the diameter of the sun?

27. By the census of 1850, the whole population of the United States was 23191876; the number of births for the previous year was 629444, and the number of deaths, 324394: supposing the births to be the only source of increase, what was the population at the beginning of the previous year?

28. A farmer purchased a farm, for which he paid 18050 dollars; he sold 50 acres for 60 dollars an acre, and the remainder stood him in 50 dollars an acre: how much land did he purchase?

29. A merchant bought a hogshead of molasses, containing 96 gallons, at 35 cents per gallon; but 26 gallons leaked out, and he sold the remainder at 50 cents per gallon: did he gain or lose, and how much?

30. Mr. James bought of Mr. Johnson two farms, one con-

taining 250 acres, for which he paid 85 dollars per acre; the second containing 175 acres, for which he paid 70 dollars an acre; he then sold them both, for 75 dollars an acre: did he make or lose, and how much?

31. A farmer has 279 dollars, with which he wishes to buy cows at 25 dollars, sheep at 4 dollars, and pigs at 2 dollars apiece, of each an equal number: how many can he buy of each sort?

32. Two persons counting their money, found that together they had 342 dollars; but one had 14 dollars more than one-half of it: how many dollars had each?

33. Mr. Bailey has 7 calves, worth 4 dollars apiece, 9 sheep, worth 3 dollars apiece, and a fine horse, worth 175 dollars. He exchanges them for a yoke of oxen, worth 125 dollars, and a colt, worth 65 dollars, and takes the balance in hogs, at 8 dollars apiece: how many hogs does he take?

34. How many pounds of coffee, worth 12 cents a pound, must be given for 368 pounds of sugar, worth 9 cents a pound?

35. If 600 barrels of flour cost 4800 dollars, what will 2172 barrels cost?

36. Mr. Snooks, the tailor, bought of Mr. Squires, the merchant, 4 pieces of cloth; the first and second pieces each measured 45 yards, the third, 47 yards, and the fourth, 53 yards; for the whole he paid 760 dollars: what did he pay for 35 yards?

37. What is the difference between the cost of a flock of sheep containing 175, at 4 dollars apiece, and a drove of 97 cattle, at 85 dollars apiece?

38. A miller bought 320 bushels of wheat for 576 dollars, and sold 256 bushels for 480 dollars: what did the remainder cost him per bushel?

39. A merchant bought 117 yards of cloth for 702 dollars, and sold 76 yards of it at the same price for which he bought it: what was the value of the cloth sold?

40. If 46 acres of land produce 2484 bushels of corn, *how many bushels* will 120 acres produce?

41. Mr. Gill, a drover, purchased 36 head of cattle, at 64 dollars a head, and 88 sheep, at 5 dollars a head: he sold the cattle for 40 dollars a head, and the sheep for 4 dollars apiece: did he make or lose, and how much?

42. Mrs. Louisa Wilsie has 3 houses, valued at 12530 dollars, 11324 dollars, and 9875 dollars: also a farm, worth 6720 dollars. She has a daughter and 2 sons. To the daughter she gives one-third the value of the houses and one-fourth the value of the farm, and then divides the remainder equally among the boys: how much did each receive?

43. Mr. Jones has a farm of 250 acres, worth 125 dollars per acre, and offers to exchange with Mr. Cushing, whose farm contains 185 acres, provided Mr. Cushing will pay him 20150 dollars difference: what was Mr. Cushing's farm valued at, per acre?

44. Mr. Sparks bought a third part of neighbor Spendthrift's farm for 2750 dollars: what would he have paid for the whole farm at the same rate?

45. George Wilson bought 24 barrels of pork, at 14 dollars a barrel; one-fourth of it proved damaged, and he sold it at half price, and the remainder he sold at an advance of 3 dollars a barrel: did he make or lose by the operation, and how much?

46. A gentleman, having 50000 dollars, spent half of it in buying 5 houses, which, after repairing at an expense of 1250 dollars, he sold at 6520 dollars each: how much money had he after the transaction?

47. A gentleman bought 3 houses for 15850 dollars. For two he paid an equal price; and for the third, 850 dollars more than for either of the others: what did he pay for each?

48. Mr. J. Williams went into business with a capital of 25000 dollars: in the first year he gained 2000; in the second year, 3500; in the third year, 4000 dollars: he then invested the whole in a cargo of tea and doubled his money: what was then the value of his fortune?

PROPERTIES OF NUMBERS.

Exact Divisors.

80. An EXACT DIVISOR of a number, is any number, except 1 and the number itself, that will divide it without a remainder. The dividend is then said to be *divisible* by the divisor.

81. An ODD NUMBER is one not divisible by 2.

82. An EVEN NUMBER is one divisible by 2.

1. Three, is an exact divisor of any number, the sum of whose digits is divisible by 3.
2. Four, is an exact divisor of a number, when it will exactly divide the number expressed by the two right-hand digits.
3. Five, is an exact divisor of every number whose right-hand figure is 0 or 5.
4. Six, is an exact divisor of any even number of which 3 is an exact divisor.
5. Nine, is an exact divisor of any number, the sum of whose digits is divisible by 9.
6. Ten, is an exact divisor of every number whose right-hand figure is 0.

83. A PRIME NUMBER is one which has no exact divisor:

1, 2, 3, 5, 7, 11, 13, 17, 19, &c.,

are prime numbers.

84. A COMPOSITE NUMBER being the product of two or more numbers, has two or more exact divisors.

85. A FACTOR of a composite number, is always an exact divisor.

---

80. What is an exact divisor of any number? What is then said of the dividend?—81. What is an odd number?—82. What is an even number?—83. What is a prime number?—84. What is a *composite* number?—85. What is a factor?

CASE I.

86. **To find the prime factors of a composite number.**

1. What are the prime factors of 2310 ?

OPERATION.

$$\begin{array}{r|r} 2 & 2310 \\ \hline 3 & 1155 \\ \hline 5 & 385 \\ \hline 7 & 77 \\ \hline & 11 \end{array}$$

Analysis.—We first divide by 2, the least prime factor of the given number. We then divide the quotient by 3, then the quotient by 5, and then by 7, when we obtain the quotient 11, which is prime. Hence, the prime factors of 2310 are, 2, 3, 5, 7, and 11. Hence, the following

Rule.

Divide the given number by any prime number that will exactly divide it: then divide the quotient in the same manner, and so on, till a quotient is found which is a prime number: the several divisors and the last quotient will be the prime factors.

Examples.

What are the prime factors of the following numbers?

1. Of the number 9?
2. Of the number 15?
3. Of the number 24?
4. Of the number 16?
5. Of the number 18?
6. Of the number 32?
7. Of the number 48?
8. Of the number 56?
9. Of the number 63?
10. Of the number 76?

CASE II.

87. **To find the prime factors common to two or more composite numbers.**

1. What are the common prime factors of 70, 210, and 280?

OPERATION.

$$\begin{array}{r|l} 2 & 70 \;.\; 210 \;.\; 280 \\ \hline 5 & 35 \;.\; 105 \;.\; 140 \\ \hline 7 & 7 \;.\; 21 \;.\; 28 \\ \hline & 1 \;.\; 3 \;.\; 4 \end{array}$$

Analysis.—It is plain that 2 is an exact divisor of all the numbers, and hence, a common factor: 5 is an exact divisor of the first set of quotients, 35, 105, and 140; hence, it is a common factor: 7 is an exact divisor of the second set of quotients; hence, it is a common factor, and the third set of quotients have no exact divisor. Hence, the following

RULE.

I. *Write the numbers in a row, and then divide them by any prime number that is an exact divisor of all of them:*

II. *Divide each set of quotients in the same manner until a set is found which has no exact divisor. The divisors will be the common prime factors.*

NOTE.—The *product* of the prime factors, is the *greatest factor* common to all the numbers. Thus, 2 × 5 × 7 = 70, is the greatest factor common to 70, 210, 280.

EXAMPLES.

1. What are the prime factors common to 6, 9, and 24?
2. What are the prime factors common to 21, 63, and 84?
3. What are the prime factors common to 21, 63, and 105?
4. What are the common prime factors of 28, 42, and 70?
5. What are the prime common factors of 84, 126, and 210?
6. What are the prime factors of 210, 315, and 525?

CANCELLATION.

88. CANCELLATION is a process of shortening Arithmetical operations in Division, by omitting, or *canceling*, factors common to the dividend and divisor.

89. Cancellation depends upon the principle that

If the dividend and divisor be both divided by the same number, the quotient will not be changed.

---

86. How do you find the factors of a composite number?

87. How do you find the prime factors common to two or more composite numbers? What is the greatest factor common to all the numbers?

8. What is Cancellation?

On what principle does Cancellation depend?

1. Divide 63 by 21.

Analysis.—Resolve the dividend and divisor into factors, then cancel those which are common, and mark the canceled figures.

OPERATION.

$$\frac{63}{21} = \frac{\cancel{7}\times 9}{\cancel{7}\times 3} = 3.$$

2. In 7 times 56, how many times 8?

Analysis.—Resolve 56 into the two factors 7 and 8, and then cancel the 8.

OPERATION.

$$\frac{56\times 7}{8} = \frac{\cancel{8}\times 7\times 7}{\cancel{8}} = 49.$$

3. In 36 times 15, how many times 45?

Analysis.—We see that 9 is a factor of 36 and 45. Divide by this factor, and write the quotient 4 over 36, and the quotient 5 below 45. Again, 5 is a factor of 15 and 5. Divide 15 by 5, and write the quotient 3 over 15, and the quotient of 5 by 5, under 5. Dividing 5 by 5, reduces the divisor to 1: hence, the true quotient is, $\frac{4\times 3}{1} = \frac{12}{1} = 12.$

OPERATION.

$$\frac{\overset{4}{\cancel{36}}\times\overset{3}{\cancel{15}}}{\underset{\underset{1}{\cancel{5}}}{\cancel{45}}} = \frac{12}{1}$$

90. Hence, for the operations of Cancellation, we have the following

Rule.

Cancel those factors that are common to the dividend and divisor, and then divide the product of the remaining factors of the dividend by the product of the remaining factors of the divisor.

Notes.—1. If one of the numbers contains a factor equal to the product of two or more factors of the other, they may all be canceled.

2. If the product of two or more factors of the dividend is equal to the product of two or more factors of the divisor, they may all be canceled.

3. If all the factors of the dividend are canceled, the quotient 1 must be put for the factor last canceled.

---

90. What is the rule for the operations of Cancellation?

EXAMPLES.

1. What number is equal to the product of 36 and 13, divided by the product of 4 and 9?

ANALYSIS.—We see that 4 times 9 are equal to 36: therefore, we cancel the 36, and the 4 and 9.

OPERATION.

$$\frac{\cancel{36}\times 13}{\cancel{4}\times\cancel{9}}=\frac{13}{1}=13.$$

2. Divide 960 by 480.

ANALYSIS.—We see that 10 is a common factor; then 12, then 4. We may divide mentally, by the common factors, and place the results at the right.

OPERATION.

$$\frac{960}{480}=\frac{96}{48}=\frac{8}{4}=2.$$

3. Divide the product of $6\times7\times9\times11$, by $2\times3\times7\times3\times21$.

4. Divide the product of $4\times14\times16\times24$, by $7\times8\times32\times12$.

5. Divide the product of $5\times11\times9\times7\times15\times6$, by $30\times3\times21\times3\times5$.

6. Divide 285120 by 5184.

7. Divide 5080320 by 635040.

8. How much calico, at 25 cents a yard, must be given for 8700 cents?

9. How many yards of cloth, at 46 cents a yard, can be bought for 2116 cents?

10. How much molasses, at 42 cents a gallon, can be bought for 1512 cents?

11. In a certain operation, the numbers 24, 28, 32, 49, 81, are to be multiplied together, and the product divided by $8\times4\times7\times9\times6$: what is the result?

12. How many pounds of butter, worth 15 cents a pound, may be bought for 25 pounds of tea, at 48 cents a pound?

13. How many bushels of oats, at 42 cents a bushel, must be given for 3 boxes of raisins, each containing 26 pounds, at 14 cents a pound?

14. A man buys 2 pieces of cotton cloth, each containing 33 yards, at 11 cents a yard, and pays for it in butter at 18 *cents a pound*: how many pounds of butter did he give?

15. If sugar can be bought for 7 cents a pound, how many bushels of oats, at 42 cents a bushel, must I give for 56 pounds?

16. Bought 48 yards of cloth, at 125 cents a yard: how many bushels of potatoes are required to pay for it, at 150 cents a bushel?

17. Mr. Butcher sold 342 pounds of beef, at 6 cents a pound, and received his pay in molasses at 36 cents a gallon: how many gallons did he receive?

18. Mr. Farmer sold 1323 pounds of wool, at 5 cents a pound, and took his pay in cloth at 441 cents a yard: how many yards did he take?

19. How many firkins of butter, each containing 56 pounds, at 18 cents a pound, must be given for 3 barrels of sugar, each containing 200 pounds, at 9 cents a pound?

20. How many boxes of tea, each containing 24 pounds, worth 5 shillings a pound, must be given for 4 bins of wheat, each containing 145 bushels, at 12 shillings a bushel?

21. A. worked for B. 8 days, at 6 shillings a day, for which he received 12 bushels of corn: how much was the corn worth a bushel?

22. Bought 15 barrels of apples, each containing 2 bushels, at the rate of 3 shillings a bushel: how many cheeses, each weighing 30 pounds, at 1 shilling a pound, will pay for the apples?

Least Common Multiple.

91. A Multiple of a number is any product in which the number enters as a factor: hence, any multiple of a number is exactly divisible by the number.

92. A Common Multiple of two or more numbers, is a number exactly divisible by each of them.

93. The Least Common Multiple of two or more numbers, is the least number which is divisible by each

---

91. What is a multiple of a number?
92. What is a common multiple of two or more numbers?
93. What is the least common multiple of two or more numbers?

of them. Thus, 18 is the least common multiple of 2, 6, and 9.

NOTES.—1. If a division is exact, the dividend may be resolved into two factors, one of which will be the divisor, and the other the quotient.

2. If the divisor be resolved into its prime factors, the corresponding factor of the dividend may be resolved into the same factors: hence, *the dividend will contain every factor of an exact divisor.*

3. The question of finding the least common multiple of several numbers, is, therefore, reduced to finding a number which shall contain all the prime factors of the given numbers, and none others.

1. Find the prime factors and least common multiple of 6, 12, and 18.

ANALYSIS.—Write the numbers in a line, and then divide them and the quotients which follow, by any prime number that will exactly divide two or more of them, until quotients are found which are prime with each other. It is plain, that the divisors, 2 and 3, are prime factors of 6; 2, 3, and the quotient 2, of 12; and 2, 3, and the quotient 3, of 18: hence, the prime factors are 2, 3, 2, and 3, and their product, 2 × 3 × 2 × 3 = 36, the least common multiple.

OPERATION.

$$\begin{array}{r|rrr} 2 & 6\ . & 12\ . & 18 \\ \hline 3 & 3\ . & 6\ . & 9 \\ \hline & 1 & 2 & 3 \end{array}$$

94. Hence, to find the least common multiple

RULE.

I. *Write the numbers in a line, and divide by any prime number that will exactly divide any two of them, and write down the quotients, and the undivided numbers:*

II. *Divide as before, until there is no exact divisor of any two of the qaotients: the product of the divisors and the final quotients, will be the least common multiple.*

EXAMPLES.

1. Find the least common multiple of 3, 4, and 8.
2. Find the least common multiple of 3, 8, and 9.

3. Find the least common multiple of 6, 7, 8, and 10.
4. Find the least common multiple of 21 and 49.
5. Find the least common multiple of 2, 7, 5, 6, and 8.
6. Find the least common multiple of 4, 14, 28, and 98.
7. Find the least common multiple of 13 and 6.
8. Find the least common multiple of 12, 4, and 7.
9. Find the least common multiple of 6, 9, 4, 14, and 16.
10. Find the least common multiple of 13, 12, and 4.

GREATEST COMMON DIVISOR.

95. A COMMON DIVISOR of two or more numbers, is any number that will divide each of them without a remainder.

96. The GREATEST COMMON DIVISOR of two or more numbers, is the greatest number that will divide each of them without a remainder.

97. Two numbers are *prime to each other,* when they have no common divisor.

CASE I.

98. To find the greatest common divisor of two or more numbers, when the numbers are small.

Since an exact divisor is a factor, the greatest common divisor of the given numbers, will be their greatest common factor: Hence,

RULE.

Find the prime factors common to all the numbers (Art. 87), *and their product will be the greatest common divisor.*

---

94. What is the rule for finding the least common multiple?
95. What is a common divisor of two or more numbers?
96. What is the greatest common divisor of two or more numbers?
97. When are two numbers prime to each other?
98. How do you find the greatest common divisor, when the numbers are small?

EXAMPLES.

1. What is the greatest common divisor of 24 and 30?
2. What is the greatest common divisor of 9 and 18?
3. What is the greatest common divisor of 6, 12, and 30?
4. What is the greatest common divisor of 15, 25, and 30?
5. What is the greatest common divisor of 12, 18, and 72?
6. What is the greatest common divisor of 25, 35, and 70?
7. What is the greatest common divisor of 28, 42, and 70?
8. What is the greatest common divisor of 84, 126, and 210?

CASE II.

99. To find the greatest common divisor, when the numbers are large.

The operation of finding the common divisor, depends on the following principles:

1. Any number which will exactly divide the difference of two numbers, and one of them, will exactly divide the other; else, we should have a whole number equal to a fraction, which is impossible.

ILLUSTRATION

$24 - 16 = 8$

2. Any number that will exactly divide another, will divide any multiple of that other; because, the first dividend is a factor of the multiple, and any number which will divide a factor, will divide the multiple.

1. What is the greatest common divisor of 25 and 70?

OPERATION.

```
25 ) 70 ( 2
     50
     --
     20 ) 25 ( 1
          20
          --
           5 ) 20 ( 4
               20
               --
```

ANALYSIS.—Divide the greater number, 70, by the less, 25; we find a quotient, 2, and a remainder 20. Then divide the divisor 25 by the remainder 20; the quotient is 1, and the remainder 5. Then divide the divisor 20 by the remainder 5; the quotient is 4, and the division exact.

Now, the remainder 5, exactly divides itself and 20; hence, by the first principle, it will exactly divide 25. Since 5 divides 25, it will, by the second principle, divide 50, a multiple of 25; and since it divides the difference, 20, and one number, 50, it will divide 70; hence, it is a common divisor of 25 and 70; and since there is no other common factor, it is the greatest common divisor.

Hence, to find the greatest common divisor,

Rule.

Divide the greater number by the less, and then divide the divisor by the remainder, and continue to divide the last divisor by the remainder, till nothing remains: the last divisor will be the greatest common divisor.

Examples.

1. What is the greatest common divisor of 216 and 408?
2. Find the greatest common divisor of 408 and 740.
3. Find the greatest common divisor of 315 and 810.
4. Find the greatest common divisor of 4410 and 5670.
5. Find the greatest common divisor of 3471 and 1869.
6. Find the greatest common divisor of 1584 and 2772.

Note.—If it be required to find the greatest common divisor of more than two numbers, first find the greatest common divisor of two of them, then of that common divisor and one of the remaining numbers, and so on for all the numbers: the last common divisor will be the greatest common divisor of all the numbers.

7. What is the greatest common divisor of 492, 744, and 1044?
8. What is the greatest common divisor of 944, 1488, and 2088?
9. What is the greatest common divisor of 216, 408, and 740?
10. What is the greatest common divisor of 945, 1560, and 22683?

---

99. How do you find the greatest common divisor, when the numbers are large?

COMMON FRACTIONS.

100. A UNIT is a single thing; as, 1 apple, 1 chair, 1 pound of tea; and is denoted by 1.

If a unit be divided into two equal parts, each part is called, *one-half.*

If a unit be divided into three equal parts, each part is called, *one-third.*

If a unit be divided into four equal parts, each part is called, *one-fourth.*

If a unit be divided into twelve equal parts, each part is called, *one-twelfth;* and if it be divided into *any number* of equal parts, we have a like expression for each part.

The parts are thus written:

$\frac{1}{2}$	is read, one-half.	$\frac{1}{7}$	is read, one-seventh.
$\frac{1}{3}$	. . . one-third.	$\frac{1}{8}$	. . . one-eighth.
$\frac{1}{4}$	. . . one-fourth.	$\frac{1}{10}$	. . . one-tenth.
$\frac{1}{5}$	. . . one-fifth.	$\frac{1}{15}$	. . . one-fifteenth.
$\frac{1}{6}$	. . . one-sixth.	$\frac{1}{50}$	. . . one-fiftieth.

101. The UNIT OF A FRACTION is the single thing that is divided into equal parts.

102. A FRACTIONAL UNIT is one of the equal parts of the unit that is divided.

NOTE.—In every fraction, let the pupil distinguish carefully between the *unit of the fraction* and the *fractional unit.* The first is the *whole thing* from which the fractions are derived; the second, *one of the equal parts* into which that thing is divided.

---

100. What is a unit? By what is it denoted? What is one-half? One-third? One-fourth? One-twelfth?

101. What is the unit of a fraction?

102. What is a fractional unit? What is the difference between the unit of a fraction and a fractional unit?

103. Every whole number, except 1, has a fractional unit corresponding to it: thus, the numbers

2, 3, 4, 5, 6, 7, 8, 9, 10, &c.,

have, corresponding to them, the fractional units

$\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, $\frac{1}{9}$, $\frac{1}{10}$, &c.

If we suppose a class of boys each to have an apple, and that the apple of each be divided into equal parts corresponding to his class number, the first boy will have the whole apple, or the unit of the fraction; the second boy will have the whole apple in the two fractional units, one-half; the third, in the three fractional units, one-third; the fourth, in the four fractional units, one-fourth; and each boy of a higher number, will have the whole apple in as many fractional units as are denoted by his number in the class.

The fractional units of the fourth boy may be derived from those of the second, by dividing each half by 2, giving 4 fourths; the units of the 6th boy may be derived from those of the 2d, by dividing each by 3, or from those of the 3d, by dividing each by 2; and similarly for any of the higher numbers which are multiples of the lower.

104. An INTEGER, or WHOLE NUMBER, is the unit 1, or a collection of such units.

105. A FRACTION is a fractional unit, or a collection of fractional units.

---

103. What is the fractional unit corresponding to 3? To 4? To 6? To 12? To 65? If each of a class of boys has an apple divided into parts corresponding to his number, what will be the fractional unit of the 4th boy? How many fractional units will he have? How may they be derived from those of the second boy? What will be the fractional unit of the 12th boy? From those of what other boys may they be derived? How from the 2d? How from the 3d? How from the 4th? How from the 6th?

104. What is an integer, or whole number?

105. What is a fraction?

106. Any collection of fractional units, is thus written:

$\frac{2}{2}$	which is read,	2 halves	$= \frac{1}{2} \times 2.$
$\frac{2}{3}$	" "	2 thirds	$= \frac{1}{3} \times 2.$
$\frac{3}{4}$	" "	3 fourths	$= \frac{1}{4} \times 3.$
$\frac{4}{5}$	" "	4 fifths	$= \frac{1}{5} \times 4.$
$\frac{5}{8}$	" "	5 eighths	$= \frac{1}{8} \times 5.$
$\frac{7}{11}$	" "	7 elevenths	$= \frac{1}{11} \times 7.$
$\frac{12}{15}$	" "	12 fifteenths	$= \frac{1}{15} \times 12.$
&c.,	&c.,	&c.,	&c.

Hence, we see that *every fraction may be divided into two factors; one of which is the fractional unit, and the other, the number denoting how many times the fractional unit is taken.*

107. The DENOMINATOR is the number written below the line, and shows into how many equal parts the unit of the fraction is divided.

108. The NUMERATOR is the number written above the line, and shows how many fractional units are taken.

109. The TERMS of a fraction are the numerator and denominator, taken together; hence, every fraction has two terms.

110. The VALUE of a fraction is the quotient of the numerator divided by the denominator.

111. The ANALYSIS of a fraction is the naming of its unit, its fractional unit, and the number of fractional units taken: Thus, in the fraction $\frac{3}{4}$, the unit of the fraction is 1; the fractional unit, $\frac{1}{4}$; and the number of fractional units taken is 3.

---

106. Explain the manner of writing fractional units. Into how many factors may every fraction be divided? What are they?

107. What is the denominator? What does it show?

108. What is the numerator? What does it show?

109. What are the terms of a fraction? How many terms has every fraction?

110. What is the value of a fraction?

111. What is the analysis of a fraction?

112. A whole number may be expressed fractionally, by writing 1 under it for a denominator. Thus,

3 may be written $\frac{3}{1}$ and is read, 3 ones.
5 . . . . . $\frac{5}{1}$ . . . . 5 ones.
6 . . . . . $\frac{6}{1}$ . . . . 6 ones.
8 . . . . . $\frac{8}{1}$ . . . . 8 ones.

113. Properties of Fractions.

1. All the parts of the unit 1, however divided, make up the unit itself. When all the parts are taken, the numerator is equal to the denominator; hence, when the numerator is equal to the denominator, the value of the fraction is 1.

2. If the numerator is less than the number of parts, the value of the fraction is less than 1.

3. If the numerator is greater than the number of parts, some of the fractional units must have come from a second unit; and hence, the value of the fraction will be greater than 1.

EXAMPLES IN WRITING AND READING FRACTIONS.

1. Analyze the following fractions:

$$\frac{5}{12},\quad \frac{5}{9},\quad \frac{16}{7},\quad \frac{7}{10},\quad \frac{3}{8},\quad \frac{9}{50},\quad \frac{65}{117}.$$

2. Write 12 of the 17 equal parts of 1.

3. If the unit of the fraction is 1, and the fractional unit one-twentieth, express 6 fractional units; express, also, 12 and 18.

4. If the fractional unit is one 36th, express 32 fractional units; also, 35, 38, 54, 6, 8.

5. If the fractional unit is one-fortieth, express 9 fractional units; also, 16, 25, 69, 75.

6. Write forty-nine, one hundred and fifteenths.

7. Write three hundred and sixty-one, forty-sevenths.

---

112. How may a whole number be expressed fractionally?

113. When is a fraction equal to 1? When less than 1? When greater than 1?

8. Write seven thousand six hundred and fifteen, nine hundred and fifteenths.

9. Write six thousand four hundred, elevenths.

10. Write six thousand two hundred and forty-two, three hundred and fifty-thirds.

Analyze each of the above fractions, when written.

114. **There are six kinds of fractions:**

1. A PROPER FRACTION is one whose numerator is less than the denominator.

The following are proper fractions:

$\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{3}{7}$, $\frac{5}{8}$, $\frac{9}{10}$, $\frac{8}{9}$, $\frac{5}{6}$.

2. An IMPROPER FRACTION is one whose numerator is equal to, or exceeds the denominator.

The following are improper fractions:

$\frac{3}{3}$, $\frac{3}{2}$, $\frac{5}{3}$, $\frac{6}{5}$, $\frac{8}{7}$, $\frac{9}{8}$, $\frac{12}{6}$, $\frac{14}{7}$, $\frac{19}{7}$.

NOTE.—Such a fraction is called *improper*, because its value equals or exceeds 1.

3. A SIMPLE FRACTION is one whose numerator and denominator are both whole numbers.

The following are simple fractions:

$\frac{1}{4}$, $\frac{3}{2}$, $\frac{5}{6}$, $\frac{8}{7}$, $\frac{9}{2}$, $\frac{8}{3}$, $\frac{6}{5}$, $\frac{7}{5}$.

NOTE.—A simple fraction may be either proper or improper.

4. A COMPOUND FRACTION is a fraction of a fraction, or several fractions connected by the word *of*, or ×.

The following are compound fractions:

$\frac{1}{2}$ of $\frac{1}{4}$, $\frac{1}{3}$ of $\frac{1}{2}$ of $\frac{1}{3}$, $\frac{1}{6} \times 3$, $\frac{1}{7} \times \frac{1}{8} \times 4$.

5. A MIXED NUMBER is a number expressed by an integer and a fraction.

The following are mixed numbers:

$3\frac{1}{2}$, $4\frac{1}{3}$, $6\frac{2}{3}$, $5\frac{3}{5}$, $6\frac{5}{8}$, $3\frac{1}{7}$.

---

114. How many kinds of fractions are there? Name them.

6. A COMPLEX FRACTION is one whose numerator or denominator is fractional; or, in which both are fractional. The following are complex fractions:

$$\frac{\frac{1}{7}}{5},\qquad \frac{2}{19\frac{1}{2}},\qquad \frac{\frac{2}{3}}{\frac{4}{5}},\qquad \frac{45\frac{1}{8}}{69\frac{1}{7}}.$$

FUNDAMENTAL PROPOSITIONS.

115. Let it be required to multiply $\frac{5}{6}$ by 3.

OPERATION.

$$\frac{5}{6} \times 3 = \frac{5 \times 3}{6} = \frac{15}{6}.$$

ANALYSIS.—In $\frac{5}{6}$ there are 5 fractional units, each of which is $\frac{1}{6}$, and these are to be taken 3 times. But 5 things taken 3 times, gives 15 things of the *same kind;* that is, 15 sixths; that is, a number three times as great as 5 sixths; hence,

PROPOSITION I.—*If the numerator of a fraction be multiplied by any number, the value of the fraction will be increased as many times as there are units in the multiplier.*

EXAMPLES.

1. Multiply $\frac{3}{8}$ by 8.
2. Multiply $\frac{7}{8}$ by 5.
3. Multiply $\frac{1}{7}$ by 9.
4. Multiply $\frac{8}{19}$ by 14.
5. Multiply $\frac{7}{6}$ by 20.
6. Multiply $\frac{167}{81}$ by 25.

116. Let it be required to multiply $\frac{4}{6}$ by 3.

OPERATION.

$$\frac{4}{6} \times 3 = \frac{4}{6 \div 3} = \frac{4}{2}.$$

ANALYSIS.—In $\frac{4}{6}$ there are 4 fractional units, each of which is $\frac{1}{6}$. If we divide the denominator by 3, we change the fractional unit from $\frac{1}{6}$ to $\frac{1}{2}$, which is 3 times as great as $\frac{1}{6}$. If we take this fractional unit, 4 times, as before, the result, $\frac{4}{2}$, is 3 times as great as $\frac{4}{6}$: therefore, we have

PROPOSITION II.—*If the denominator of a fraction be divided by any number, the value of the fraction will be increased as many times as there are units in the divisor.*

---

115. What is Proposition I.?—116. What is Proposition II.?

of them. Thus, 18 is the least common multiple of 2, 6, and 9.

NOTES.—1. If a division is exact, the dividend may be resolved into two factors, one of which will be the divisor, and the other the quotient.

2. If the divisor be resolved into its prime factors, the corresponding factor of the dividend may be resolved into the same factors: hence, *the dividend will contain every factor of an exact divisor.*

3. The question of finding the least common multiple of several numbers, is, therefore, reduced to finding a number which shall contain all the prime factors of the given numbers, and none others.

1. Find the prime factors and least common multiple of 6, 12, and 18.

ANALYSIS.—Write the numbers in a line, and then divide them and the quotients which follow, by any prime number that will exactly divide two or more of them, until quotients are found which are prime with each other. It is plain, that the divisors, 2 and 3, are prime factors of 6; 2, 3, and the quotient 2, of 12; and 2, 3, and the quotient 3, of 18: hence, the prime factors are 2, 3, 2, and 3, and their product, 2 × 3 × 2 × 3 = 36, the least common multiple.

OPERATION.

$$\begin{array}{r|ccc} 2 & 6 . & 12 . & 18 \\ \hline 3 & 3 . & 6 . & 9 \\ \hline & 1 & 2 & 3 \end{array}$$

94. Hence, to find the least common multiple

RULE.

I. *Write the numbers in a line, and divide by any prime number that will exactly divide any two of them, and write down the quotients, and the undivided numbers:*

II. *Divide as before, until there is no exact divisor of any two of the quotients: the product of the divisors and the final quotients, will be the least common multiple.*

EXAMPLES.

1. Find the least common multiple of 3, 4, and 8.
2. Find the least common multiple of 3, 8, and 9.

3. Find the least common multiple of 6, 7, 8, and 10.
4. Find the least common multiple of 21 and 49.
5. Find the least common multiple of 2, 7, 5, 6, and 8.
6. Find the least common multiple of 4, 14, 28, and 98.
7. Find the least common multiple of 13 and 6.
8. Find the least common multiple of 12, 4, and 7.
9. Find the least common multiple of 6, 9, 4, 14, and 16.
10. Find the least common multiple of 13, 12, and 4.

GREATEST COMMON DIVISOR.

95. A COMMON DIVISOR of two or more numbers, is any number that will divide each of them without a remainder.

96. The GREATEST COMMON DIVISOR of two or more numbers, is the greatest number that will divide each of them without a remainder.

97. Two numbers are *prime to each other,* when they have no common divisor.

CASE I.

98. To find the greatest common divisor of two or more numbers, when the numbers are small.

Since an exact divisor is a factor, the greatest common divisor of the given numbers, will be their greatest common factor: Hence,

RULE.

Find the prime factors common to all the numbers (Art. 87), *and their product will be the greatest common divisor.*

---

94. What is the rule for finding the least common multiple?
95. What is a common divisor of two or more numbers?
96. What is the greatest common divisor of two or more numbers?
97. When are two numbers prime to each other?
98. How do you find the greatest common divisor, when the numbers are small?

EXAMPLES.

1. What is the greatest common divisor of 24 and 30?
2. What is the greatest common divisor of 9 and 18?
3. What is the greatest common divisor of 6, 12, and 30?
4. What is the greatest common divisor of 15, 25, and 30?
5. What is the greatest common divisor of 12, 18, and 72?
6. What is the greatest common divisor of 25, 35, and 70?
7. What is the greatest common divisor of 28, 42, and 70?
8. What is the greatest common divisor of 84, 126, and 210?

CASE II.

99. To find the greatest common divisor, when the numbers are large.

The operation of finding the common divisor, depends on the following principles:

1. Any number which will exactly divide the difference of two numbers, and one of them, will exactly divide the other; else, we should have a whole number equal to a fraction, which is impossible.

ILLUSTRATION

$24 - 16 = 8$

2. Any number that will exactly divide another, will divide any multiple of that other; because, the first dividend is a factor of the multiple, and any number which will divide a factor, will divide the multiple.

1. What is the greatest common divisor of 25 and 70?

ANALYSIS.—Divide the greater number, 70, by the less, 25; we find a quotient, 2, and a remainder 20. Then divide the divisor 25 by the remainder 20; the quotient is 1, and the remainder 5. Then divide the divisor 20 by the remainder 5; the quotient is 4, and the division exact.

OPERATION.

```
25 ) 70 ( 2
     50
     ——
     20 ) 25 ( 1
          20
          ——
           5 ) 20 ( 4
               20
               ——
```

Now, the remainder 5, exactly divides itself and 20; hence, by the first principle, it will exactly divide 25. Since 5 divides 25, it will, by the second principle, divide 50, a multiple of 25; and since it divides the difference, 20, and one number, 50, it will divide 70; hence, it is a common divisor of 25 and 70; and since there is no other common factor, it is the greatest common divisor.

Hence, to find the greatest common divisor,

RULE.

Divide the greater number by the less, and then divide the divisor by the remainder, and continue to divide the last divisor by the remainder, till nothing remains: the last divisor will be the greatest common divisor.

EXAMPLES.

1. What is the greatest common divisor of 216 and 408?
2. Find the greatest common divisor of 408 and 740.
3. Find the greatest common divisor of 315 and 810.
4. Find the greatest common divisor of 4410 and 5670.
5. Find the greatest common divisor of 3471 and 1869.
6. Find the greatest common divisor of 1584 and 2772.

NOTE.—If it be required to find the greatest common divisor of more than two numbers, first find the greatest common divisor of two of them, then of that common divisor and one of the remaining numbers, and so on for all the numbers: the last common divisor will be the greatest common divisor of all the numbers.

7. What is the greatest common divisor of 492, 744, and 1044?
8. What is the greatest common divisor of 944, 1488, and 2088?
9. What is the greatest common divisor of 216, 408, and 740?
10. What is the greatest common divisor of 945, 1560, and 22683?

---

99. How do you find the greatest common divisor, when the numbers are large?

COMMON FRACTIONS.

100. A UNIT is a single thing; as, 1 apple, 1 chair, 1 pound of tea; and is denoted by 1.

If a unit be divided into two equal parts, each part is called, *one-half.*

If a unit be divided into three equal parts, each part is called, *one-third.*

If a unit be divided into four equal parts, each part is called, *one-fourth.*

If a unit be divided into twelve equal parts, each part is called, *one-twelfth;* and if it be divided into *any number* of equal parts, we have a like expression for each part.

The parts are thus written:

$\frac{1}{2}$	is read, one-half.	$\frac{1}{7}$	is read, one-seventh.
$\frac{1}{3}$	. . . one-third.	$\frac{1}{8}$	. . . one-eighth.
$\frac{1}{4}$	. . . one-fourth.	$\frac{1}{10}$	. . . one-tenth.
$\frac{1}{5}$	. . . one-fifth.	$\frac{1}{15}$	. . . one-fifteenth.
$\frac{1}{6}$	. . . one-sixth.	$\frac{1}{50}$	. . . one-fiftieth.

101. The UNIT OF A FRACTION is the single thing that is divided into equal parts.

102. A FRACTIONAL UNIT is one of the equal parts of the unit that is divided.

NOTE.—In every fraction, let the pupil distinguish carefully between the *unit of the fraction* and the *fractional unit.* The first is the *whole thing* from which the fractions are derived; the second, *one of the equal parts* into which that thing is divided.

---

100. What is a unit? By what is it denoted? What is one-half? One-third? One-fourth? One-twelfth?

101. What is the unit of a fraction?

102. What is a fractional unit? What is the difference between the unit of a fraction and a fractional unit?

103. Every whole number, except 1, has a fractional unit corresponding to it: thus, the numbers

2, 3, 4, 5, 6, 7, 8, 9, 10, &c.,

have, corresponding to them, the fractional units

$\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, $\frac{1}{9}$, $\frac{1}{10}$, &c.

If we suppose a class of boys each to have an apple, and that the apple of each be divided into equal parts corresponding to his class number, the first boy will have the whole apple, or the unit of the fraction; the second boy will have the whole apple in the two fractional units, one-half; the third, in the three fractional units, one-third; the fourth, in the four fractional units, one-fourth; and each boy of a higher number, will have the whole apple in as many fractional units as are denoted by his number in the class.

The fractional units of the fourth boy may be derived from those of the second, by dividing each half by 2, giving 4 fourths; the units of the 6th boy may be derived from those of the 2d, by dividing each by 3, or from those of the 3d, by dividing each by 2; and similarly for any of the higher numbers which are multiples of the lower.

104. An INTEGER, or WHOLE NUMBER, is the unit 1, or a collection of such units.

105. A FRACTION is a fractional unit, or a collection of fractional units.

---

103. What is the fractional unit corresponding to 3? To 4? To 6? To 12? To 65? If each of a class of boys has an apple divided into parts corresponding to his number, what will be the fractional unit of the 4th boy? How many fractional units will he have? How may they be derived from those of the second boy? What will be the fractional unit of the 12th boy? From those of what other boys may they be derived? How from the 2d? How from the 3d? How from the 4th? How from the 6th?

104. What is an integer, or whole number?

105. What is a fraction?

106. Any collection of fractional units, is thus written:

$\frac{2}{2}$	which is read,	2 halves	$= \frac{1}{2} \times 2.$
$\frac{2}{3}$	" "	2 thirds	$= \frac{1}{3} \times 2.$
$\frac{3}{4}$	" "	3 fourths	$= \frac{1}{4} \times 3.$
$\frac{4}{5}$	" "	4 fifths	$= \frac{1}{5} \times 4.$
$\frac{5}{8}$	" "	5 eighths	$= \frac{1}{8} \times 5.$
$\frac{7}{11}$	" "	7 elevenths	$= \frac{1}{11} \times 7.$
$\frac{12}{15}$	" "	12 fifteenths	$= \frac{1}{15} \times 12.$
&c.,	&c.,	&c.,	&c.

Hence, we see that *every fraction may be divided into two factors; one of which is the fractional unit, and the other, the number denoting how many times the fractional unit is taken.*

107. The DENOMINATOR is the number written below the line, and shows into how many equal parts the unit of the fraction is divided.

108. The NUMERATOR is the number written above the line, and shows how many fractional units are taken.

109. The TERMS of a fraction are the numerator and denominator, taken together; hence, every fraction has two terms.

110. The VALUE of a fraction is the quotient of the numerator divided by the denominator.

111. The ANALYSIS of a fraction is the naming of its unit, its fractional unit, and the number of fractional units taken: Thus, in the fraction $\frac{3}{4}$, the unit of the fraction is 1; the fractional unit, $\frac{1}{4}$; and the number of fractional units taken is 3.

---

106. Explain the manner of writing fractional units. Into how many factors may every fraction be divided? What are they?

107. What is the denominator? What does it show?

108. What is the numerator? What does it show?

109. What are the terms of a fraction? How many terms has every fraction?

110. What is the value of a fraction?

111. What is the analysis of a fraction?

112. A whole number may be expressed fractionally, by writing 1 under it for a denominator. Thus,

3 may be written $\frac{3}{1}$ and is read, 3 ones.
5 . . . . . $\frac{5}{1}$ . . . . 5 ones.
6 . . . . . $\frac{6}{1}$ . . . . 6 ones.
8 . . . . . $\frac{8}{1}$ . . . . 8 ones.

113. Properties of Fractions.

1. All the parts of the unit 1, however divided, make up the unit itself. When all the parts are taken, the numerator is equal to the denominator; hence, when the numerator is equal to the denominator, the value of the fraction is 1.

2. If the numerator is less than the number of parts, the value of the fraction is less than 1.

3. If the numerator is greater than the number of parts, some of the fractional units must have come from a second unit; and hence, the value of the fraction will be greater than 1.

EXAMPLES IN WRITING AND READING FRACTIONS.

1. Analyze the following fractions:

$\frac{5}{12}$, $\frac{5}{9}$, $\frac{16}{7}$, $\frac{7}{10}$, $\frac{3}{8}$, $\frac{9}{50}$, $\frac{65}{117}$.

2. Write 12 of the 17 equal parts of 1.

3. If the unit of the fraction is 1, and the fractional unit one-twentieth, express 6 fractional units; express, also, 12 and 18.

4. If the fractional unit is one 36th, express 32 fractional units; also, 35, 38, 54, 6, 8.

5. If the fractional unit is one-fortieth, express 9 fractional units; also, 16, 25, 69, 75.

6. Write forty-nine, one hundred and fifteenths.

7. Write three hundred and sixty-one, forty-sevenths.

---

112. How may a whole number be expressed fractionally?

113. When is a fraction equal to 1? When less than 1? When greater than 1?

8. Write seven thousand six hundred and fifteen, nine hundred and fifteenths.

9. Write six thousand four hundred, elevenths.

10. Write six thousand two hundred and forty-two, three hundred and fifty-thirds.

Analyze each of the above fractions, when written.

114. **There are six kinds of fractions:**

1. A Proper Fraction is one whose numerator is less than the denominator.

The following are proper fractions:

$\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{3}{7}$, $\frac{5}{8}$, $\frac{9}{10}$, $\frac{8}{9}$, $\frac{5}{6}$.

2. An Improper Fraction is one whose numerator is equal to, or exceeds the denominator.

The following are improper fractions:

$\frac{3}{3}$, $\frac{3}{2}$, $\frac{5}{3}$, $\frac{6}{5}$, $\frac{8}{7}$, $\frac{9}{8}$, $\frac{12}{6}$, $\frac{14}{7}$, $\frac{19}{7}$.

Note.—Such a fraction is called *improper*, because its value equals or exceeds 1.

3. A Simple Fraction is one whose numerator and denominator are both whole numbers.

The following are simple fractions:

$\frac{1}{4}$, $\frac{3}{2}$, $\frac{5}{6}$, $\frac{8}{7}$, $\frac{9}{2}$, $\frac{8}{3}$, $\frac{6}{5}$, $\frac{7}{5}$.

Note.—A simple fraction may be either proper or improper.

4. A Compound Fraction is a fraction of a fraction, or several fractions connected by the word *of*, or ×.

The following are compound fractions:

$\frac{1}{2}$ of $\frac{1}{4}$, $\frac{1}{3}$ of $\frac{1}{2}$ of $\frac{1}{3}$, $\frac{1}{6} \times 3$, $\frac{1}{7} \times \frac{1}{8} \times 4$.

5. A Mixed Number is a number expressed by an integer and a fraction.

The following are mixed numbers:

$3\frac{1}{2}$, $4\frac{1}{3}$, $6\frac{2}{3}$, $5\frac{3}{5}$, $6\frac{5}{8}$, $3\frac{1}{7}$.

---

114. **How many kinds of fractions are there? Name them.**

6. A COMPLEX FRACTION is one whose numerator or denominator is fractional; or, in which both are fractional.

The following are complex fractions:

$$\frac{\frac{1}{7}}{5}, \quad \frac{2}{19\frac{1}{2}}, \quad \frac{\frac{2}{3}}{\frac{4}{5}}, \quad \frac{45\frac{1}{8}}{69\frac{1}{7}}.$$

FUNDAMENTAL PROPOSITIONS.

115. Let it be required to multiply $\frac{5}{6}$ by 3.

OPERATION.

$$\frac{5}{6} \times 3 = \frac{5 \times 3}{6} = \frac{15}{6}.$$

ANALYSIS.—In $\frac{5}{6}$ there are 5 fractional units, each of which is $\frac{1}{6}$, and these are to be taken 3 times. But 5 things taken 3 times, gives 15 things of the *same kind;* that is, 15 sixths; that is, a number three times as great as 5 sixths; hence,

PROPOSITION I.—*If the numerator of a fraction be multiplied by any number, the value of the fraction will be increased as many times as there are units in the multiplier.*

EXAMPLES.

1. Multiply $\frac{3}{8}$ by 8.
2. Multiply $\frac{7}{5}$ by 5.
3. Multiply $\frac{1}{7}$ by 9.
4. Multiply $\frac{8}{19}$ by 14.
5. Multiply $\frac{7}{6}$ by 20.
6. Multiply $\frac{167}{81}$ by 25.

116. Let it be required to multiply $\frac{4}{6}$ by 3.

OPERATION.

$$\frac{4}{6} \times 3 = \frac{4}{6 \div 3} = \frac{4}{2}.$$

ANALYSIS.—In $\frac{4}{6}$ there are 4 fractional units, each of which is $\frac{1}{6}$. If we divide the denominator by 3, we change the fractional unit from $\frac{1}{6}$ to $\frac{1}{2}$, which is 3 times as great as $\frac{1}{6}$. If we take this fractional unit, 4 times, as before, the result, $\frac{4}{2}$, is 3 times as great as $\frac{4}{6}$: therefore, we have

PROPOSITION II.—*If the denominator of a fraction be divided by any number, the value of the fraction will be increased as many times as there are units in the divisor.*

---

115. What is Proposition I.?—116. What is Proposition II.?

Hence, *to multiply a fraction, divide its denominator.*

Examples.

1. Multiply $\frac{3}{4}$ by 2, by 4.
2. Multiply $\frac{16}{32}$ by 2, 4, 8.
3. Multiply $\frac{4}{48}$ by 2, 4, 6.
4. Multiply $\frac{12}{84}$ by 2, 4, 6.
5. Multiply $\frac{37}{42}$ by 2, 6, 7.
6. Multiply $\frac{151}{200}$ by 5, 10.

117. Let it be required to divide $\frac{9}{11}$ by 3.

OPERATION.

$$\frac{9}{11} \div 3 = \frac{9 \div 3}{11} = \frac{3}{11}.$$

Analysis.—In $\frac{9}{11}$ there are 9 fractional units, each of which is $\frac{1}{11}$, and these are to be divided by 3. But 9 things, divided by 3, give 3 *things of the same kind* for a quotient; hence, the quotient is 3 elevenths, a number one-third as great as $\frac{9}{11}$: hence, we have

Proposition III.—*If the numerator of a fraction be divided by any number, the value of the fraction will be diminished as many times as there are units in the divisor.*

Examples.

1. Divide $\frac{28}{15}$ by 2, by 7.
2. Divide $\frac{112}{56}$ by 56.
3. Divide $\frac{400}{117}$ by 25, by 8.
4. Divide $\frac{640}{530}$ by 8, 16, 10.
5. Divide $\frac{64}{127}$ by 2, 4, 8.
6. Divide $\frac{42}{114}$ by 3, 21, 7.

118. Let it be required to divide $\frac{9}{11}$ by 3.

OPERATION.

$$\frac{9}{11} \div 3 = \frac{9}{11 \times 3} = \frac{9}{33}.$$

Analysis.—In $\frac{9}{11}$ there are 9 fractional units, each of which is $\frac{1}{11}$. Now, if we multiply the denominator by 3, it becomes 33, and the fractional unit becomes $\frac{1}{33}$, which is only $\frac{1}{3}$ of $\frac{1}{11}$, because 33 is 3 times as great as 11. If we take this fractional unit 9 times, the result, $\frac{9}{33}$, is exactly $\frac{1}{3}$ of $\frac{9}{11}$: hence, we have

Proposition IV.—*If the denominator of a fraction be multiplied by any number, the value of the fraction will be diminished as many times as there are units in the multiplier.*

---

117. What is Proposition III.?—118. What is Proposition IV.?

Hence, *to divide a fraction, multiply the denominator.*

EXAMPLES.

1. Divide $\frac{1}{2}$ by 2.
2. Divide $\frac{1}{5}$ by 7.
3. Divide $\frac{17}{15}$ by 8.
4. Divide $\frac{41}{65}$ by 17.

119. Let it be required to multiply both terms of the fraction $\frac{3}{5}$ by 4.

ANALYSIS.—In $\frac{3}{5}$ the fractional unit is $\frac{1}{5}$, and it is taken 3 times. By multiplying the denominator by 4, the fractional unit becomes $\frac{1}{20}$, the value of which is $\frac{1}{4}$ times as great as $\frac{1}{5}$. By multiplying the numerator by 4, we increase the *number* of fractional units taken, 4 times; that is, we *increase the number just as many times* as we *decrease the value;* hence, the value of the fraction is not changed; therefore, we have

OPERATION.

$$\frac{3 \times 4}{5 \times 4} = \frac{12}{20}.$$

PROPOSITION V.—*If both terms of a fraction be multiplied by the same number, the value of the fraction will not be changed.*

EXAMPLES.

1. Multiply the numerator and denominator of $\frac{5}{7}$ by 7: this gives, $\frac{5 \times 7}{7 \times 7} = \frac{35}{49}$.

2. Multiply the numerator and denominator of $\frac{7}{12}$ by 3, by 4, by 5, by 6, by 9.

3. Multiply each term of $\frac{75}{84}$ by 2, by 3, by 4, by 5, by 6.

120. Let it be required to divide the numerator and denominator of $\frac{6}{15}$ by 3.

ANALYSIS.—In $\frac{6}{15}$ the fractional unit is $\frac{1}{15}$, and is taken 6 times. By dividing the denominator by 3, the fractional unit beecomes $\frac{1}{5}$, the value of which is 3 *times as great as* $\frac{1}{15}$. By dividing the numerator by 3, we diminish the number of fractional units taken, 3 times; that is, we *diminish the number just as many times as we increase the value;* hence, the value of the fraction is not changed: therefore, we have

OPERATION.

$$\frac{6 \div 3}{15 \div 3} = \frac{2}{5}.$$

---

119. What is Proposition V?

PROPOSITION VI.—*If both terms of a fraction be divided by the same number, the value of the fraction will not be changed.*

EXAMPLES.

1. Divide both terms of the fraction $\frac{8}{16}$ by 2: this gives $\frac{8 \div 2}{16 \div 2} = \frac{4}{8}$ *Ans.*

2. Divide both terms by 8: this gives $\frac{8 \div 8}{16 \div 8} = \frac{1}{2}$.

3. Divide both terms of the fraction $\frac{32}{128}$ by 2, by 4, by 8, by 16.

4. Divide both terms of the fraction $\frac{60}{180}$ by 2, by 3, by 4, by 5, by 6, by 10, by 12.

REDUCTION OF FRACTIONS.

121. REDUCTION OF FRACTIONS is the operation of changing the fractional unit, without altering the value of the fraction.

122. The LOWEST TERMS of a fraction, are those terms which are prime to each other.

CASE I.

123. To reduce a whole number to a fraction having a given denominator.

1. Reduce 6 to a fraction whose denominator shall be 4.

ANALYSIS.—This question requires us to reduce 6, to fourths. In 1 unit, there are 4 fourths; in 6 units, there are 6 times 4 fourths, or 24 fourths: therefore, $6 = \frac{24}{4}$. Hence, the following

OPERATION.

$6 \times 4 = 24.$

$\frac{24}{4}.$

RULE.

Multiply the whole number and denominator together, and write the product over the required denominator.

---

120. What is Proposition VI.?
121. What is Reduction of Fractions?
122. What are the lowest terms of a fraction?
123. What is Case I.? What is the rule?

EXAMPLES.

1. Reduce 12 to a fraction whose denominator shall be 9.
2. Reduce 46 to a fraction whose denominator shall be 15.
3. Change 26 to 7ths.
4. Change 178 to 40ths.
5. Reduce 240 to 114ths.
6. Change 54 to quarters.
7. Change 96 to quarters.
8. Change 426 to 16ths.

CASE II.

124. **To reduce a mixed number to its equivalent improper fraction.**

1. Reduce $4\frac{4}{5}$ to its equivalent improper fraction.

ANALYSIS.—Since in any number there are 5 times as many fifths as units 1, in 4 there will be 5 times 4 fifths, or 20 fifths, to which add 4 fifths, and we have 24 fifths. Hence, the following

OPERATION.

$4 \times 5 = 20$ fifths.
Add . . . $\underline{4}$ fifths,
gives . $\frac{24}{5} = 24$ fifths.

RULE.

Multiply the whole number by the denominator of the fraction: to the product add the numerator, and place the sum over the given denominator.

EXAMPLES.

1. Reduce $47\frac{5}{6}$ to its equivalent fraction.
2. In $17\frac{3}{8}$ yards, how many eighths of a yard?
3. In $42\frac{9}{20}$ rods, how many twentieths of a rod?
4. Reduce $625\frac{4}{13}$ to an improper fraction.
5. How many 112ths in $205\frac{46}{112}$?
6. In $84\frac{17}{24}$ days, how many twenty-fourths of a day?
7. In $15\frac{142}{365}$ years, how many 365ths of a year?
8. Reduce $916\frac{459}{600}$ to an improper fraction.
9. Reduce $25\frac{9}{76}$ and $156\frac{22}{60}$ to their equivalent fractions.

---

124. What is Case II.? How do you reduce a mixed number to its equivalent improper fraction?

CASE III.

125. **To reduce an improper fraction to its equivalent whole or mixed number.**

1. In $\frac{59}{8}$, how many entire units?

ANALYSIS.—Since there are 8 eighths in the unit 1, in $\frac{59}{8}$ there are as many units, as 8 is contained times in 59, which is $7\frac{3}{8}$ times. Hence, the following

OPERATION.

$$8\,)\,\underline{59}$$
$$7\tfrac{3}{8}$$

RULE.

Divide the numerator by the denominator, and the quotient will be the whole or mixed number.

EXAMPLES.

1. Reduce $\frac{84}{4}$ and $\frac{67}{9}$ to their equivalent whole or mixed numbers.

OPERATION.

$$4\,)\,\underline{84}$$
$$21$$

OPERATION.

$$9\,)\,\underline{67}$$
$$7\tfrac{4}{9}$$

2. Reduce $\frac{22}{8}$ to a whole or mixed number.

3. In $\frac{12}{7}$ yards of cloth, how many yards?

4. In $\frac{61}{9}$ bushels, how many bushels?

5. If I give $\frac{1}{3}$ of an apple to each one of 15 children, how many apples do I give?

6. Reduce $\frac{387}{125}$, $\frac{3678}{165}$, $\frac{60287}{6941}$, $\frac{987625}{72301}$, to their whole or mixed numbers.

7. If I distribute 878 quarter-apples among a number of boys, how many whole apples do I use?

8. Reduce $\frac{62587}{3114}$, $\frac{4927}{109}$, $\frac{2641674}{278436}$, to their whole or mixed numbers.

9. Reduce $\frac{147254149}{4674}$, $\frac{145260}{108}$, $\frac{62015735}{7803}$, to their whole or mixed numbers.

---

125. What is Case III.? What is the rule?

CASE IV.

126. To reduce a fraction to its lowest terms.

1. Reduce $\frac{70}{175}$ to its lowest terms.

ANALYSIS.—By inspection, it is seen that 5 is a common factor of the numerator and denominator. Dividing by it, we have $\frac{14}{35}$. We then see that 7 is a common factor of 14 and 35: dividing by it, we have $\frac{2}{5}$; and 2 and 5 are prime to each other: hence, 2 and 5 are the *lowest terms*.

1ST OPERATION.

$5\,)\ \frac{70}{175} = \frac{14}{35}.$

$7\,)\ \frac{14}{35} = \frac{2}{5}.$

2D OPERATION.

$35\,)\ \frac{70}{175} = \frac{2}{5}.$

The greatest common divisor of 70 and 175 is 35 (Art. 96); if we divide both terms of the fraction by it, we obtain $\frac{2}{5}$. The value of the fraction is not changed in either operation, since the numerator and denominator are both divided by the same number (Art. 120): Hence, the following

RULE.

Divide the numerator and denominator by each of their common prime factors, in succession:

Or, *Divide the numerator and denominator by their greatest common divisor.*

EXAMPLES.

Reduce the following fractions to their lowest terms:

1. Reduce $\frac{12}{15}$.
2. Reduce $\frac{18}{24}$.
3. Reduce $\frac{27}{36}$.
4. Reduce $\frac{36}{144}$.
5. Reduce $\frac{84}{96}$.
6. Reduce $\frac{144}{864}$.
7. Reduce $\frac{288}{2592}$.
8. Reduce $\frac{85}{165}$.
9. Reduce $\frac{175}{375}$.
10. Reduce $\frac{104}{312}$.
11. Reduce $\frac{1049}{8392}$.
12. Reduce $\frac{275}{440}$.
13. Reduce $\frac{351}{795}$.
14. Reduce $\frac{172}{1118}$.
15. Reduce $\frac{63}{81}$.
16. Reduce $\frac{315}{405}$.

---

126. What is Case IV.? How do you reduce a fraction to its lowest terms? Is the value of the fraction altered? Why not?

CASE V.

127. To reduce a compound fraction to a simple one.

1. What is the value of $\frac{3}{4}$ of $\frac{5}{7}$?

ANALYSIS.—Three-fourths of $\frac{5}{7}$, is 3 times 1 fourth of $\frac{5}{7}$; 1 fourth of $\frac{5}{7}$, is $\frac{5}{28}$ (Art. 118); 3 fourths of $\frac{5}{7}$, is 3 times $\frac{5}{28}$, or $\frac{15}{28}$: therefore, $\frac{3}{4}$ of $\frac{5}{7}=\frac{15}{28}$. Hence,

OPERATION.

$$\frac{3\times5}{4\times7}=\frac{15}{28}.$$

RULE.

Multiply the numerators together for a new numerator, and the denominators together for a new denominator.

NOTE.—If there are mixed numbers, reduce them to their equivalent improper fractions.

EXAMPLES.

Reduce the following fractions to simple ones:

1. Reduce $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{5}{7}$.
2. Reduce $\frac{5}{5}$ of $\frac{3}{6}$ of $\frac{6}{7}$.
3. Reduce $\frac{6}{8}$ of $\frac{8}{9}$ of $\frac{9}{16}$.
4. Reduce $2\frac{1}{4}$ of $6\frac{1}{2}$ of 7.
5. Reduce 5 of $\frac{1}{2}$ of $\frac{1}{7}$ of 6.
6. Reduce $6\frac{1}{3}$ of $7\frac{1}{4}$ of $6\frac{3}{2}\frac{1}{7}$.

METHOD BY CANCELING.

128. Since the numerators are factors of a dividend, and the denominators factors of a divisor, the common factors may be canceled. When they are all canceled, the compound fraction will be reduced to a simple fraction in its lowest terms.

EXAMPLES.

1. Reduce $\frac{6}{8}$ of $\frac{8}{9}$ of $\frac{9}{15}$ to a simple fraction.

OPERATION.

$$\frac{\overset{2}{\cancel{6}}}{\cancel{8}}\text{ of }\frac{\cancel{8}}{\cancel{9}}\text{ of }\frac{\cancel{9}}{\underset{5}{\cancel{15}}}=\frac{2}{5}.$$

---

127. What is Case V.? How do you reduce a compound fraction to a simple one?

128. How are compound fractions reduced by cancellation?

2. Reduce $\frac{4}{5}$ of $\frac{3}{6}$ of $\frac{6}{7}$ to a simple fraction.

Reduce the following:

3. $\frac{3}{4}$ of $\frac{5}{8}$ of $\frac{5}{9}$ of $\frac{27}{100}$ of $\frac{5}{13}$.
4. $\frac{42}{110}$ of $\frac{3}{19}$ of $\frac{40}{108}$ of $\frac{3}{7}$.
5. $3\frac{3}{8}$ of $\frac{5}{9}$ of $\frac{27}{315}$ of 49.
6. $\frac{5}{12}$ of $6\frac{1}{2}$ of $\frac{26}{15}$.
7. $\frac{7}{9}$ of $\frac{24}{21}$ of $\frac{1}{3}$ of $2\frac{4}{7}$.
8. $\frac{10}{21}$ of $\frac{36}{100}$ of $\frac{14}{28}$ of $\frac{3}{25}$.

CASE VI.

129. To reduce fractions having different denominators to equivalent fractions having a common denominator.

Fractions have a *common denominator*, when their denominators are alike.

1. Reduce $\frac{1}{2}$, $\frac{7}{3}$, and $\frac{4}{5}$, to a common denominator.

OPERATION.

$1 \times 3 \times 5 = 15$ 1st num.
$7 \times 2 \times 5 = 70$ 2d num.
$4 \times 2 \times 3 = 24$ 3d num.
$2 \times 3 \times 5 = 30$ com. den.

ANALYSIS.—The numerator and denominator of each fraction must be multiplied by the same number, else the value will be changed. Take the product of the denominators as the common denominator. Since any one of these denominators, multiplied by the product of the other two, will give this common denominator, each numerator must be multiplied by the same product. Multiplying the terms of $\frac{1}{2}$ by 3 and 5, the denominators of the other fractions, we have $\frac{15}{30}$; multiplying the terms of $\frac{7}{3}$ by 2 and 5, the denominators of the other fractions, we have $\frac{70}{30}$; and multiplying the terms of $\frac{4}{5}$ by 2 and 3, the denominators of the other fractions, we have $\frac{24}{30}$: Hence, the

RULE.

Multiply the numerator of each fraction by all the denominators except its own, for the new numerators, and all the denominators together for a common denominator.

---

129. What is Case VI.? What is a common denominator? How do you reduce fractions of different denominators to fractions having a common denominator? When the numbers are small, how may the work be performed?

Notes.—1. Before multiplying, reduce all fractions to simple fractions.

2. When the numbers are small, the work may be performed mentally: Thus:

$$\frac{1}{2},\ \frac{1}{4},\ \frac{2}{5} = \frac{20}{40},\ \frac{10}{40},\ \frac{16}{40}.$$

Examples.

Reduce the following fractions to common denominators:

1. Reduce $\frac{3}{5}$, $\frac{2}{3}$, and $\frac{1}{7}$.
2. Reduce $\frac{2}{3}$, $\frac{4}{11}$, and $\frac{3}{5}$.
3. Reduce $\frac{5}{7}$, $\frac{1}{8}$, and $\frac{5}{9}$.
4. Reduce $2\frac{1}{3}$, and $\frac{1}{2}$ of $\frac{1}{7}$.
5. Reduce $5\frac{1}{2}$, $\frac{4}{7}$ of $\frac{1}{3}$, and 4.
6. Reduce $3\frac{1}{6}$ of $\frac{1}{2}$, and $\frac{3}{5}$.
7. Reduce $\frac{7}{8}$, $\frac{135}{75}$, and 37.
8. Reduce 4, $\frac{21}{25}$, and $\frac{62}{3}$.
9. Reduce $7\frac{1}{2}$, $\frac{31}{18}$, and $6\frac{1}{4}$.
10. Reduce $4\frac{1}{5}$, $8\frac{1}{7}$, and $2\frac{1}{2}$.

Note.—We may often shorten the work by multiplying the numerator and denominator of each fraction by such a number as will make the denominators the same in all.

11. Reduce $\frac{1}{2}$ and $\frac{1}{3}$, to a common denominator.

OPERATION.

$$\frac{1}{2} = \frac{3}{6}$$
$$\frac{1}{3} = \frac{2}{6}$$

Analysis.—Multiply both terms of the first by 3, and both terms of the second by 2.

12. Reduce $\frac{1}{5}$ and $\frac{1}{3}$.
13. Reduce $\frac{1}{6}$, $\frac{1}{12}$, and $\frac{3}{4}$.
14. Reduce $\frac{3}{7}$, $\frac{8}{28}$, and $\frac{4}{14}$.
15. Reduce $\frac{5}{8}$, $3\frac{5}{6}$, and $\frac{3}{4}$.
16. Reduce $6\frac{5}{12}$, $9\frac{1}{2}$, and 5.
17. Reduce $7\frac{5}{6}$, $\frac{4}{9}$, $\frac{1}{4}$, and $\frac{1}{9}$.

CASE VII.

130. **To reduce fractions to their least common denominator.**

The Least Common Denominator is the number which contains all the prime factors of the denominators, and none others.

---

130. What is the least common denominator of several fractions? How do you reduce fractions to their least common denominator?

1. Reduce $\frac{1}{3}$, $\frac{5}{6}$, and $\frac{3}{4}$, to their least common denominator.

Analysis.—The least common denominator will be the least common multiple of all the denominators. Therefore, find the least common multiple of the denominators. In this case, it is 12. Then multiply both terms of each fraction by it, and cancel common factors. This is easiest done by dividing the least common multiple, 12, by each denominator—then, multiplying the quotient by the numerator, and writing each product over the common denominator.

OPERATION.

$$\begin{array}{r|l} 3 & 3\ .\ 6\ .\ 4 \\ \hline 2 & 1\ .\ 2\ .\ 4 \\ \hline & 1\ .\ 1\ .\ 2 \end{array} \qquad \begin{array}{l} (12 \div 3) \times 1 = 4, \text{ 1st numerator.} \\ (12 \div 6) \times 5 = 10, \text{ 2d } `` \\ (12 \div 4) \times 3 = 9, \text{ 3d } `` \end{array}$$

$3 \times 2 \times 2 = 12$; least com. denom.; hence, the fractions are,

$\frac{4}{12}$, $\frac{10}{12}$, and $\frac{9}{12}$.

Rule.

I. *Find the least common multiple of the denominators* (Art. 94), *which will be the least common denominator of the fractions:*

II. *Divide the least common denominator by the denominator of each fraction, separately; multiply the numerator by the corresponding quotient, and place each product over the least common denominator.*

Note.—Before beginning the operation, reduce every fraction to a simple fraction, and to its lowest terms.

Examples.

Reduce the following fractions to their least common denominator:

1. Reduce $\frac{4}{5}$, $\frac{8}{9}$, $\frac{3}{15}$.
2. Reduce $14\frac{5}{4}$, $6\frac{3}{8}$, $5\frac{1}{2}$.
3. Reduce $\frac{3}{15}$, $\frac{4}{24}$, $\frac{8}{9}$.
4. Reduce $\frac{67}{120}$, $\frac{6}{40}$, $\frac{5}{2}$.
5. Reduce $\frac{41}{60}$, $3\frac{6}{20}$, 4.
6. Reduce $3\frac{1}{8}$, $4\frac{4}{12}$, $8\frac{6}{18}$.
7. Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{6}$.
8. Reduce $2\frac{1}{2}$ of $\frac{1}{6}$, $3\frac{1}{7}$ of 2.
9. Reduce $\frac{2}{5}$, $\frac{4}{6}$, $\frac{5}{9}$, and $\frac{7}{10}$.
10. Reduce $\frac{1}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, $\frac{11}{16}$.

ADDITION OF FRACTIONS.

131. Addition of Fractions is the operation of finding the sum of two or more fractions.

1. What is the sum of $\frac{1}{2}$, $\frac{3}{2}$, and $\frac{5}{2}$?

Analysis.—The fractional unit is the same in each fraction, viz.: $\frac{1}{2}$; the numerator of each fraction shows how many such units are taken: hence, *the sum of the numerators, written over the common denominator, expresses the sum of the fractions.*

OPERATION.

$1 + 3 + 5 = 9.$

Ans. $\frac{9}{2} = 4\frac{1}{2}$.

2. What is the sum of $\frac{1}{2}$ and $\frac{2}{3}$?

Analysis.—In the first, the fractional unit is $\frac{1}{2}$, in the second it is $\frac{1}{3}$. These units, not being of the same kind, cannot be expressed in the same collection. But the $\frac{1}{2} = \frac{3}{6}$, and $\frac{2}{3} = \frac{4}{6}$, in each of which the unit is $\frac{1}{6}$: hence, their sum is $\frac{7}{6} = 1\frac{1}{6}$.

OPERATION.

$\frac{1}{2} = \frac{3}{6}$

$\frac{2}{3} = \frac{4}{6}$

$\frac{3}{6} + \frac{4}{6} = \frac{7}{6} = 1\frac{1}{6}$.

Note.—*Only units of the same kind, whether fractional or integral, can be expressed in the same collection.*

From the above analysis, we have the following

Rule.

I. *When the fractions have the same denominator, add the numerators, and place their sum over the common denominator:*

II. *When they have not the same denominator, reduce them to their least common denominator, and then add as before.*

---

131. What is Addition of Fractions? When the fractional unit is the same, what is the sum of the fractions? What units may be expressed in the same collection? What is the rule for the addition of fractions?

EXAMPLES.

1. Add $\frac{1}{2}$, $\frac{3}{2}$, $\frac{6}{2}$, and $\frac{8}{2}$.
2. Add $\frac{1}{7}$, $\frac{5}{7}$, and $\frac{9}{7}$.
3. Add $\frac{3}{9}$, $\frac{4}{9}$, $\frac{6}{9}$, $\frac{13}{9}$, and $\frac{16}{9}$.
4. Add $\frac{3}{14}$, $\frac{8}{14}$, $\frac{9}{14}$, and $\frac{5}{14}$.
5. Add $\frac{4}{5}$, $\frac{3}{10}$, and $\frac{2}{15}$.
6. Add $\frac{1}{2}$, $\frac{3}{4}$, $\frac{2}{8}$, and $\frac{3}{16}$.
7. Add $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{7}{12}$.
8. Add $\frac{3}{4}$, $\frac{7}{8}$, $\frac{1}{5}$, and $\frac{9}{10}$.
9. Add 9, $\frac{2}{5}$, $\frac{1}{15}$, $\frac{5}{6}$, and $\frac{2}{5}$.
10. Add $\frac{1}{2}$, $\frac{3}{7}$, $\frac{6}{7}$, $\frac{1}{8}$, and $\frac{4}{9}$.
11. Add $\frac{5}{12}$, $\frac{5}{8}$, $\frac{6}{10}$, and $\frac{3}{5}$.
12. Add $\frac{1}{7}$, $\frac{2}{9}$, and $\frac{5}{6}$.
13. Add $\frac{1}{16}$, $\frac{3}{7}$, $\frac{2}{8}$, and $\frac{4}{9}$.
14. Add $\frac{6}{12}$, $\frac{3}{5}$, $\frac{4}{8}$, and $\frac{6}{30}$.

15. What is the sum of $19\frac{1}{7}$, $6\frac{2}{3}$, and $4\frac{4}{5}$?

OPERATION.

Whole numbers. *Fractions.*

$$19 + 6 + 4 = 29; \qquad \frac{1}{7} + \frac{2}{3} + \frac{4}{5} = \frac{169}{105} = 1\frac{64}{105};$$

$$\text{sum} = 29 + 1\frac{64}{105} = 30\frac{64}{105}.$$

132. NOTE.—When there are mixed numbers, *add the whole numbers and fractions separately, and then add their sums.*

16. Add $3\frac{1}{4}$, $7\frac{9}{10}$, $12\frac{4}{5}$, $1\frac{7}{8}$.
17. Add 16, $9\frac{3}{4}$, $25\frac{7}{9}$, $\frac{11}{12}$.
18. Add $\frac{1}{2}$ of $\frac{2}{5}$, $\frac{4}{7}$ of 9, $14\frac{9}{10}$.
19. Add $2\frac{8}{11}$, $6\frac{1}{5}$, and $12\frac{19}{22}$.
20. Add $900\frac{1}{10}$, $450\frac{4}{5}$, $75\frac{19}{25}$.
21. Add $\frac{1}{3}$ of $\frac{3}{11}$ of $\frac{11}{12}$ to $\frac{1}{2}$ of $\frac{2}{9}$.
22. Add $17\frac{2}{3}$ to $\frac{4}{5}$ of $7\frac{5}{6}$.
23. Add $\frac{4}{5}$, $7\frac{1}{2}$, $8\frac{3}{4}$.

24. What is the sum of $\frac{3}{4}$ of $12\frac{2}{3}$ of $7\frac{4}{5}$, and $\frac{8}{9}$ of 25?
25. What is the sum of $\frac{4}{25}$ of $9\frac{3}{5}$, and $\frac{4}{21}$ of $328\frac{2}{3}$?

133. 1. What is the sum of $\frac{1}{5}$ and $\frac{1}{6}$?

NOTE.—If each of two fractions has 1 for a numerator, the sum of the fractions will be equal to the sum of their denominators divided by their product.

OPERATION.

$$\frac{1}{5} + \frac{1}{6} = \frac{6}{30} + \frac{5}{30} = \frac{11}{30}.$$

$$\frac{1}{5} + \frac{1}{6} = \frac{5+6}{5 \times 6} = \frac{11}{30}.$$

2. What is the sum of $\frac{1}{8}$ and $\frac{1}{9}$? Of $\frac{1}{6}$ and $\frac{1}{10}$?

3. What is the sum of $\frac{1}{7}$ and $\frac{1}{13}$? Of $\frac{1}{10}$ and $\frac{1}{12}$? Of $\frac{1}{72}$ and $\frac{1}{5}$?

4. What is the sum of $\frac{1}{4}$ and $\frac{1}{12}$? Of $\frac{1}{6}$ and $\frac{1}{9}$? Of $\frac{1}{3}$ and $\frac{1}{15}$?

SUBTRACTION.

134. Subtraction of Fractions is the operation of finding the difference between two fractions.

1. What is the difference between $\frac{5}{8}$ and $\frac{3}{8}$?

Analysis.—In this example, the fractional unit is $\frac{1}{8}$: there are 5 such units in the minuend and 3 in the subtrahend: their difference is 2 eighths; therefore, 2 is written over the common denominator 8.

OPERATION.

$\frac{5}{8} - \frac{3}{8} = \frac{2}{8} = \frac{1}{4}$.

Ans. $\frac{1}{4}$.

2. From $\frac{15}{7}$ take $\frac{10}{7}$.

3. From $\frac{5}{9}$ take $\frac{2}{9}$.

4. From $\frac{125}{365}$ take $\frac{67}{365}$.

5. From $\frac{335}{105}$ take $\frac{169}{105}$.

6. What is the difference between $\frac{5}{6}$ and $\frac{1}{3}$?

Analysis.—Reduce both to the same fractional unit, $\frac{1}{12}$; then, there are 10 such units in the minuend and 4 in the subtrahend: hence, the difference is 6 twelfths.

OPERATION.

$\frac{5}{6} = \frac{10}{12}$.

$\frac{1}{3} = \frac{4}{12}$.

$\frac{10}{12} - \frac{4}{12} = \frac{6}{12} = \frac{1}{2}$.

Ans. $\frac{1}{2}$.

From the above analyses, we have the following

Rule.

I. *When the fractions have the same denominator, subtract the less numerator from the greater, and place the difference over the common denominator:*

II. *When they have not the same denominator, reduce them to their least common denominator, and then subtract as before.*

---

132. When there are mixed numbers, how do you add?
133. When two fractions have 1 for a numerator, what is their sum equal to?
134. What is Subtraction of Fractions? What is the rule?

EXAMPLES.

1. From $\frac{5}{7}$ take $\frac{4}{9}$.
2. From $\frac{6}{7}$ take $\frac{5}{8}$.
3. From $\frac{6}{13}$ take $\frac{5}{17}$.
4. From 1 take $\frac{67}{100}$.
5. From $\frac{1}{8}$ of 12, take $\frac{16}{17}$ of $\frac{1}{2}$.
6. From $\frac{3}{7}$ of $1\frac{1}{2}$ of 7, take $\frac{5}{4}$ of $\frac{3}{8}$.
7. From $\frac{8}{9}$ of $\frac{3}{4}$ of $\frac{1}{2}$, take $\frac{3}{11}$ of $\frac{2}{3}$ of 1.
8. From $\frac{5}{8}$ of $\frac{4}{5}$ of $6\frac{1}{3}$, take $\frac{3}{5}$ of $\frac{5}{3}$ of $\frac{5}{7}$.
9. From $\frac{4}{11}$ of $\frac{22}{18}$ of $\frac{1}{3}$, take $\frac{3}{12}$ of $\frac{3}{7}$.

10. What is the difference between $4\frac{1}{6}$ and $2\frac{1}{7}$?

OPERATION.

$$\begin{aligned} 4\tfrac{1}{6} &= \tfrac{25}{6} = \tfrac{175}{42} \\ 2\tfrac{1}{7} &= \tfrac{15}{7} = \underline{\tfrac{90}{42}} \\ & \qquad \tfrac{85}{42} = 2\tfrac{1}{42}\ \textit{Ans.} \end{aligned} \qquad \text{Or,} \qquad \left\{\begin{aligned} 4\tfrac{1}{6} &= 4\tfrac{7}{42} \\ 2\tfrac{1}{7} &= \underline{2\tfrac{6}{42}} \\ & \quad 2\tfrac{1}{42}\ \textit{Ans.} \end{aligned}\right.$$

135. Therefore: *When there are mixed numbers, change both to improper fractions, and subtract as in* Art. 134: Or, *subtract the integral and fractional numbers separately.*

11. From $84\frac{7}{15}$ take $16\frac{1}{3}$. | 12. From $246\frac{3}{8}$ take $164\frac{1}{5}$.

13. From $7\frac{2}{7}$ take $4\frac{1}{3}$: $\frac{2}{7} = \frac{6}{21}$ and $\frac{1}{3} = \frac{7}{21}$.

NOTE.—Since we cannot take $\frac{7}{21}$ from $\frac{6}{21}$, we borrow 1, or $\frac{21}{21}$, from the minuend, which, added to $\frac{6}{21} = \frac{27}{21}$; then, $\frac{7}{21}$ from $\frac{27}{21}$, leaves $\frac{20}{21}$. We must now carry 1 to the next figure of the subtrahend, and proceed as in subtraction of simple numbers.

OPERATION.

$$\begin{aligned} 7\tfrac{2}{7} &= 7\tfrac{6}{21} \\ 4\tfrac{1}{3} &= \underline{4\tfrac{7}{21}} \\ \textit{Ans.}\ & 2\tfrac{20}{21}. \end{aligned}$$

14. From $16\frac{5}{8}$ take $5\frac{8}{9}$.
15. From $26\frac{3}{7}$ take $19\frac{7}{9}$.
16. From $36\frac{3}{5}$ take $27\frac{8}{11}$.
17. From $400\frac{5}{12}$ take $327\frac{5}{8}$.

18. From the fraction $\frac{1}{8}$, take the fraction $\frac{1}{11}$.

NOTE.—When the numerators are 1, the difference of the two fractions is equal to the difference of the denominators divided by their product.

OPERATION.

$$\frac{1}{8} - \frac{1}{11} = \frac{11}{88} - \frac{8}{88} = \frac{3}{88}.$$
$$\frac{1}{8} - \frac{1}{11} = \frac{11-8}{11 \times 8} = \frac{3}{88}.$$

19. What is the difference between $\frac{1}{6}$ and $\frac{1}{8}$? Between $\frac{1}{5}$ and $\frac{1}{12}$? $\frac{1}{9}$ and $\frac{1}{17}$? $\frac{1}{20}$ and $\frac{1}{37}$? $\frac{1}{11}$ and $\frac{1}{15}$? $\frac{1}{40}$ and $\frac{1}{112}$?

MULTIPLICATION.

136. Multiplication of Fractions is the operation of taking one number as many times as there are units in another, when one or both of the numbers are fractional.

CASE I.

137. To multiply a fraction by a whole number.

1. If one yard of cloth cost $\frac{5}{8}$ of a dollar, what will 4 yards cost?

Analysis.—Four yards will cost 4 times as much as one yard. Since 1 yard costs 5 *eighths* of a dollar, 4 yards will cost 4 times 5 *eighths* of a dollar, which is 20 *eighths:* therefore, if one yard cost $\frac{5}{8}$ of a dollar, 4 yards will cost $\frac{20}{8} = 2\frac{1}{2}$ dollars.

OPERATION.

$$\frac{5}{8} \times 4 = \frac{5 \times 4}{8} = \frac{20}{8} = 2\frac{1}{2}.$$

2d. If we divide the denominator by 4, the fraction will be multiplied by 4 (Prop. II.): performing the operation, we obtain $\frac{5}{2}$, which $= 2\frac{1}{2}$: Hence,

OPERATION.

$$\frac{5}{8} \times 4 = \frac{5}{8 \div 4} = \frac{5}{2}.$$

To multiply a fraction by a whole number,

Multiply the numerator, or divide the denominator by the whole number.

Examples.

1. Multiply $\frac{37}{144}$ by 12.
2. Multiply $\frac{47}{49}$ by 7.
3. Multiply $\frac{175}{45}$ by 9.
4. Multiply $\frac{127}{15}$ by 5.
5. Multiply $\frac{369}{147}$ by 49.
6. Multiply $\frac{175}{273}$ by 26.

---

135. When there are mixed numbers, how do you subtract? Explain the case when the fractional part of the subtrahend is the greater.

136. What is the Multiplication of Fractions?

137. What is Case I.? What is the rule?

7. If 1 dollar will buy $\frac{5}{8}$ of a cord of wood, how much will 15 dollars buy?

8. At $\frac{5}{8}$ of a dollar a pound, what will 12 pounds of tea cost?

9. If a horse eats $\frac{3}{4}$ of a bushel of oats in a day, how much will 18 horses eat?

10. What will 64 pounds of cheese cost, at $\frac{3}{32}$ of a dollar a pound?

11. At $2\frac{3}{5}$ cents a pound, what will 8 pounds of chalk cost?

Note.—When the multiplicand is a mixed number, multiply the fraction and integer separately, and add the results; or, reduce the mixed number to an improper fraction, and multiply.

12. If a man receives $3\frac{9}{10}$ dollars a day, how much will he receive in 15 days?

13. If a family consumes $5\frac{4}{7}$ barrels of flour in 1 year, how much would they consume in 9 years?

CASE II.

138. To multiply a whole number by a fraction.

1. At 15 dollars a ton, what will $\frac{4}{5}$ of a ton of hay cost?

Analysis.—1st. Four-fifths of a ton will cost 4 times as much as 1 fifth of a ton; if 1 ton cost 15 dollars, 1 fifth will cost $\frac{1}{5}$ of 15 dollars, or 3 dollars, and $\frac{4}{5}$ will cost 4 times 3 dollars, which are 12 dollars.

Or, 2d. 4 fifths of a ton will cost 1 fifth of 4 times the cost of 1 ton; 4 times 15 is 60, and 1 fifth of 60 is 12: Hence,

OPERATION.

$$\frac{15}{5} \times 4 = 12.$$

OPERATION.

$$\frac{\overset{3}{\cancel{15}} \times 4}{\cancel{5}} = 12.$$

Rule.

Divide the whole number by the denominator of the fraction, and multiply the quotient by the numerator:

Or, *Multiply the whole number by the numerator of the fraction, and divide the product by the denominator.*

Note.—Cancel, when possible.

EXAMPLES.

1. Multiply 24 by $\frac{7}{8}$.
2. Multiply 42 by $\frac{11}{12}$.
3. Multiply 105 by $\frac{5}{7}$.
4. Multiply 64 by $\frac{15}{16}$.

5. What is the cost of $\frac{2}{3}$ of a yard of cloth, at 8 dollars a yard?

6. If an acre of land is valued at 75 dollars, what is $\frac{7}{12}$ of it worth?

7. If a house is worth 320 dollars, what is $\frac{9}{16}$ of it worth?

8. If a man travels 46 miles a day, how far does he travel in $\frac{4}{5}$ of a day?

9. At 18 dollars a ton, what is the cost of $\frac{9}{10}$ of a ton of hay?

10. If a man earn 480 dollars in a year, how much does he earn in $\frac{11}{12}$ of a year?

CASE III.

139. To multiply one fraction by another.

1. If a bushel of corn costs $\frac{3}{4}$ of a dollar, what will $\frac{5}{6}$ of a bushel cost?

ANALYSIS.—5 sixths of a bushel will cost $\frac{5}{6}$ times as much as 1 bushel, or 5 times 1 sixth as much: $\frac{1}{6}$ of $\frac{3}{4}$ is $\frac{3}{24}$ (Art. 127), and 5 times $\frac{3}{24}$, is $\frac{15}{24} = \frac{5}{8}$: Hence,

OPERATION.

$$\frac{3}{4} \times \frac{5}{6} = \frac{15}{24} = \frac{5}{8};$$

Or,

$$\frac{\cancel{3}}{4} \times \frac{5}{\underset{2}{\cancel{6}}} = \frac{5}{8}.$$

RULE.

Multiply the numerators together for a new numerator, and the denominators together for a new denominator.

NOTES.—1. When the multiplier is less than 1, we do not take the whole of the multiplicand, but only such a part of it as the multiplier is of 1.

2. When the multiplier is a proper fraction, multiplication does

---

138. What is Case II.? What is the rule?
139. What is Case III.? What is the rule?

not imply *increase*, as in the multiplication of whole numbers. The product is the same part of the multiplicand which the multiplier is of 1.

3. If the multiplicand or multiplier, or both, be whole or mixed, the whole number may be reduced to a fractional form, and the mixed numbers reduced to improper fractions; and then the last rule will apply to all examples.

EXAMPLES.

1. Multiply $\frac{7}{8}$ by $\frac{4}{5}$.

2. Multiply $\frac{9}{13}$ by $\frac{15}{27}$.

3. Find the product of $\frac{3}{4}$, $\frac{5}{8}$, and $\frac{7}{15}$.

4. Find the product of $\frac{6}{7}$, $\frac{9}{14}$, and $\frac{21}{27}$.

5. If silk is worth $\frac{9}{10}$ of a dollar a yard, what is $\frac{5}{6}$ of a yard worth?

6. If I own $\frac{6}{7}$ of a farm, and sell $\frac{2}{3}$ of my share, what part of the whole farm do I sell?

7. At $\frac{4}{5}$ of a dollar a pound, what will $\frac{7}{16}$ of a pound of tea cost?

8. If a knife costs $\frac{5}{8}$ of a dollar, and a slate $\frac{6}{7}$ as much, what does the slate cost?

9. Multiply $5\frac{1}{4}$ by $\frac{1}{6}$ of $\frac{8}{9}$.

Note.—Before multiplying, reduce both fractions to the form of simple fractions.

OPERATION.

$5\frac{1}{4} = \frac{21}{4}$; $\frac{1}{6}$ of $\frac{8}{9} = \frac{8}{54}$.

$$\frac{\overset{7}{\cancel{21}}}{\cancel{4}} \times \frac{\overset{2}{\cancel{8}}}{\underset{\underset{9}{\cancel{18}}}{\cancel{54}}} = \frac{7}{9}.$$

GENERAL EXAMPLES.

1. Mult. $\frac{1}{2}$ of $\frac{7}{8}$ of $\frac{4}{5}$, by $\frac{9}{14}$.
2. Mult. $\frac{9}{10}$ by $\frac{2}{5}$ of $1\frac{1}{4}$.
3. Mult. $\frac{1}{8}$ of 3, by $\frac{1}{6}$ of $15\frac{1}{9}$.
4. Mult. 5 of $\frac{2}{3}$ of $\frac{3}{5}$, by $4\frac{1}{6}$.
5. Mult. 14 of $\frac{5}{6}$ of 9, by $6\frac{3}{7}$.
6. Mult. $\frac{3}{7}$ of 6 of $\frac{5}{4}$, by $\frac{8}{5}$ of 4.

---

139. How do you multiply one fraction by another? When the multiplier is less than 1, what part of the multiplicand is taken? If the fraction is proper, does multiplication imply increase? What part is the product of the multiplicand?

140. When the multiplicand is a whole, and the multiplier a mixed number.

7. What is the product of 48 by $8\frac{1}{6}$?

OPERATION.

$$48 \times \frac{1}{6} = 8$$
$$48 \times 8 = 384$$
$$\overline{392}$$

NOTE.—First multiply 48 by $\frac{1}{6}$, which gives 8; then by 8, which gives 384, and the sum 392 is the product: Hence,

RULE.

Multiply first by the fraction, and then by the whole number, and add the products.

8. Multiply 67 by $9\frac{1}{2}$.
9. Multiply 9 by $12\frac{2}{3}$.
10. Multiply 108 by $12\frac{4}{5}$.
11. Multiply $5\frac{6}{7}$ by $3\frac{1}{2}$.

12. What is the product of $6\frac{1}{5}$, $2\frac{7}{8}$, and $\frac{1}{4}$ of 12?

13. What will 24 yards of cloth cost, at $3\frac{3}{4}$ dollars a yard?

14. What will $6\frac{2}{3}$ bushels of wheat cost, at $3\frac{3}{4}$ dollars a bushel?

15. A horse eats $\frac{3}{14}$ of $\frac{7}{9}$ of 12 tons of hay in three months: how much does he consume?

16. If $\frac{2}{3}$ of $\frac{5}{8}$ of a dollar buy a bushel of corn, what will $\frac{7}{10}$ of $\frac{6}{11}$ of a bushel cost?

17. What is the cost of $5\frac{2}{3}$ gallons of molasses, at $96\frac{1}{2}$ cents a gallon?

18. What will $7\frac{1}{12}$ dozen candles cost, at $\frac{3}{16}$ of a dollar per dozen?

19. What must be paid for 175 barrels of flour, at $7\frac{2}{5}$ dollars a barrel?

20. If $\frac{3}{8}$ of $\frac{6}{7}$ of 2 yards of cloth can be bought for one dollar, how much can be bought for $\frac{7}{8}$ of $13\frac{1}{3}$ dollars?

21. What is the cost of $15\frac{2}{3}$ cords of wood, at $3\frac{6}{7}$ dollars a cord?

---

140. How may you multiply, when the multiplicand is a whole, and the multiplier a mixed number?

DIVISION.

141. Division of Fractions is the operation of finding how many times one number is contained in another, when one or both, are fractional.

1. What is the quotient of 5 divided by $\frac{1}{6}$?

Analysis.—One-sixth is contained in 1, 6 times, because there are 6 sixths in 1: one-sixth is contained in 5, 5 times as many times as in 1: hence, $\frac{1}{6}$ is contained in 5, 30 times.

OPERATION.

$$5 \div \frac{1}{6} = 5 \times 6 = 30.$$

2. How many times is $\frac{1}{5}$ contained in 8?

3. How many times is $\frac{1}{7}$ contained in 6?

4. How many• times is $\frac{1}{9}$ contained in 9?

CASE I.

142. To divide a fraction by a whole number.

1. If 4 bushels of apples cost $\frac{8}{9}$ of a dollar, what will 1 bushel cost?

Analysis.—Since 4 bushels cost $\frac{8}{9}$ of a dollar, 1 bushel will cost $\frac{1}{4}$ of $\frac{8}{9}$ of a dollar. Dividing the numerator of the fraction by 4, we have $\frac{2}{9}$ (Art. 117).

OPERATION.

$$\frac{8}{9} \div 4 = \frac{2}{9}.$$

Multiplying the denominator by 4, will produce the same result (Art. 118): Hence,

$$\frac{8}{9} \div 4 = \frac{8}{36} = \frac{2}{9}.$$

Rule.

Divide the numerator, or multiply the denominator, by the divisor.

---

141. What is Division of Fractions? What is the quotient of 8 divided by $\frac{1}{9}$?

142. What is Case I.? What is the rule!

EXAMPLES.

1. Divide $\frac{15}{11}$ by 6.
2. Divide $\frac{18}{37}$ by 9.
3. Divide $\frac{405}{19}$ by 15.
4. Divide $\frac{450}{531}$ by 75.
5. Divide $\frac{18}{19}$ by 6.
6. Divide $\frac{42}{91}$ by 12.
7. Divide $\frac{15}{33}$ by 20.
8. Divide $\frac{126}{651}$ by 27.

9. If 6 horses eat $\frac{9}{10}$ of a ton of hay in 1 month, how much will one horse eat?

10. If 9 yards of ribbon cost $\frac{6}{7}$ of a dollar, what will 1 yard cost?

11. If 1 yard of cloth cost 4 dollars, how much can be bought for $\frac{8}{9}$ of a dollar?

12. If 5 pounds of coffee cost $\frac{15}{16}$ of a dollar, what will 1 pound cost?

13. At $6 a barrel, what part of a barrel of flour can be bought for $\frac{3}{5}$ of a dollar?

14. If 10 bushels of barley cost $3\frac{1}{3}$ dollars, what will 1 bushel cost?

NOTE.—Reduce the mixed number to an improper fraction, and divide as in the case of a simple fraction.

15. If 21 pounds of raisins cost $4\frac{2}{3}$ dollars, what will 1 pound cost?

16. If 12 men consume $6\frac{2}{5}$ pounds of meat in a day, how much does 1 man consume?

CASE II.

143. To divide a whole number by a fraction.

1. At $\frac{4}{5}$ of a dollar apiece, how many hats can be bought for 6 dollars?

OPERATION.

$$6 \div \frac{4}{5} = 6 \div \frac{1}{5} \times 4$$
$$= 6 \times \frac{5}{4} = \frac{30}{4} = 7\frac{1}{2} \text{ hats.}$$

ANALYSIS.—As many as $\frac{4}{5}$ of a dollar is contained times in 6 dollars. $\frac{4}{5} = \frac{1}{5} \times 4$. To divide 6 by $\frac{4}{5}$, is to divide it by $\frac{1}{5}$, and then the quotient by 4 (Art. 76). $6 \div \frac{1}{5} = 6 \times 5 = 30$; then, dividing by 4, we have $\frac{30}{4} = 7\frac{1}{2}$ for the answer. Hence,

RULE.

Invert the terms of the divisor, and multiply the whole number by the new fraction.

EXAMPLES.

1. Divide 14 by $\frac{7}{8}$.

2. Divide 212 by $\frac{15}{16}$.

3. Divide 63 by $\frac{18}{27}$.

4. Divide 420 by $\frac{9}{11}$.

5. At $\frac{11}{12}$ of a dollar a yard, how many yards of cloth can be bought for 9 dollars?

6. If a man travel $\frac{7}{8}$ of a mile in 1 hour, how long will it take him to travel 10 miles?

7. If $\frac{5}{8}$ of a ton of hay is worth 9 dollars, what is a ton worth?

CASE III.

144. To divide one fraction by another.

1. At $\frac{2}{5}$ of a dollar a gallon, how much molasses can be bought for $\frac{7}{8}$ of a dollar?

ANALYSIS.—As many gallons as $\frac{2}{5}$ of a dollar is contained times in $\frac{7}{8}$ of a dollar: $\frac{2}{5} = \frac{1}{5} \times 2$; hence, to divide by $\frac{2}{5}$, is to divide by $\frac{1}{5}$ and 2. $\frac{7}{8} \div \frac{1}{5} = \frac{35}{8}$; then dividing by 2, we have, $\frac{35}{16} = 2\frac{3}{16}$ gallons. Hence,

OPERATION.

$$\frac{7}{8} \div \frac{2}{5} = \frac{7}{8} \div \frac{1}{5} \times 2$$
$$= \frac{35}{8} \div 2 = \frac{35}{16} = 2\frac{3}{16} \text{ gall.}$$

RULE.

I. *Invert the terms of the divisor:*

II. *Then multiply the numerators together for the numerator of the quotient, and the denominators together for the denominator of the quotient.*

NOTES.—1. Cancel all common factors.

2. If the dividend and divisor have a common denominator, they will cancel, and the answer will be the quotient of their numerators.

3. When the dividend or divisor contains a whole or mixed number, or compound fractions, reduce to the form of simple fractions, before dividing.

EXAMPLES.

1. Divide $\frac{9}{10}$ by $\frac{15}{16}$.
2. Divide $\frac{4}{11}$ by $\frac{12}{13}$.
3. Divide $3\frac{7}{9}$ by $\frac{17}{21}$.
4. Divide $\frac{3}{4}$ of $\frac{7}{9}$ by $\frac{7}{12}$ of $1\frac{1}{2}$.
5. Divide $\frac{8}{9}$ of 21 by $\frac{4}{5}$ of $3\frac{2}{3}$.
6. Divide $6\frac{1}{8}$ by $2\frac{1}{3}$.

7. At $\frac{1}{8}$ of a dollar a pound, how much butter can be bought for $\frac{11}{16}$ of a dollar?

8. If 1 man consume $1\frac{1}{3}$ pounds of meat in a day, how many men would $8\frac{2}{3}$ pounds supply?

9. If 6 pounds of tea cost $4\frac{1}{2}$ dollars, what does it cost a pound?

10. At $\frac{4}{5}$ of a dollar a basket, how many baskets of peaches can be bought for $11\frac{1}{5}$ dollars?

11. If $\frac{5}{8}$ of a ton of coal cost $6\frac{2}{3}$ dollars, what will 1 ton cost, at the same rate?

12. How much cheese can be bought for $\frac{19}{24}$ of a dollar, at $\frac{1}{3}$ of a dollar a pound?

13. A man divided $2\frac{4}{5}$ dollars among his children, giving them $\frac{7}{10}$ of a dollar apiece: how many children had he?

14. How many times will $\frac{11}{12}$ of a gallon of beer fill a vessel holding $\frac{1}{3}$ of $\frac{4}{5}$ of a gallon?

15. How many times is $\frac{1}{3}$ of $\frac{4}{5}$ of 27 contained in $\frac{7}{8}$ of $\frac{1}{2}$ of $42\frac{2}{3}$?

16. If $5\frac{1}{3}$ bushels of potatoes cost $2\frac{2}{3}$ dollars, how much do they cost a bushel?

17. If John can walk 21 miles in $\frac{7}{8}$ of a day, how far can he walk in 1 day?

18. If a turkey cost $1\frac{3}{8}$ dollars, how many can be bought for $12\frac{3}{8}$ dollars?

19. At $\frac{4}{5}$ of $\frac{1}{3}$ of a dollar a yard, how many yards of ribbon can be bought for $\frac{24}{25}$ of a dollar?

---

143. What is Case II.? What is the rule?
144. What is Case III.? What is the rule?

COMPLEX FRACTIONS.

145. Complex fractions are reduced to their simplest form by the operations of Reduction and Division.

1. Reduce $\frac{2\frac{2}{5}}{4\frac{1}{2}}$ to its simplest form.

OPERATION.

$$\frac{2\frac{2}{5} = \frac{12}{5}}{4\frac{1}{2} = \frac{9}{2}} = \frac{12}{5} \div \frac{9}{2} = \frac{\overset{4}{\cancel{12}}}{5} \times \frac{2}{\underset{3}{\cancel{9}}} = \frac{8}{15}, \textit{ Ans.}$$

Hence, for the reduction of a complex fraction to its simplest form, we have the following

RULE.

Reduce each term to a simple fraction, and then perform the division.

EXAMPLES.

Reduce the following complex fractions to their simplest form:

1. Reduce $\frac{\frac{4}{5}}{\frac{6}{7}}$.
2. Reduce $\frac{6\frac{2}{3}}{\frac{2}{3}}$.
3. Reduce $\frac{7\frac{11}{12}}{3\frac{5}{6}}$.
4. Reduce $\frac{3}{4}$ of $\frac{\frac{1}{2}}{\frac{3}{7}}$.
5. Reduce $\frac{\frac{2}{9} \text{ of } \frac{3}{5}}{\frac{1}{4} \text{ of } 2\frac{4}{9}}$.
6. Reduce $\frac{25}{8\frac{3}{7}}$.
7. Reduce $\frac{14\frac{9}{10}}{\frac{7}{9} \text{ of } 15}$.
8. Reduce $\frac{214\frac{3}{4}}{25\frac{11}{12}}$.
9. Reduce $\frac{\frac{21}{25}}{4\frac{9}{10}}$.
10. Reduce $\frac{\frac{4}{7} \text{ of } \frac{2}{3} \text{ of } 5\frac{1}{4}}{\frac{15}{16} \text{ of } 48}$.

---

145. What is a complex fraction? How are complex fractions reduced to their simplest forms? What is the rule for Reduction?

MISCELLANEOUS EXAMPLES.

1. A man, having 9$\frac{6}{7}$ dollars, paid 3$\frac{4}{5}$ dollars for boots, and 4$\frac{5}{8}$ dollars for a hat: how much had he left?

2. A retailer gave his customer 1$\frac{7}{8}$ dollars in change, which, he afterwards found, was $\frac{4}{5}$ of a dollar too much: what was the exact amount of change due?

3. A young clerk, having charged $\frac{5}{8}$ of a dollar too much for some cloth, gave in change, 1$\frac{5}{6}$ dollars: what was the exact amount that he ought to have given?

4. A bank of issue failed, and was able to redeem its notes by paying $\frac{3}{5}$ of a dollar on a dollar: how much would he who has a 10-dollar bill, receive from the bank?

5. The sum of two numbers is 12$\frac{7}{16}$; one of the numbers is 7$\frac{4}{7}$: what is the other?

6. James, Joseph, and Daniel owned three farms, whose total area was 475$\frac{9}{10}$ acres. Daniel had 15$\frac{2}{3}$ acres more than Joseph, and Joseph 24$\frac{7}{8}$ acres more than James: how many acres had each in his farm?

7. A housekeeper bought 6 mahogany chairs, at 3$\frac{4}{5}$ dollars each, and gave for them, 2 ten-dollar and 1 five-dollar bill: what change ought she to receive?

8. A mechanic that was fond of reading, wished to buy Macaulay's History, worth 6$\frac{1}{2}$ dollars, Irving's Columbus, worth 4$\frac{2}{3}$ dollars, and Prescott's Philip II., worth 5$\frac{5}{8}$ dollars; his daily wages were 1$\frac{3}{4}$ dollars a day: how many days' wages would pay for the books?

9. If 12 barrels of flour were given for a piece of cloth, measuring 31$\frac{1}{4}$ yards, and valued at 2$\frac{2}{5}$ dollars a yard, what would be the value of one barrel?

10. A grocer having $\frac{7}{8}$ of a barrel of sugar, sold $\frac{2}{7}$ of it for 4$\frac{5}{6}$ dollars: what was the value of the barrel, at the same rate?

11. The product of $\frac{6}{7}$ of 2$\frac{1}{3}$, by $\frac{2}{3}$ of $\frac{3}{4}$ of 9, is how much greater than the quotient of 7$\frac{1}{3}$ divided by $\frac{7}{8}$ of 6$\frac{1}{2}$?

12. The cost of a barrel of flour is $6\frac{1}{5}$ dollars, and it will buy 2 barrels of apples, each of which is worth $1\frac{3}{4}$ barrels of potatoes: how many pounds of butter, at $\frac{5}{8}$ of a dollar for 3 pounds, would pay for a barrel of potatoes?

13. The product of 3 numbers is $\frac{5}{7}$: two of the numbers are $2\frac{1}{2}$ and $\frac{7}{9}$: what is the third?

14. A father and son, working an equal number of days, earned $54\frac{7}{8}$ dollars: the father received $1\frac{3}{5}$ dollars, and the son $\frac{9}{16}$ of a dollar, a day: how many days did they work?

15. A regiment lost in battle 250 men, which was $\frac{2}{5}$ of the regiment: what was the number of men before the battle?

16. A merchant owning $\frac{4}{15}$ of a vessel, sold $\frac{1}{8}$ of his share for 1640 dollars: what was the value of the ship, at that rate?

17. How many lemons, at $\frac{6}{25}$ of a dollar a dozen, will pay for 81 oranges at $2\frac{1}{3}$ cents each?

18. A lad, multiplying by $\frac{5}{6}$ instead of $\frac{5}{16}$, obtained $\frac{5}{9}$ for a result: what result ought he to have obtained?

19. Reduce $\frac{\frac{7}{9}\text{ of }2\frac{1}{7}}{2\frac{1}{9}} \times \frac{\frac{2}{3}}{\frac{1}{2}\text{ of }\frac{1}{6}}$ to a simple fraction.

20. If $\frac{3}{5}$ of a yard of cloth cost $\frac{7}{9}$ of a dollar, what will be the cost of $2\frac{5}{6}$ yards?

21. If $23\frac{1}{4}$ dollars are required to pay 18 men for 1 day's wages, how much would be required to pay 33 men for $15\frac{3}{4}$ days' labor?

22. If A. can mow an acre of ground in 3 days, and B. in 2 days, how long would it take them both to mow it?

23. If A. and B. can do a piece of work in 10 days, and A. alone can do it in 16 days, in what time can B. do it?

24. Multiply $\frac{5\frac{1}{3}}{18\frac{2}{5}}$ by $\frac{11\frac{11}{12}}{12\frac{1}{10}}$.

25. In a piece of cloth there were $36\frac{1}{4}$ yards. The piece cost $65\frac{1}{8}$ dollars. For what must the cloth be sold per yard, that there may be a gain of $18\frac{2}{25}$ dollars?

DECIMAL FRACTIONS.

146. There are two kinds of Fractions: *Common Fractions,* and *Decimal Fractions.*

147. A COMMON FRACTION, is one in which the unit is divided into any number of equal parts. In such fractions, the denominators may be any numbers whatever.

148. A DECIMAL FRACTION, is one in which the unit is divided into 10 equal parts, then each of these parts is again divided into 10 equal parts, and so on, using 10, constantly, as a divisor. Hence, a Decimal Fraction is but a particular case of a Common Fraction.

When the unit is divided into 10 equal parts, there are 10 such parts of the unit, and each part is called, *one-tenth.*

If each tenth be divided into 10 equal parts, there will be 100 equal parts in the unit, and each part will be $\frac{1}{10}$ of $\frac{1}{10}$ $= \frac{1}{100}$.

If each hundredth be divided into 10 equal parts, there will be 1000 equal parts in the unit, and each part will be $\frac{1}{10}$ of $\frac{1}{100} = \frac{1}{1000}$; and smaller parts may be obtained, by still dividing by 10.

NOTATION AND NUMERATION.

149. A period (.), called the *decimal point,* written before a figure, denotes that its unit is 1 tenth:

Thus, .1 is read, 1 tenth $= \frac{1}{10}$.
.4 " 4 tenths $= \frac{4}{10}$.
.7 " 7 tenths $= \frac{7}{10}$.
&c., &c.

---

146. How many kinds of Fractions are there? What are they?

147. What is a Common Fraction?

148. What is a Decimal Fraction? When the unit is divided into 10 equal parts, what is each part called? What is each part called, when it is divided into 100 equal parts?

The second place from the decimal point, is the place of hundredths:

Thus,	.01	is read,	1 hundredth $= \frac{1}{100}$.
	.04	"	4 hundredths $= \frac{4}{100}$.
	.07	"	7 hundredths $= \frac{7}{100}$.
	&c.,		&c.

The third place is the place of thousandths:

Thus,	.001	is read,	1 thousandth $= \frac{1}{1000}$.
	.004	"	4 thousandths $= \frac{4}{1000}$.
	.007	"	7 thousandths $= \frac{7}{1000}$.

The fourth place is the place of ten-thousandths; the fifth, of hundred-thousandths; the sixth, of millionths, &c.

Thus, 4, written in the different places, is read,

Four tenths, ...	.4
Four hundredths, ...	.04
Four thousandths, ...	.004
Four ten-thousandths, ...	.0004
Four hundred-thousandths, ...	.00004
Four millionths, ...	.000004
Four ten-millionths, ...	.0000004

150. We numerate from the decimal point to the right, and read in the *lowest fractional unit of the decimal.* Thus, we numerate, tenths, hundredths, &c.; and read, 4 tenths, 4 hundredths, 4 thousandths, &c., &c., &c.

151. From the nature of decimals, and the manner of writing them, we see,

1st. *That the denominator belonging to any decimal fraction, is* 1, *with as many ciphers annexed as there are places of figures in the decimal.*

---

149. What is the decimal point? Where is it written? What does it denote? What is the first place to the right called? The second? The third?

150. How do you numerate decimals? How do you read them?

2d. *That the unit of any place, is ten times as great as the unit of the next place to the right—the same as in whole numbers: hence, whole numbers and decimals may be written together, by placing the decimal point between them, as in the following*

NUMERATION TABLE.

&c., &c., &c.	Tens of millions.	Millions.	Hundreds of thousands.	Tens of thousands.	Thousands.	Hundreds.	Tens.	UNITS.	Tenths.	Hundredths.	Thousandths.	Ten-thousandths.	Hundred-thousandths.	Millionths.	Ten-millionths.	&c., &c., &c.
	8	3	6	3	0	6	4	2·	0	4	7	8	9	7	6	
	Whole numbers.								Decimals.							

152. A MIXED NUMBER, in decimals, is composed of a whole number and a decimal: Thus, 67.0478 is a MIXED NUMBER.

RULE FOR NOTATION AND NUMERATION.

153. I. *Write the decimal as if it were a whole number, and then prefix as many ciphers as may be necessary to give the true name to the last significant figure; and then prefix the decimal point:*

II. *Read the decimal in terms of its lowest unit, the same as if it were a whole number.*

---

151. What is the first principle which follows from the nature of decimals? What is the second principle? What follows from this principle?

152. What is a mixed number?

153. Give the rule for Notation. Give the rule for Numeration.

EXAMPLES.

Write the following numbers decimally:

(1.) $\frac{3}{100}$ (2.) $\frac{16}{1000}$ (3.) $\frac{17}{10000}$ (4.) $\frac{32}{100}$ (5.) $\frac{165}{10000}$

(6.) $18\frac{3}{100}$ (7.) $12\frac{9}{1000}$ (8.) $16\frac{12}{1000}$ (9.) $9\frac{565}{100}$ (10.) $10\frac{121}{10}$

Write the denominators belonging to the following decimals:

(11.) .0479 (12.) .4756 (13.) .0001 (14.) .674124

(15.) .2700 (16.) .47043 (17.) .270496 (18.) .000047

Numerate and read the following:

(19.) 67.0472 (20.) 2.00498 (21.) 6.010406 (22.) 1.890470

Write the following numbers in figures, and then numerate them: also, write the denominator of each:

23. Forty-one, and three-tenths.
24. Sixteen, and three millionths.
25. Five, and nine hundredths.
26. Sixty-five, and fifteen thousandths.
27. Eighty, and three millionths.
28. Two, and three hundred millionths.
29. Four hundred, and ninety-two thousandths.
30. Three thousand, and twenty-one ten thousandths.
31. Forty-seven, and twenty-one hundred thousandths.
32. Fifteen hundred, and three millionths.
33. Thirty-nine, and six hundred and forty thousandths.
34. Three thousand, and eight hundred and forty millionths.
35. Six hundred and fifty thousandths.

154. Annexing Ciphers.

ANNEXING a cipher is placing it on the right of a number.

If a cipher is annexed to a decimal, it makes *one more decimal place;* and therefore, a cipher must also be *annexed to the denominator* (Art. 151).

The numerator and denominator will therefore have been multiplied by the same number, and consequently the value of the fraction will not be changed (Art. 119): Hence,

Annexing ciphers to a decimal does not alter its value.

We may take as an example, $.3 = \frac{3}{10}$.

If we annex a cipher to the numerator, 3, we must, at the same time, annex one to the denominator, 10; thus,

$.3 = \frac{3}{10} = \frac{30}{100} = .30$, by annexing one cipher.
$.3 = \frac{3}{10} = \frac{30}{100} = \frac{300}{1000} = .300$, by annexing two ciphers.

In like manner, any decimal number may be changed from a higher to a lower fractional unit, without altering its value.

Also, if a decimal point be placed on the right of an integral number, and ciphers be then annexed, the value will not be changed: thus, $5 = 5.0 = 5.00 = 5.000$, &c.

155. Prefixing Ciphers.

PREFIXING a cipher, is placing it on the left of a number.

By prefixing a cipher to a decimal, each decimal place is removed one place to the right; and hence, its unit is di-

---

154. When is a cipher annexed to a number? Does the annexing of ciphers to a decimal alter its value? Why not? What does three-tenths become by annexing a cipher? What, by annexing two ciphers? Three ciphers? What does 8 tenths become by annexing a cipher? By annexing two ciphers? By annexing three ciphers? What is the effect of placing a decimal point on the right of an integral number and then adding ciphers?

minished ten times (Art. 151); and the same takes place for every cipher that is prefixed: Hence,

Prefixing ciphers to a decimal fraction diminishes its value ten times for every cipher prefixed.

Take, for example, the fraction, $.2 = \frac{2}{10}$.

$.2$ becomes $.02 = \frac{02}{100}$, by prefixing one cipher,
$.2$ becomes $.002 = \frac{002}{1000}$, by prefixing two ciphers,
$.2$ becomes $.0002 = \frac{0002}{10000}$, by prefixing three ciphers.

NOTE.—See Art. 103. Which boy of the class has the unit of the fraction? Which has the tenths? Which the hundredths? Which the thousandths? &c., &c.

◆

ADDITION OF DECIMALS.

156. ADDITION OF DECIMALS is the operation of finding the sum of two or more decimal numbers.

Only units of the same kind can be added together. Therefore, in setting down decimal numbers for addition, figures having the same unit value must be placed in the same column.

The addition of decimals is then made in the same manner as that of whole numbers.

---

155. When is a cipher prefixed to a number? When prefixed to a decimal, does it increase the numerator? Does it increase the denominator? What effect, then, has it on the value of the fraction?. What does .2 become by prefixing a cipher? By prefixing two ciphers? By prefixing three? What does .07 become by prefixing a cipher? By prefixing two? By prefixing three? By prefixing four? What is the effect of moving the decimal point one place to the left? Two places? Three places? What is the effect of moving it one place to the right? Two places? Four places?

1. Find the sum of 37.04, 704.3, and .0376.

Analysis.—Place the decimal points in the same column: this brings units of the same value in the same column: then add as in whole numbers: Hence,

OPERATION.

$$\begin{array}{r} 37.04 \\ 704.3 \\ .0376 \\ \hline 741.3776 \end{array}$$

Rule.

I. *Write the numbers to be added, so that figures of the same unit value shall stand in the same column:*

II. *Add as in simple numbers, and place the decimal point in the sum, directly under those in the numbers added.*

Proof.—The same as in simple numbers.

Examples.

1. Add 4.035, 763.196, 445.3741, and 91.3754 together.

2. Add 365.103113, .76012, 1.34976, .3549, and 61.11 together.

3. 67.407 + 97.004 + 4 + .6 + .06 + .3.

4. .0007 + 1.0436 + .4 + .05 + .047.

5. .0049 + 47.0426 + 37.0410 + 360.0039 = 444.0924.

6. What is the sum of 27, 14, 49, 126, 999, .469, and .2614?

7. Add 15, 100, 67, 1, 5, 33, .467, and 24.6 together.

8. What is the sum of 99, 99, 31, .25, 60.102, .29, and 100.347?

9. Add together .7509, .0074, 69.8408, and .6109.

---

156. What is Addition of Decimals? What kinds of units may be added together? How do you set down the numbers for addition? How will the decimal points fall? How do you then add? How many decimal places do you point off in the sum?

10. Required, the sum of twenty-nine and 3 tenths, four hundred and sixty-five, and two hundred and twenty-one thousandths.

11. What is the sum of one tenth, one hundredth, and one thousandth ?

12. Find the sum of twenty-five hundredths, three hundred and sixty-five thousandths, six tenths, and nine millionths.

13. What is the sum of twenty-three millions and ten, one thousand, four hundred-thousandths, twenty-seven, nineteen-millionths, seven, and five tenths?

14. What is the sum of six millionths, four ten-thousandths, 19 hundred-thousandths, sixteen hundredths, and four tenths?

15. Find the sum of the following numbers: Sixty-nine thousand and sixty-nine thousandths, forty-seven hundred and forty-seven thousandths, eighty-five and eighty-five hundredths, six hundred and forty-nine and six hundred and forty-nine ten-thousandths.

16. A gentleman bought 6 houses, for which he paid, as follows: 1st, 2785.625 dollars; 2d, 3964.75 dollars; 3d, 5762.1875 dollars; 4th, 4960.50 dollars; 5th, 6912.375 dollars; 6th, 9156.3125 dollars: what did the six houses cost?

17. A farmer sold, at different times, the following quantities of hay: 3.75 tons, 14.165 tons, 375.16247 tons, 54.8125 tons, 18.5 tons, 21.75 tons, and 25 tons: how much hay did he sell?

18. A vessel sailed, in 9 successive days, the following distances: 240.17 miles, 315.875 miles, 87.416 miles, 195.125 miles, 269.1875 miles, 291.06 miles, 197.0106 miles, 300.47925 miles, and 200 miles: what distance did the vessel sail?

19. Add 475.62; nine hundred and twelve thousandths; four hundred and sixty thousandths; thirty-seven thousand, eight hundred and ninety-nine; one hundred and ninety-nine millionths; 176942.125, and two hundred and ninety-six ten-thousandths.

SUBTRACTION.

157. Subtraction of Decimals is the operation of finding the difference between two decimal numbers.

1. From 3.275 take .0879.

Analysis.—The subtraction is performed as in whole numbers, because the units of place in decimals have the same relative values as in whole numbers.

OPERATION.

$$\begin{array}{r} 3.2750 \\ .0879 \\ \hline 3.1871 \end{array}$$

In this example, a cipher is annexed to the minuend, to make the number of decimal places equal to the number in the subtrahend. This does not alter the value of the minuend (Art. 154): Hence,

Rule.

I. *Write the less number under the greater, so that figures of the same unit value shall fall in the same column:*

II. *Subtract as in simple numbers, and place the decimal point, in the remainder, directly under that of the subtrahend.*

Proof.—The same as in whole numbers.

Examples.

1. From 3295 take .0879.
2. From 291.10001 take 41.375.
3. From 10.000001 take .111111.
4. From 396 take 8 ten-thousandths.
5. From 1 take one thousandth.
6. From 6378 take one tenth.
7. From 365.0075 take 3 millionths.
8. From 21.004 take 97 ten-thousandths.
9. From 260.4709 take 47 ten-millionths.

---

157. What is Subtraction of Decimals? How do you set down the numbers for subtraction? How do you then subtract? How many decimal places do you point off in the remainder?

10. From 10.0302 take 19 millionths.
11. From 2.01 take 6 ten-thousandths.
12. From thirty-five thousand take thirty-five thousandths.
13. From 4262.0246 take 23.41653.
14. From 346.523120 take 219.691245943.
15. From 64.075 take .195326.
16. What is the difference between 107 and .0007 ?
17. What is the difference between 1.5 and .3785 ?
18. From 96.71 take 96.709.

◆

MULTIPLICATION.

158. Multiplication of Decimals is the operation of taking one decimal number as many times as there are units in another.

1. Multiply 3.05 by 4.102.

Analysis.—We may change the multiplicand and multiplier into common fractions, and then multiply them together; the product of the numerators will be the product of the decimals. Since each denominator contains as many ciphers as there are places in the numerator (Art. 151); and since the product of the denominators will contain as many ciphers as both the factors, it follows that the product of the numerators must have as many places of figures as there are in both factors: Hence, the following

OPERATION.

$$3.05 = 3\tfrac{5}{100} = \tfrac{305}{100}$$

$$4.102 = 4\tfrac{102}{1000} = \tfrac{4102}{1000}$$

$$\tfrac{305}{100} \times \tfrac{4102}{1004} = \tfrac{1251110}{100000}$$

$$\begin{array}{r} 3.05 \\ \underline{4.102} \\ 610 \\ 305 \\ \underline{12.20} \\ 12.51110 \end{array}$$

Rule.

Multiply as in simple numbers, and point off in the product, from the right hand, as many figures for decimals as there are decimal places in both factors; and if there be not so many in the product, supply the deficiency by prefixing ciphers.

EXAMPLES.

1. Multiply the number 3.049 by .012.

2. Multiply the number 365.491 by .001.

3. Multiply the number 496.0135 by 1.496.

4. Multiply one and one millionth by one thousandth.

5. Multiply one hundred and forty-seven millionths by one millionth.

6. Multiply three hundred, and twenty-seven hundredths by 31.

7. Multiply 31.00467 by 10.03962.

8. What is the product of five tenths by five tenths?

9. What is the product of five tenths by five thousandths?

10. Multiply 596.04 by 0.00004.

11. Multiply 38049.079 by 0.00008.

12. What will 6.29 weeks' board come to, at 2.75 dollars per week?

13. What will 61 pounds of sugar come to, at 0.234 of a dollar per pound?

14. If 12.836 dollars are paid for one barrel of flour, what will .354 barrels cost?

15. Multiply 49000 by .0049.

16. Bought 1234 oranges for 4.6 cents apiece: how much did they cost?

17. What will 375.6 pounds of coffee cost, at .125 dollar per pound?

18. If I buy 36.251 pounds of indigo at 0.029 of a dollar per pound, what will it come to?

19. Multiply $89.3421001 by .0000028.

20. Multiply $341.45 by .007.

21. What is the product of the decimal .004 by the decimal .004?

22. Multiply .007853 by .035.

23. What is the product of $26.000375 multiplied by .00007?

---

158. What is Multiplication of Decimals? What is the rule for multiplication?

CONTRACTIONS IN MULTIPLICATION.

159. Removing the decimal point one place to the right, increases the unit of each place ten times; two places, one hundred times, &c. Therefore, when a decimal number is to be multiplied by 10, 100, 1000, &c., the multiplication may be made by removing the decimal point as many places to the right as there are ciphers in the multiplier; and if there be not so many figures on the right of the decimal point, supply the deficiency by annexing ciphers.

EXAMPLES.

24. Multiply the number 6.79 by 10; by 100.
25. Multiply the number .2694 by 10; by 1000.
26. Multiply the number .075 by 100; by 100000.
27. Multiply the number 1.0049 by 10000000.

DIVISION.

160. DIVISION OF DECIMALS is the operation of finding how many times one decimal number is contained in another.

1. Let it be required to divide 1.38483 by 60.21.

ANALYSIS.—The dividend must be equal to the product of the divisor and quotient (Art. 72); and hence, it must contain as many decimal places as both of them: therefore,

There must be as many decimal places in the quotient as the number of places in the dividend exceeds the number in the divisor: Hence, the following

OPERATION.

60.21) 1.38483 (23
1.2042
18063
18063
Ans. .023

---

159. How do you multiply a decimal number by 10, 100, 1000, &c.? If there are not as many decimal figures as there are ciphers in the multiplier, what do you do?

RULE.

Divide as in simple numbers, and point off in the quotient, from the right hand, as many places for decimals as the decimal places in the dividend exceed those in the divisor; and if there are not so many, supply the deficiency by prefixing ciphers.

EXAMPLES.

1. Divide 2.3421 by 2.11.
2. Divide 12.82561 by 3.01.
3. Divide 33.66431 by 1.01.
4. Divide .010001 by .01.
5. Divide 8.2470 by .002.
6. Divide 94.0056 by .08.

7. What is the quotient of 37.57602, divided by 3; by .3; by .03; by .003; by .0003?

8. What is the quotient of 129.75896, divided by 8; by .08; by .008; by .0008; by .00008?

161. NOTES.—1. When there are more decimal places in the divisor than in the dividend, annex ciphers to the dividend until the decimal places are equal; *all the figures of the quotient will then be whole numbers.*

2. When it is necessary to continue the division further than the figures of the dividend will allow, we annex ciphers, and consider them as decimal places of the dividend.

When the division does not terminate, we annex the plus sign to the quotient, to show that the division may be continued: thus, .2 divided by .3 = .666+.

3. When any decimal number is to be divided by 10, 100, 1000, &c., the division is made by removing the decimal point *as many places to the left as there are 0's in the divisor;* and if there be not so many figures on the left of the decimal point, the deficiency is supplied by prefixing ciphers.

---

160. What is Division of Decimals? How does the number of decimal places in the dividend compare with that in the divisor and quotient? How do you determine the number of decimal places in the quotient? If the divisor contains four places and the dividend six, how many in the quotient? If the divisor contains three places and the dividend five, how many in the quotient? Give the rule for the division of decimals

9. Divide 27.69 by 10.
10. Divide 6.479 by 100.
11. Divide .056 by 1000.
12. Divide 4397.4 by 3.49.
13. Divide 1 by .0001.
14. Divide 10 by .15.
15. Divide .2 by .6.
16. Divide 37.4 by 4.5.
17. Divide 5.864 by 375.
18. Divide 1 by 475.6.
19. Divide 10 by 12.75.
20. Divide 145.764 by 100.
21. Divide 15.64 by .1.
22. Divide 16.495 by 1000.
23. Divide the number 2194.02194 by .100001.
24. Divide the number 9811.0047 by .325947.
25. Divide the number 6. by .6; by .06; by .006.
26. Divide the number 6 by .5; by .05; by .005.
27. Divide the number 7296.4135 by 9647.1895.
28. Divide the number 126.45637 by 716493.256.

162. To change a common to a decimal fraction.

The value of a fraction, is the quotient of the numerator divided by the denominator (Art. 110).

1. Reduce $\frac{5}{8}$ to a decimal.

OPERATION.

$$8\,\overline{)\,5.000}$$
$$.625$$

ANALYSIS.—If we place a decimal point after the 5, and then write any number of 0's after it, the value of the numerator will not be changed (Art. 154).

If, then, we divide by the denominator, the quotient will be the decimal number: Hence, the following

RULE.

Annex decimal ciphers to the numerator, and then divide by the denominator, pointing off as in division of decimals.

---

161. NOTES.—1. If there are more decimal places in the divisor than in the dividend, what do you do? What will the figures of the quotient then be?

2. How do you continue the division after you have brought down all the figures of the dividend? What sign do you place after the quotient? What does it show?

3. How do you divide a decimal fraction by 10, 100, 1000, &c.?

162. How do you change a common to a decimal fraction? Is the value of the fraction altered?

EXAMPLES.

1. Reduce $\frac{3}{8}$ to a decimal.
2. Reduce $\frac{15}{17}$ to a decimal.
3. Reduce $\frac{3}{35}$ to a decimal.
4. Reduce $\frac{1}{4}$ and $\frac{9}{1129}$.
5. Reduce $\frac{12}{480}$, $\frac{29}{39}$, $\frac{3}{1000}$.
6. Reduce $\frac{1}{2}$ and $\frac{5}{1785}$.
7. Reduce $\frac{314957123}{210456891}$.
8. Reduce $\frac{8}{6}$, $\frac{1375}{8436}$, $\frac{3265}{4121}$.
9. Reduce $\frac{3}{4}$ of $\frac{17}{20}$.
10. Reduce $4\frac{3}{40}$ to a decimal.
11. Reduce $8\frac{17}{125}$.
12. Reduce $\frac{7}{800}$.
13. Reduce $\frac{372}{1250}$.
14. Reduce $\frac{11}{1600}$.
15. Reduce $\frac{15}{1280}$.
16. Reduce $\frac{347}{2560}$.
17. Reduce $\frac{1}{10000}$.
18. Reduce $\frac{3476}{15625}$.

163. To change decimal to common fractions.

Write the denominator of the decimal, and reduce the fraction to its lowest terms.

EXAMPLES.

1. Reduce .04 to a common fraction.
2. Reduce .5, .25, and .125, to common fractions.
3. Reduce 4.2, .875, and .375, to common fractions.
4. Reduce 3.067 and 8.275 to common fractions.
5. What common fraction is equal to .00049 ?
6. What common fractions are equal to .3125 and .75 ?
7. What common fraction is equal to $.31\frac{1}{4}$?
8. What common fraction is equal to $.45\frac{3}{8}$?

APPLICATIONS IN THE PRECEDING RULES.

1. What is the sum of $4\frac{1}{2}$, $7\frac{5}{8}$, $21\frac{7}{16}$, and $12\frac{1}{8}$, when expressed in decimals?

2. Add $6\frac{3}{4}$, 14.375, 14.3125, $18\frac{7}{32}$, and $4627\frac{9}{14}$.

3. A merchant sold 4 parcels of cloth: the first contained 127 and 3 thousandths yards; the second, 6 and 3 tenths yards; the third, 4 and one-hundredth yards; the fourth, 90 and one millionth yards: how many yards did he sell in all?

4. A merchant buys three chests of tea: the first contains 60 and one thousandth pounds; the second, 39 and
231 ~~one~~ ten-thousandth pounds; the third, 26 and one tenth pounds: how much does he buy in all?

5. If one man can remove 5.91 cubic yards of earth in a day, how much could 19 men remove?

6. What is the cost of 8.3 yards of cloth, at 5.47 dollars per yard?

7. What will be the cost of 375 thousandths of a cord of wood, at 2 dollars a cord?

8. A farmer sells to a merchant 13.12 cords of wood at \$4.25 per cord, and 13 bushels of wheat at \$1.06 per bushel: he is to take in payment 13 yards of broadcloth at \$4.07 per yard, and the remainder in cash: how much money did he receive?

9. A gentleman, having 5456.75 dollars, not in use, bought a house for $4896\frac{7}{8}$ dollars: what was left?

10. Ascertain, by decimals, how much $3\frac{7}{16}$ dollars exceeds $2\frac{7}{8}$ dollars.

11. Multiply, decimally, $25\frac{3}{4}$ by $74\frac{2}{3}$.

12. A merchant buys $3\frac{7}{12}$ yards of cloth, at 1.25 dollar a yard: what was the cost of the cloth?

13. What will be the cost of $9\frac{3}{4}$ miles of railroad, at $45675\frac{3}{8}$ dollars a mile?

14. I gave 28 dollars to 267 persons: how much was that apiece?

15. Divide a dollar into 12 equal parts.

16. How many times will .35 of 35 be contained in .0024 of 24?

17. At .75 of a dollar a bushel, how many bushels of rye can be bought for 141 dollars?

18. Divide one millionth by one billionth.

19. Reduce $\dfrac{4.375}{\frac{2}{5}\text{ of }4\frac{1}{2}}$ to a decimal fraction.

20. Reduce $\dfrac{\frac{5}{8}\text{ of }16.125}{4\frac{7}{8}}$ to a decimal fraction.

UNITED STATES CURRENCY.

164. In the year 1792, Congress established the Decimal Currency, as the Currency of the United States.

The unit of this currency, is 1 dollar, denoted by $1: 10 dollars make 1 Eagle; one-tenth of a dollar, 1 dime; one-tenth of a dime, or one-hundredth of a dollar, 1 cent; one-tenth of a cent, or one-thousandth of a dollar, 1 mill; as shown in the following

TABLE.

10 Mills (*m*) .	make	1 Cent, .	marked	*ct.*
10 Cents . .	“	1 Dime, .	“	*d.*
10 Dimes . .	“	1 Dollar, .	“	$.
10 Dollars . .	“	1 Eagle, .	“	*E.*

Table Reversed.

Eagles.	Dollars.	Dimes.	Cents.	Mills.
			1 =	10.
		1 =	10 =	100.
	1 =	10 =	100 =	1000.
1 =	10 =	100 =	1000 =	10000.

NOTE.—Dimes are generally read in cents: thus, 4 dimes is read 40 cents; and 4 dimes and 6 cents, 46 cents.

Coins.

165. COINS, are pieces of metal, whose values are fixed by law.

The coins of the United States are of gold, silver, copper, and nickel, and are of the following denominations:

1. Gold: Eagle, double-eagle, half-eagle, three dollars, quarter-eagle, and dollar.
2. Silver: Dollar, half-dollar, quarter-dollar, dime, half-dime, and three-cent piece.
3. Copper: Cent, half-cent.
4. Nickel: Cent.

Expressing Currency decimally.

1. Express $39 and 39 cents and 7 mills, decimally.
2. Express $12 and 3 mills, decimally.
3. Express $147 and 4 cents, decimally.
4. Express $148, 4 mills, decimally.
5. Express $4, 6 mills, decimally.
6. Express $9, 6 cents, 9 mills, decimally.
7. Express $10, 13 cents, 2 mills, decimally.
8. Express 25 cents, decimally.
9. Express 18¾ cents, decimally.
10. Express 4 cents and 7 mills, decimally.
11. Read in dollars, dimes, cents, and mills, $24.185.
12. Read $135.3125; $607.437, and $0.634.

Reduction.

166. Reduction is the operation of changing the unit of a number, without altering the value of the number. We see, from the foregoing Table, that 1 unit of any denomination is equal to 10 units of the next lower.

167. To change from a greater unit to a less.

1. To change from any denomination to the next lower, multiply by 10.

2. To change from any denomination to the second lower, multiply by 100.

3. To change from any denomination to the third lower, multiply by 1000.

Note.—If there be no decimal point in the number, perform the operation by annexing ciphers. If there is a decimal point,

---

164. What is the Currency of the United States? What is the unit of United States Currency? What are its denominations? Give the Table.

165. What are coins? What metals are used in the coins of the United States? What are the gold coins? What the silver? Copper?

166. What is Reduction?

remove it as many places to the right as there are 0's in the multiplier, annexing ciphers, if necessary. Omit the $ mark, when the answer is in dimes, cents, or mills.

Examples.

1. How many dimes, in $56? In $96.47? In $1.06?
2. How many cents, in $16? In $45.625? In $1.0875?
3. How many mills, in $4.156? In $69? In $0.75?

168. **To change from a less unit to a greater.**

1. To change from any denomination to the next greater, divide by 10.

2. To change from any denomination to the second next higher, divide by 100.

3. To change from any denomination to the third next higher, divide by 1000.

Examples.

1. Reduce 45689 mills to dollars.
2. In 6794 cents, how many dollars? How many dimes?
3. How many dollars are there in 376594 cents?
4. How many cents in 47546 mills?
5. How many Eagles in 67506 mills?
6. How many dollars in 37496 mills?
7. How many dollars in 47049 mills?
8. In 8756 cents, how many dimes? How many dollars?

◆

ADDITION OF CURRENCY.

169. Since Decimal Currency follows the decimal system, it may be added, subtracted, multiplied, and divided by the rules of decimals.

---

167. How do you change a greater unit to a less?
168. How do you change a less unit to a greater?

1. John gives \$1.37½ for a pair of shoes, 25 cents for a penknife, and 12½ cents for a pencil: how much does he pay for all?

Analysis.—We place the numbers so that units of the same value may be in the same column, having reduced the half-cents to decimals. The addition is performed as in addition of decimals: Hence,

OPERATION.

$$\begin{array}{r} \$1.375 \\ .25 \\ .125 \\ \hline \$1.750 \end{array}$$

Rule.

Write the numbers so that units of the same value shall fall in the same column, and then add as in decimal fractions.

Proof.—Same as in decimals.

Examples.

1. Add \$0.047, \$6.210, \$0.47, and 3 mills.
2. Add 4 dollars, 16 cents, 87 cents, and 95 mills.
3. Add 120 dimes, 374 cents, and 956 mills.
4. Add \$67.214, \$10.049, \$6.041, \$0.271, together.
5. Add \$59.316, \$87.425, \$48.872, \$56.708, together.
6. Add \$81.053, \$67.412, \$95.376, \$87.064, together.

Applications.

1. A grocer purchased a box of candles, for 6 dollars 89 cents; a box of cheese, for 25 dollars 4 cents and 3 mills; a keg of raisins, for 1 dollar 12½ cents (or 12 cents and 5 mills); and a cask of wine, for 40 dollars 37 cents 8 mills: what did the whole cost him?

2. A farmer purchased a cow, for which he paid 30 dollars and 4 mills; a horse, for which he paid 104 dollars 60 cents and 1 mill; a wagon, for which he paid 85 dollars and 9 mills: how much did the whole cost?

---

169. How do you set down the numbers for addition? How do you add up the columns? How do you place the separating point? How do you prove addition?

3. Mr. Jones sold farmer Sykes 6 chests of tea, for $75.641; 9 yards of broadcloth, for $27.41; a plow, for $9.75, and a harness, for $19.674: what was the amount of the bill?

4. A grocer sold Mrs. Williams 18 hams, for $26.497; a bag of coffee, for $17.419; a chest of tea, for $27.047, and a firkin of butter, for $28.147: what was the amount of her bill?

5. A father bought a suit of clothes for each of his four boys: the suit of the eldest cost $15.167; of the second, $13.407; of the third, $12.75, and of the youngest, $11.047: how much did he pay in all?

6. A father has six children: to the first two he gives each 375.416 dollars; to each of the second two, $287.55; to each of the third two, $259.004: how much does he give to them all?

7. A man is indebted to A, $630.49; to B, $25; to C, 87½ cents; to D, 4 mills: how much does he owe?

8. Bought 1 gallon of molasses, at 28 cents; a half-pound of tea, for 78 cents; a piece of flannel, for 12 dollars 6 cents and 3 mills; a plow, for 8 dollars 1 cent and 1 mill; and a pair of shoes, for 1 dollar and 20 cents: what did the whole cost?

9. Bought 6 pounds of coffee, for 1 dollar 12½ cents; a wash-tub, for 75 cents 6 mills; a tray, for 26 cents 9 mills; a broom, for 27 cents; a box of soap, for 2 dollars 65 cents 7 mills; a cheese, for 2 dollars 87½ cents: what is the whole amount?

10. What is the entire cost of the following articles, viz.: 2 gallons of molasses, 57 cents; half a pound of tea, 37½ cents; 2 yards of broadcloth, $3.37½ cents; 8 yards of flannel, $9.875; two skeins of silk, 12½ cents, and 4 sticks of twist, 8½ cents?

11. A person paid $569.75 for muslin, $1256⅞ for silk, $674.12½ for calico, $962¾ for flannel, $169.1875 for thread: *what did* he pay for all?

SUBTRACTION.

170. 1. A man buys a cow for $26.37, and a calf for $4.50; how much more does he pay for the cow than for the calf?

ANALYSIS.—The operation is performed as in subtraction of decimals.

OPERATION.

$$\begin{array}{r} \$26.37 \\ \underline{4.50} \\ \$21.87 \end{array}$$

RULE.—Same as in Subtraction of Decimals.

PROOF.—The same as in Subtraction of Decimals.

EXAMPLES.

	(1.)		(2.)
From	$204.679	From	$8976.400
Take	98.714	Take	610.098
Remainder,		Remainder,	

(3.)	(4.)	(5.)
$620.000	$327.001	$2349
19.021	2.090	29.33

6. What is the difference between $6 and 1 mill? Between $9.75 and 8 mills? Between 75 cents and 6 mills? Between $87.354 and 9 mills?

7. From $107.003 take $0.479.

8. From $875.043 take $704.987.

9. From $904.273 take $859.896.

10. A man's income is $3000 a year; he spends $487.50: how much does he lay up?

11. A man purchased a yoke of oxen for $78, and a cow for $26.003: how much more did he pay for the oxen than for the cow?

12. A man buys a horse for $97.50, and gives a hundred-dollar bill: how much ought he to receive back?

13. How much must be added to $60.039, to make the sum $1005.40?

14. A man sold his house for $3005, this sum being $98.039 more than he gave for it: what did it cost him?

15. A man bought a pair of oxen for $100, and sold them again for $75.37½: did he make or lose by the bargain, and how much?

16. A man starts on a journey with $100; he spends $87.57: how much has he left?

17. How much must you add to $40.173, to make $100?

18. A merchant stocked a store, at the beginning of a year, at an expense of $10500; during the year he bought goods to the amount of $9475⅝, and sold to the amount of $14500.625; his expenses were $1267.31¼; and the goods in the store, at the close of the year, are worth $8965.459: what was his gain during the year?

19. A farmer had a horse worth $147.49. He traded him for a colt worth $35.048, and a cow worth $47\frac{5}{16}$, and received the rest in cash: what cash did he receive?

20. My house is worth $8975.034; my barn, $695.879: what is the difference of their values, and what would be gained if both were sold for $10000?

21. What is the difference between nine hundred and sixty-nine dollars eighty cents and 1 mill, and thirty-six dollars ninety-nine cents and 9 mills?

◆

MULTIPLICATION.

171. 1. A farmer sells 8 sheep for $1.25 each: how much does he receive for the whole?

Analysis.—The price of one sheep is multiplied by the number of sheep. The multiplicand is the price of one, in decimals; and the multiplier is an abstract number, denoting the number of things whose value is required. The product, however, will be the same, whichever number is used as the multiplier (Art. 51): Hence,

OPERATION.

```
$ 1.25
     8
------
$10.00
```

RULE.

Multiply and point off, as in decimal fractions.

EXAMPLES.

1. What will 8 barrels of flour cost, at $6.375 per barrel?

2. What will 55 yards of cloth come to, at 37½ cents per yard?

3. What will 300 bushels of wheat come to, at $1.25 per bushel?

4. What will 85.25 pounds of tea cost, at $1.37½ per pound?

5. What is the cost of a cask of wine containing 29¾ gallons, at $2.75 per gallon?

6. What will be the cost of 47 barrels of apples, at $1¾ per barrel?

7. What is the cost of a box of oranges, containing 450, at 2½ cents apiece?

8. What is the cost of 307 yards of linen, at 68½ cents per yard?

9. If 1 pound of butter cost 12½ cents, what will 4 firkins cost, each weighing 56 pounds?

10. At $1.33⅓ a foot, what will it cost to dig a well 78 feet deep?

11. If it cost $2.3125 to keep a horse for 1 week, what would be the cost of keeping 3 horses for $18\frac{4}{7}$ weeks?

12. A flour merchant bought 125 barrels of flour at $5⅛ each, and sold them at $5.9375 per barrel: what was the amount of gain?

13. A speculator sold 16 houses at $2463.12½ each, and received in exchange 25 lots of ground, each worth, on an average, $872¾, and the rest in cash: what cash did he receive?

14. A cloak required 6⅜ yards of cloth. What would be the cost of the cloak, if the cloth cost $3⅘ a yard, and as many yards of lining at $0.98¾ a yard, and the making cost $3.45?

DIVISION.

172. 1. Bought 9 pounds of tea for $5.85: what was the price per pound?

Analysis.—One pound will cost $\frac{1}{9}$ as much as 9 pounds. $\frac{1}{9}$ of $5.85 is found by dividing $5.85 by 9, according to the principles of Division of Decimals: Hence the

Operation.

$$9\overline{)\,5.85}$$
$$.65$$

Rule.

Arrange the numbers for division, and proceed as in division of decimals.

Proof.—The same as in division of decimals.

Examples.

1. Divide $56.16 by 16.
2. Divide $495.704 by 129.
3. Divide $12 by 200.
4. What is $\frac{1}{600}$ of $400?
5. What is $\frac{1}{61}$ of $857?
6. What is $\frac{12}{157}$ of $6578.95?

7. Paid $29.68 for 14 barrels of apples: what was the price per barrel?

8. If 27 bushels of potatoes cost $10.125, what is the price of a bushel?

9. If a man receive $29.25 for a month's work, how much is that a day, allowing 26 working days to the month?

10. A produce-dealer bought 3 barrels of eggs, each containing 150 dozen, for which he paid $63: how much did he pay a dozen?

11. A man bought a piece of cloth containing 72 yards, for which he paid $252: what did he pay per yard?

12. If $600 be equally divided among 26 persons, what will be each one's share?

13. A lady bought 17$\frac{1}{2}$ yards of silk for $15.75: what was the price per yard?

---

172. What is the rule for division of United States Money?

14. If $47.31¼ paid for the board of a family for 5¾ weeks, what was the price per week?

15. If I pay $4.50 a ton for coal, how much can I buy for $67.50?

16. At $7 a barrel, how much flour can be bought for $178.50?

17. How many pounds of tea can be bought for $6.75, at 75 cents a pound?

18. What number of barrels of apples can be bought for $47.50, at $2.37½ a barrel?

19. At 44 cents a bushel, how many bushels of oats can be bought for $14.30?

20. At 34 cents a bushel, how many barrels of apples can I buy for $13.60, allowing 2½ bushels to the barrel?

21. A farmer receives $840 for the wool of 1400 sheep: how much does each sheep produce him?

22. A merchant buys a piece of goods containing 105 yards, for which he pays $262.50; he wishes to sell it so as to make $52.50: how much must he ask per yard?

23. If 1 acre of land cost $28.75, how much can be bought for $3220?

24. Paid $40.50 for a pile of wood, at the rate of $3.37½ a cord: how much was there in the pile?

APPLICATIONS.

173. An ALIQUOT PART of a number is any exact divisor of it, whether integral or fractional. Thus, 3 months is an aliquot part of a year, being one-fourth of it; and 12½ cents is an aliquot part of 1 dollar, being one-eighth of it.

ALIQUOT PARTS OF A DOLLAR.

$1 = 100 cents.
½ of a dollar = 50 cents.
⅓ of a dollar = 33⅓ cents.
¼ of a dollar = 25 cents.
⅕ of a dollar = 20 cents.

⅛ of a dollar = 12½ cents.
⅒ of a dollar = 10 cents.
$\frac{1}{16}$ of a dollar = 6¼ cents.
$\frac{1}{20}$ of a dollar = 5 cents.
½ of a cent = 5 mills.

CASE I.

174. **To find the cost of several things, when the price of a single thing is an aliquot part of $1.**

1. What will be the cost of 69 yards of cotton sheeting at $33\frac{1}{3}$ cents a yard?

ANALYSIS.—$33\frac{1}{3}$ cents $= \frac{1}{3}$ of a dollar; 69 yards, at $1 a yard, would cost $69: at $\frac{1}{3}$ of a dollar a yard, it will cost $\frac{1}{3}$ of $69, which is $23: Hence,

OPERATION.

$$\begin{array}{r} 3\,\underline{)\,69} \\ \$23 \end{array}$$

RULE.—*Take such a part of the number of things, as the price of a single thing is of* **$1.**

EXAMPLES.

1. What will 100 cocoa-nuts cost, at $12\frac{1}{2}$ cts.$=\frac{1}{8}$ of a dollar apiece?
2. What will 250 water-melons cost, at 25 cents apiece?
3. What will 55 tops cost, at $6\frac{1}{4}$ cents each?
4. What will 650 yards of muslin cost, at $12\frac{1}{2}$ cents a yard?
5. What will 450 melons cost, at 5 cents apiece?
6. What will 6640 lemons cost, at $6\frac{1}{4}$ cents each?
7. What will 136 gallons of molasses cost, at $33\frac{1}{3}$ cents a gallon?

CASE II.

175. **To find the cost, when the price of one, and the number of things are given.**

1. What is the cost of 36 oranges at 5 cents apiece?

ANALYSIS.—Since 5 cents taken 36 times, is the same as 36 taken 5 times (Art. 51), the cost will be the product of 5 by 36, or of 36 by 5: Hence,

OPERATION.

$$\begin{array}{r} 36 \\ \underline{5} \\ \$1.80 \end{array}$$

RULE.—*Multiply the price of 1 by the number of things, or the number of things by the price of 1, and the product will be the cost.*

EXAMPLES.

1. What will 367 hats cost, at $1.12½ each?

2. What will 2479 bushels of wheat cost, at $1.50 a bushel?

3. What will 4204 yards of cloth cost, at $2.37½ a yard?

4. What will 3270 barrels of flour cost, at $8.45 a barrel?

5. What will be the cost of 3204 cheeses, at $2.39 apiece?

6. What will be the cost of 694 sheep, at $4.29 apiece?

CASE III.

176. **To find the cost of things sold by the hundred or thousand.**

1. What will be the cost of 936 feet of lumber, at 3 dollars per hundred?

ANALYSIS.—At 3 dollars a foot, the cost would be 936 × 3 = 2808 dollars; but as 3 dollars is the price of 100 feet, 2808 dollars is 100 times the cost of the lumber: therefore, if we divide 2808 dollars by 100, which we do by cutting off two of the right-hand figures (Art. 77), we shall obtain the cost.

OPERATION.

$$\begin{array}{r} 936 \\ 3 \\ \hline \$28.08 \end{array}$$

NOTE.—Had the price been so much per *thousand*, we should have divided by 1000, or cut off three of the right-hand figures.

RULE.—*Multiply the number and price together, and point off in the product, two places of decimals more than there are in both factors, when sold by the hundred, and three places more, when sold by the thousand.*

EXAMPLES.

1. What will 4280 bricks cost, at $5 per 1000?

2. What will 2673 feet of timber cost, at $2.25 per C?

3. What will be the cost of 576 feet of boards, at $10.62 per M?

---

174. What is Case I.? Give the rule.
175. What is Case II.? Give the rule.
176. What is Case III.? Give the rule.

4. What is the value of 1200 feet of lathing, at 7 dollars per M?

5. What will be the freight of 6727 pounds, from Buffalo to New York, at $2.45 per 100 pounds?

6. What will 27097 pounds of butter cost, at $28¼ per C?

CASE IV.

177. **To find the cost of articles sold by the ton of 2000 pounds, when the price of a ton is known.**

1. What is the cost of 1684 pounds of hay, at $10.50 per ton?

ANALYSIS.—The cost of 1000 pounds will be one-half the cost of 2000 pounds, or $5.25. Then, by the last case, if 1000 pounds cost $5 25, 1684 pounds will cost $8.84100.

OPERATION.

$$\begin{array}{r} 2\,)\,\underline{10.50} \\ 5.25 \\ \underline{1684} \\ \$8.84100, \textit{ Ans.} \end{array}$$

RULE.—*Divide the price by 2, and then find the cost of the quantity by the last Case.*

EXAMPLES.

1. What will 3426 pounds of plaster cost, at $3.48 per ton?

2. What is the cost of the transportation of 6742 pounds of iron from Buffalo to New York, at 7 dollars per ton?

3. What is the cost of 6527 pounds of oats, at $30.25 per ton?

4. What is the cost of 1678 pounds of coal, at $8.75 per ton?

5. What is the transportation of 37941 pounds of railroad iron from New York to Buffalo, at $7.37½ per ton?

CASE V.

178. **When the number of things is known, and their cost: to find the price of 1 thing.**

---

177. What is Case IV.? Give the rule.

1. If 7 pounds of tea cost $4.55: what is the price per pound?

Analysis.—1 pound will cost one-seventh as much as 7 pounds: one-seventh of $4.55 is 65 cents; therefore, 1 pound will cost 65 cents.

OPERATION.

7) 4.55

$0.65

Rule.—*Divide the entire cost by the number of things.*

Examples.

1. Divide $3769.25 into 50 equal parts: what is one part?

2. A farmer purchased a farm containing 725 acres, for which he paid $18306.25: what did it cost him per acre?

3. A merchant buys 15 bales of goods at auction, for which he pays $1000: what do they cost him per bale?

4. A drover pays $1250 for 500 sheep: what shall he sell them for apiece, that he may neither make nor lose by the bargain?

CASE VI.

179. **When the cost of a number of things is given, and the price of 1: to find the number.**

1. If I pay $4.50 a ton for coal, how much can I buy for $67.50?

Analysis.—As many tons as $4.50 is contained times in $67.50, which is 15.

OPERATION.

4.50) 67.50 (15 tons.

67.50

Rule.—*Divide the entire cost by the cost of 1 thing.*

Examples.

1. If 1 acre of land cost $38.75, how much can be bought for $3560?

2. How many sheep can be bought for $132, at $1.37½ a head?

3. At $4.25 a yard, how many yards of cloth can be bought for $68?

---

178. What is Case V.? Give the rule.

179. What is Case VI.? Give the rule.

Miscellaneous Examples.

1. If 12 tons of hay cost $150, what will 50 tons cost?

2. If 9 dozen spelling-books cost $7.875, what will 6 dozen cost? 8 dozen? 15¾ dozen?

3. If 75⅞ bushels of wheat cost $131.25, how much will 9¼ bushels cost? 37.375 bushels?

4. If 320 pounds of coffee cost $44.80, how much will 575 pounds cost?

5. Mr. James B. Smith bought 9 barrels of sugar, each weighing 216 pounds, for which he paid $116.64: how much did he pay a pound?

6. Mr. Wilson spent $\frac{5}{16}$ of 120 dollars for his wood, and the remainder for coal: the wood was worth $3.75 per cord, and the coal $4.37½ per ton: what quantity of each was bought?

7. A gentleman bought 1 pound of coffee for $.18¾; one pound of tea, for $.87½; one pound of butter, for $.31¼; and one bushel of potatoes, for $$\frac{7}{16}$. He gave a two-dollar bill: what change must he receive?

8. A farmer sold a yoke of oxen for $80.75; 6 cows, for $29 each; 30 sheep, at $2.50 a head; and 3 colts, one for $25, the other two for $30 apiece: what did he receive for the whole lot?

9. A merchant buys 6 bales of goods, each containing 20 pieces of broadcloth, and each piece of broadcloth contained 29 yards: the whole cost him $15660: how many yards of cloth did he purchase, and how much did it cost him per yard?

10. A person sells 3 cows, at $25 each; and a yoke of oxen for $65: he agrees to take in payment 60 sheep: how much do his sheep cost him per head?

11. A man dies, leaving an estate of $33000, to be equally divided among his 4 children, after his wife shall have taken her third. What was the wife's portion, and what the part of each child?

12. A person settling with his butcher, finds that he is

charged with 126 pounds of beef, at 9 cents per pound; 85 pounds of veal, at 6 cents per pound; 6 pair of fowls, at 37 cents a pair, and three hams, at $1.50 each: how much does he owe him?

13. A farmer agrees to furnish a merchant 40 bushels of rye, at 62 cents per bushel, and to take his pay in coffee, at 16 cents per pound: how much coffee will he receive?

14. A farmer has 6 ten-acre lots, in each of which he pastures 6 cows; each cow produces 112 pounds of butter, for which he receives 18½ cents per pound; the expenses of each cow are 5 dollars and a half: how much does he make by his dairy?

15. Bought a farm of W. N. Smith, for 2345 dollars; a span of horses, for 375 dollars; 6 cows, at 36 dollars each. I paid him 520 dollars in cash, and a village lot, worth 1500 dollars; how many dollars remain unpaid?

16. Divide .5 by .5; 12½ by ⅛; 81¼ by .8125.

17. A man leaves an estate of $1473.194, to be equally divided among 12 heirs: what is each one's portion?

18. If flour is $9.25 a barrel, how many barrels can I buy for $1637.25?

19. Bought 26 yards of cloth at $4.37½ a yard, and paid for it in flour at $7.25 a barrel: how much flour will pay for the cloth?

20. How much molasses, at 22½ cents a gallon, must be given for 46 bushels of oats, at 45 cents a bushel?

21. How many days' work, at $1.25 a day, must be given for 6 cords of wood, worth $4.12½ a cord?

22. DAVID TRUSTY, *Bought of* PETER BIGTREE,

2462	Feet of	boards,	at $7	per M.	. . . .	
4520	"	"	" 9.50	"	. . . .	
600	"	scantling,	" 11.37	"	. . . .	
900	"	timber,	" 15	"	. . . .	
1464	"	lathing,	" .75	per C.	. . . .	
1012	"	plank,	" 1.25	"	. . . .	

Received payment,

PETER BIGTREE.

23. New York, May 1, 1862.

Mr. James Spendthrift,

Bought of Benj. Saveall,

16 Pounds of tea, at $.85 per pound . . . . . .
27 " " coffee, at $.15½ per pound . . . . .
15 Yards of linen, at $.66 per yard . . . .

Amount, ... $

Received payment,
Benjamin Saveall.

---

24. Albany, June 2, 1862.

Mr. Jacob Johns,

Bought of Gideon Gould,

36 Pounds of sugar, at 9½ cents per pound . . . .
3 Hogsheads of molasses, 63 galls. each, at 27 cents a gallon . . . .
5 Casks of rice, 285 pounds each, at 5 cts. per pound
2 Chests of tea, 86 pounds each, at 96 cts. per pound

Total cost, ... $

Received payment, for Gideon Gould,
Charles Clark.

---

25. Hartford, November 21, 1862.

Gideon Jones,

Bought of Jacob Thrifty,

69 Chests of tea, at $55.65 per chest . . . . . .
126 Bags of coffee, 100 pounds each, at 12½ cents per pound . . . .
167 Boxes of raisins, at $2.75 per box . . . . .
800 Bags of almonds, at $18.50 per bag . . . . .
9004 Barrels of shad, at $7.50 per barrel . . . . .
60 Barrels of oil, 32 gallons each, at $1.08 per gall.

Amount, ... $

Received the above in full,
Jacob Thrifty.

DENOMINATE NUMBERS.

180. A Denominate Number is a denominate unit, or a collection of such units.

181. Two numbers are of the *same* denomination when they have the same unit; and of different denominations, when they have different units.

182. A Compound Denominate Number is one expressed by two or more different units; as, 3 yards 2 feet 3 inches.

183. A Scale is a series of numbers expressing the law of relation between the different units of any number.

A uniform scale is one in which the law of relation is the same at every step of the scale.

A varying scale is one in which the law of relation is different, at different steps of the scale.

The units of a scale, at any step, are denoted by the number of units in the lower denomination which make one unit of the next higher.

184. Kinds of Units.

There are eight different Units of Arithmetic:

I. Units of Abstract Number;
II. Units of Currency;
III. Units of Length;
IV. Units of Surface;
V. Units of Volume, or Capacity;
VI. Units of Weight;
VII. Units of Time;
VIII. Units of Circular Measure.

I. ABSTRACT NUMBERS.

185. An Abstract Number is one whose unit is not named.

Table.

10 Units	make	1 Ten.
10 Tens		1 Hundred.
10 Hundred		1 Thousand.
10 Thousand		1 Ten-thousand.
&c.,		&c.

Table Reversed.

						Ten.		Units.
				Hund.		1	=	10.
		Thous.		1	=	10	=	100.
Ten-thous.		1	=	10	=	100	=	1000.
1	=	10	=	100	=	1000	=	10000.

SCALE.—Uniform. Units at each step, 10.

II. CURRENCY.

I. UNITED STATES CURRENCY.

186. The United States Currency, is the Decimal Currency established by a law of Congress, in 1792.

TABLE.

10 Mills (*m.*) . . .	make 1 Cent, . . .	marked *ct.*
10 Cents	1 Dime, ...	*d.*
10 Dimes	1 Dollar, ...	$.
10 Dollars	1 Eagle, ...	*E.*

Table Reversed.

						ct.		m.
				d.		1	=	10.
		$		1	=	10	=	100.
E		1	=	10	=	100	=	1000.
1	=	10	=	100	=	1000	=	10000.

SCALE.—Uniform. Units at each step, 10.

---

180. What is a denominate number?

181. When are two numbers of the same denomination? When of different denominations?

182. What is a compound denominate number?

183. What is a scale? What is a uniform scale? What is a varying scale? What denotes the units of a scale at any step?

184. How many kinds of units are there in Arithmetic? Name them.

185. What is an abstract number? Repeat the Table. What is the scale, in Abstract Numbers? Is it uniform, or variable?

186. What is United States Currency? What are its denominations? Repeat the Table. What is the scale?

II. ENGLISH CURRENCY.

187. The English Currency, is the Currency of Great Britain.

TABLE.

4 Farthings (*far.*)	make	1 Penny,	marked *d.*
12 Pence		1 Shilling,	*s.*
20 Shillings		1 Pound, or sovereign,	£.
21 Shillings		1 Guinea.	

Table Reversed.

$$\begin{array}{rcrcrcr} & & & & \text{d.} & & \text{far.} \\ & & \text{s.} & & 1 & = & 4. \\ \pounds & & 1 & = & 12 & = & 48. \\ 1 & = & 20 & = & 240 & = & 960. \end{array}$$

SCALE.—1. The units of the scale, beginning at the lowest, are 4, 12, and 20. When the scale is read from the lowest unit to the highest, it is said to be *ascending*. If we begin at the highest unit, the order of reading is reversed, and the units of the scale are 20, 12, and 4. When read in this way, the scale is said to be *descending*. The connecting steps, or units of the scale, are, however, the same in both cases.

2. The scales are uniform in abstract and decimal numbers; also, in duodecimals; in all other cases they are variable.

3. Farthings are generally expressed in fractions of a penny: Thus, 1far. = $\frac{1}{4}$d.; 2far. = $\frac{1}{2}$d.; 3far. = $\frac{3}{4}$d.

4. By reading the second table from left to right, we can see the value of any unit expressed in each of the lower denominations. Thus, 1d. = 4far.; 1s. = 12d. =48far.; £1 = 20s. = 240d. = 960far.

REDUCTION.

188. REDUCTION is the operation of changing a number from one unit to another, without altering its value.

---

187. What is English Currency? What are its denominations? Repeat the Table. What are the units of the scale? Is the scale uniform, or variable?

188. What is Reduction?

189. REDUCTION DESCENDING is the operation of changing a number from a greater unit to a less.

190. REDUCTION ASCENDING is the operation of changing a number from a less unit to a greater.

191. REDUCTION DESCENDING.

1. Reduce £27 6s. 8½d., to the denomination of farthings.

ANALYSIS.—Since there are 20 shillings in £1, in £27 there are 27 times 20 shillings, or 540 shillings, and 6 shillings added, make 546s. Since 12 pence make 1 shilling, we next multiply by 12, and then add 8d. to the product, giving 6560 pence. Since 4 farthings make 1 penny, we next multiply by 4, and add 2 farthings to the product, giving 26242 farthings for the answer: Hence, the following

OPERATION.

£27 6s. 8d. 2far.
20
546s.
12
6560d.
4
26242, *Ans.*

RULE.

I. *Multiply the highest denomination by the units of the scale at that step, and add the units, if any, of the next lower denomination:*

II. *Proceed in the same manner through all the denominations, till the number is brought to the required unit.*

192. REDUCTION ASCENDING.

1. In 26242 farthings, how many pounds, shillings, and pence?

ANALYSIS.—Since 4 farthings make 1 penny, we first divide by 4. Since 12 pence make 1 shilling, we next divide by 12. Since 20 shillings make 1 pound, we next divide by 20, and find that 26242 farthings = £27 6s. 8d. 2 far.: Hence, the following

OPERATION.

4) 26242
12) 6560 . 2far. rem.
2|0) 54|6 . 8d. rem.
27 . . 6s. rem.
Ans. £27 6s. 8d. 2far.

Rule.

I. *Divide the given number by the units of the scale at that step, and set down the remainder, if there be one:*

II. *Divide the quotient by the units of the scale at the second step, and set aside the remainder; proceed in the same way, through all the denominations; and the last quotient, with the several remainders annexed, will be the answer.*

Proof.—The proof, in either case, is made by reversing the operation.

Examples.

1. Reduce £15 7s. 6d., to pence.

OPERATION.	PROOF.
£15 7s. 6d.	12) 3690
20	2|0) 30|7 . 6d. rem.
307	15 . . 7s. rem.
12	*Ans.* £15 7s. 6d.
3690	

2. In £31 8s. 9d. 3far., how many farthings? Proof.
3. In £87 14s. 8½d., how many farthings? Also proof.
4. In £407 19s. 11¾d., how many farthings?
5. In 80 guineas, how many pounds?
6. In 1549far., how many pounds, shillings, and pence?
7. In 6169 pence, how many pounds?

III. UNITS OF LENGTH.

I. LONG MEASURE.

193. This measure is used to measure distances, lengths, breadths, heights, depths, &c.

---

189. What is Reduction Descending?
190. What is Reduction Ascending?
191. What is the rule for Reduction Descending?
192. What is the rule for Reduction Ascending?
193. What is Long Measure used for? What are its Units? Repeat the Table. What are the units of the Scale?

TABLE.

12	Inches (*in.*) . . . make	1 Foot,	marked *ft.*
3	Feet	1 Yard,	*yd.*
5½	Yards, or 16½ Feet	1 Rod,	*rd.*
40	Rods	1 Furlong,	*fur.*
8	Furlongs, or 320 Rods	1 Mile,	*mi.*
3	Miles	1 League,	*L.*
69½ 60	Statute Miles (nearly), or Geographical Miles	1 Degree of the Equator,	*deg.* or °.
360	Degrees	a Circumference of the Earth.	

Table Reversed.

mi.	fur.	rd.	yd.	ft.	in.
				1 =	12.
			1 =	3 =	36.
		1 =	5½ =	16½ =	198.
	1 =	40 =	220 =	660 =	7920.
1 =	8 =	320 =	1760 =	5280 =	63360.

NOTES.—1. A fathom is a length of six feet, and is generally used to measure the depth of water. A pace is three feet.

2. A hand is 4 inches, used to measure the height of horses.

3. SCALE.—The units of the Scale, beginning at the lowest unit, are 12, 3, 5½, 40, and 8.

4. The geographical mile is equal to a minute of one of the great circles of the earth.

EXAMPLES.

1. How many inches in 6 rd. 4 yd. 2 ft. 9 in.?

OPERATION.

6 rd. 4 yd. 2 ft. 9 in.
5½
3
34
37 yards.
3
113 feet.
12
1365 inches.

2. In 1365 inches, how many rods?

OPERATION.

12) 1365
3) 113 feet 9 in.
5½) 37 yds. 2 feet.
11) 74
6 rd. 8 half yd. = 4 yd.

Ans. 6 rd. 4 yd. 2 ft. 9 in.

3. In 59 mi. 7 fur. 38 rd., how many feet?
4. In 115188 rods, how many miles?
5. In 79 mi. 16 rd. 6 yd., how many feet?
6. In 178°, how many miles?
7. In 54° 45 mi. 7 fur. 20 rd. 4 yd. 2 ft. 10 in., how many inches?
8. In 481401716 inches, how many degrees, &c.?
9. If a river is 65 fathoms 5 feet 4 inches deep, what is its depth in inches? what is its height?
10. If a horse is 15½ hands high, how much in feet and inches?

II. SURVEYORS' MEASURE.

194. The Surveyors' or Gunters' Chain is generally used in surveying land. It is 4 rods, or 66 feet in length, and is divided into 100 links.

Table.

7.92 Inches	make	1 Link,	marked	*l.*
100 Links, or 66 feet,		1 Chain,		*c.*
80 Chains		1 Mile,		*mi.*

Table Reversed.

m.		c.		ft.		l.		in.
						1	=	7.92.
		1	=	66	=	100	=	792.
1	=	80	=	5280	=	8000	=	63360.

Scale.—The units of the Scale, beginning at the lowest unit, are 7.92, 100, and 80.

Examples.

1. In 560 l., how many chains?
2. In 40 c. 65 l., how many feet and inches?
3. A field, regular in form, is 65 c. 15 l. long, and 21 c. 14 l. broad: what is the distance around it?

---

194. What chain is used in land-surveying? What is its length? How is it divided? Repeat the Table. What are the units of the Scale?

III. CLOTH MEASURE.

195. CLOTH MEASURE, is used for measuring all kinds of cloth, ribbons, and other things sold by the yard.

TABLE.

2¼	Inches (*in.*) .	make	1 Nail, . . .	marked *na.*
4	Nails		1 Quarter of a yard, .	*qr.*
8	Quarters		1 Ell Flemish, . .	*E. Fl.*
4	Quarters		1 Yard, ...	*yd.*
5	Quarters		1 Ell English, . .	*E. E.*
6	Quarters		1 Ell French, . .	*E. F.*

Table Reversed.

na. in.
qr. 1 = 2¼.
yd. 1 = 4 = 9.
E. Fl. 1 = 4 = 16 = 36.
E. E. 1 = ¾ = 3 = 12 = 27.
E. F. 1 = 1⅔ = 1¼ = 5 = 20 = 45.
1 = 1⅕ = 2 = 1½ = 6 = 24 = 54.

SCALE.—1. The units of the Scale, beginning at the lowest unit, and then reckoning from the quarter-yard, are 2¼, 4, 4, 3, 5, 6.

2. The yard of Cloth Measure, is the yard of Long Measure, and is equal to 36 inches.

EXAMPLES.

1. In 35 yards 3 qr. 3 na., how many nails?
2. In 575 nails, how many yards?
3. In 49 E. E., how many nails?
4. In 51 E. Fl. 2 qr. 3 na., how many nails?
5. In 3278 nails, how many yards?
6. In 340 nails, how many Ells Flemish?
7. In 4311 inches, how many E. E.?

---

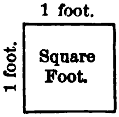
195. For what is Cloth Measure used? What are its denominations? Repeat the Table. What are the units of this measure?

IV. UNITS OF SURFACE.

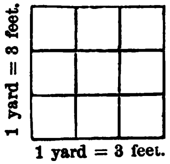
I. SQUARE MEASURE.

196. Square Measure, is used in measuring surfaces, which combine length and breadth.

The unit of this measure, is a square, constructed on the unit of length.



A square, is a figure bounded by four equal sides, at right angles to each other. If each side is one foot, the figure is called, a *square foot.*



If the sides of the square are each one yard, the square is called, a *square yard.* If two adjacent sides of the square yard be divided into feet, and through the points of division, lines be drawn parallel to the other sides, the large square will contain 9 small squares, which are square feet. Therefore, the square yard contains 9 square feet.

The number of small squares that is contained in any large square, or in any figure whose opposite sides are parallel, is always equal to the product of the length and breadth. As in the figure, 3 × 3 = 9 square feet. The number of square inches contained in a square foot, is equal to 12 × 12 = 144.

Table.

144	Square Inches (*sq. in.*) make	1	Square Foot, . marked	*sq. ft.*
9	Square Feet	1	Square Yard, ...	*sq. yd.*
$30\frac{1}{4}$	Square Yards	1	Square Rod, or Perch, .	*P.*
40	Square Rods or Perches . .	1	Rood, ...	*R*
4	Roods	1	Acre, ...	*A.*
640	Acres	1	Square Mile, ...	*M.*

---

196. For what is Square Measure used? What is the unit in this measure? What is a square? If each side is one foot, what is it called? If each side is a yard, what is it called? How many square feet does the square yard contain? How is the number of small squares contained in a large square found? Repeat the Table. What are the units of the scale?

Table Reversed.

A.	R.	P.	sq. yd.	sq. ft.	sq. in.
				1 =	144.
			1 =	9 =	1296.
		1 =	30¼ =	272¼ =	39204.
	1 =	40 =	1210 =	10890 =	1568160.
1 =	4 =	160 =	4840 =	43560 =	6272640.

SCALE.—The units of the Scale, beginning at the lowest unit, are 144, 9, 30¼, 40, and 4.

II. SURVEYORS' MEASURE.

197. Surveyors estimate the area of land in Square Miles, Acres, Roods, and Perches.

TABLE.

16 Perches . . . . make 1 Square Chain.
40 Perches, or 2½ Square Chains . . 1 Rood.
4 Roods . . . . 1 Acre.
640 Acres . . . . 1 Square Mile.

Table Reversed.

sq. mi.	A.	R.	sq. ch.	P.
			1 =	16.
		1 =	2½ =	40.
	1 =	4 =	10 =	160.
1 =	640 =	2560 =	6400 =	10240.

SCALE.—The units of the Scale, beginning at the lowest unit, are 16, 2½, 4, and 640.

EXAMPLES.

1. How many perches in 32 M. 25 A. 3 R. 19 P.?

OPERATION.

32 M. 25 A. 3 R. 19 P.
640
20505 acres.
4
82023 roods.
40
3280939 perches.

2. How many square miles, acres, &c., in 3280939 P.?

OPERATION.

40) 3280939 . 19 P.
4) 82023 . 3 R.
640) 20505 . 25 A.
32

Ans. 32 M. 25 A. 3 R. 19 P.

3. In 19 A. 2 R. 37 P., how many square rods?

4. In 175 square chains, how many square feet?

5. In 37456 square inches, how many square feet?

6. In 14972 perches, how many acres?

7. In 3674139 perches, how many square miles?

8. Mr. Wilson's farm contains 104 A. 3 R. and 19 P.; he paid for it at the rate of 75 cents a perch: what did it cost?

9. The four walls of a room are each 25 feet in length and 9 feet in height, and the ceiling is 25 feet square: how much will it cost to plaster it, at 9 cents a square yard?

V. UNITS OF VOLUME OR CAPACITY.

I. CUBIC MEASURE.

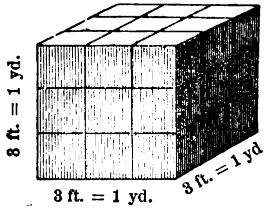
198. Cubic Measure, is used for measuring solids; as stone, timber, earth, and other things, in which the three dimensions of length, breadth, and thickness, are considered.

The unit of this measure is a cube whose edge is the unit of length.

A cube is a figure bounded by six equal squares, called *faces;* the sides of the square are called *edges.*

A cubic foot is a cube, each of whose faces is a square foot; its edges are each 1 foot.

A cubic yard is a cube, each of whose edges is 1 yard.



The base of a cube is the face on which it stands. If the edge of the cube is one yard, its base will contain 3 × 3 = 9 square feet; therefore, 9 cubic feet can be placed on the base, and hence, if the block were 1 foot thick, it would contain 9 cubic feet; if it were 2 feet thick, it would contain 2 tiers of cubes, or 18 cubic feet; if it were 3 feet thick, it would contain 27 cubic feet. Applying similar reasoning to other like solids, we conclude, that, *The contents of a body are found by multiplying the length, breadth, and thickness together.*

TABLE.

1728	Cubic Inches (*cu. in.*)	make	1 Cubic Foot,	marked *cu. ft.*
27	Cubic Feet		1 Cubic Yard,	*cu. yd.*
40	Feet of round, or		1 Ton,	*T.*
50	Feet of hewn Timber,		1 Ton,	*T.*
42	Cubic Feet		1 Ton of Shipping,	*T.*
16	Cubic Feet		1 Cord Foot,	*c. ft.*
8	Cord Feet, or		1 Cord,	*C.*
128	Cubic Feet,		1 Cord,	*C.*

Table Reversed.

Cord.	T. ship.	T. hewn T.	T. rd. T.	Cu. yd.	C. ft.	Cu. ft.	Cu. in.
						1 =	1728.
					1 =	16 =	27648.
				1 =		27 =	46656.
			1 =		2½ =	40 =	69120.
		1 =			3⅛ =	50 =	86400.
	1 =				2⅝ =	42 =	72576.
1 =					8 =	128 =	221184.

NOTES.—1. A cord of wood is a pile 4 feet wide, 4 feet high, and 8 feet long.

2. A cord foot is 1 foot in length of the pile which makes a cord.

3. A ton of *round timber*, when square, is supposed to produce 40 cubic feet; hence, *one fifth is lost by squaring.*

EXAMPLES.

1. In 15 cu. yd. 18 cu. ft. 16 cu. in., how many cubic inches?

OPERATION.

cu. yd.	cu. ft.	cu. in.
15	18	16
27		
113		
31		

$$423 \times 1728 + 16 = 730960.$$

2. In 730960 cubic inches, how many cubic yards, &c.?

OPERATION.

1728) 730960 cu. in.

27) 423 cu. ft. 16

15 cu. yd. 18

	cu. yd.	cu. ft.	cu. in.
Ans.	15	18	16

3. How many blocks, of 1 cubic inch, can be sawed from a cube of 7 feet, if there is no waste in sawing?

4. In 25 cords of wood, how many cord feet? How many cubic feet?

5. How many cords of wood in a pile 2~~8~~7 feet long, 4 feet wide, and 6 feet in height?

6. In 174964 cord feet, how many cords?

7. In 17645900 cubic inches, how many tons of hewn timber?

II. LIQUID MEASURE.

199. Liquid Measure, is used for measuring all liquids. Formerly some of them were measured by Beer Measure; but that measure is now not much used.

Table.

4	Gills (*gi.*) . . make	1	Pint, . . . marked	*pt.*
2	Pints	1	Quart, ...	*qt.*
4	Quarts.	1	Gallon, ...	*gal.*
31½	Gallons	1	Barrel, . .	*bar.* or *bbl.*
2	Barrels, or 63 Gallons	1	Hogshead, . . .	*hhd.*
2	Hogsheads	1	Pipe, ...	*pi.*
2	Pipes	1	Tun, ...	*tun.*

Table Reversed.

tun.	pi.	hhd.	bar.	gal.	qt.	pt.	gi.
						1 =	4.
					1 =	2 =	8.
				1 =	4 =	8 =	32.
			1 =	31½ =	126 =	252 =	1008.
		1 =	2 =	63 =	252 =	504 =	2016.
	1 =	2 =	4 =	126 =	504 =	1008 =	4032.
1 =	2 =	4 =	8 =	252 =	1008 =	2016 =	8064.

Note.—The *standard unit*, or gallon of Liquid Measure, in the United States, contains 231 cubic inches.

---

198. For what is Cubic Measure used? What are its denominations? What is a cord of wood? What is a cord foot? What is a cube? What is a cubic foot? What is a cubic yard? How many cubic feet is a cubic yard? What are the contents of a solid equal to? Repeat the Table. What are the units of the scale?

EXAMPLES.

1. In 5 tuns 3 hogsheads 17 gallons of wine, how many gallons?

OPERATION.

5 tuns 3 hhd. 17 gal.
4
23
63
76
139
1466 gallons.

2. In 1466 gallons, how many tuns, &c.?

OPERATION.

63) 1466
4) 23 17 gal.
5 3 hhd.

Ans. 5 tuns 3 hhd. 17 gal.

3. In 12 pipes, 1 hogshead, and 1 quart of wine, how many pints?

4. In 10584 quarts of wine, how many tuns?

5. In 201632 gills, how many tuns?

6. What will be the cost of 3 hogsheads, 1 barrel, 8 gallons, and 2 quarts of vinegar, at 4 cents a quart?

III. DRY MEASURE.

200. DRY MEASURE, is used in measuring all dry articles, such as grain, fruit, salt, coal, &c.

TABLE.

2 Pints (*pt.*) . .	make	1 Quart, . .	marked *qt.*
8 Quarts		1 Peck, ...	*pk.*
4 Pecks		1 Bushel, ...	*bu.*
36 Bushels		1 Chaldron, ...	*ch.*

Table Reversed.

ch.	bu.	pk.	qt.	pt.
			1 =	2.
		1 =	8 =	16.
	1 =	4 =	32 =	64.
1 =	36 =	144 =	1152 =	2304.

SCALE.—The units of the Scale, beginning with the lowest unit, are 2, 8, 4, and 36.

NOTES.—1. The standard bushel of the United States, is the Winchester bushel of England. It is a circular measure, 18½ inches in diameter and 8 inches deep, and contains 2150⅖ cubic inches, nearly.

2. A gallon, Dry Measure, contains 268⅘ cubic inches.

EXAMPLES.

1. How many quarts are there in 65 ch. 20 bu. 3 pk. 7 qt.?

OPERATION.

```
 65 ch. 20 bu. 3 pk. 7 qt.
 36
----
 390
197
----
2360
   4
----
9443
   8
----
75551 quarts.
```

2. How many chaldrons &c., in 75551 quarts?

OPERATION.

```
 8 ) 75551
    ------
  4 ) 9443    7 qt.
    ------
36 ) 2360     3 pk.
    ------
       65    20 bu.
```

Ans. 65 ch. 20 bu. 3 pk. 7 qt.

3. In 372 bushels, how many pints?
4. In 5 chaldrons 31 bushels, how many pecks?
5. In 17408 pints, how many bushels?
6. In 4220 pints, how many chaldrons?

VI. UNITS OF WEIGHT.

I. AVOIRDUPOIS WEIGHT.

201. By this weight all articles are weighed, except gold, silver, jewels, and liquids.

---

199. What is measured by Liquid Measure? What are its denominations? Repeat the Table. What are the units of the scale? What is the standard wine gallon?

200. What articles are measured by Dry Measure? What are its denominations? Repeat the Table. What are the units of the scale? What is the standard bushel?

TABLE.

16 Drams (*dr.*)	make	1 Ounce,	marked *oz.*
16 Ounces		1 Pound,	*lb.*
25 Pounds		1 Quarter,	*qr.*
4 Quarters		1 Hundredweight	*cwt.*
20 Hundredweight		1 Ton,	*T.*

Table Reversed.

$$\begin{array}{rrrrrr} & & & & \text{oz.} & \text{dr.} \\ & & & \text{lb.} & 1 = & 16. \\ & & \text{qr.} & 1 = & 16 = & 256. \\ & \text{cwt.} & 1 = & 25 = & 400 = & 6400. \\ \text{T.} & 1 = & 4 = & 100 = & 1600 = & 25600. \\ 1 = & 20 = & 80 = & 2000 = & 32000 = & 512000. \end{array}$$

SCALE.—The units of the Scale, beginning with the lowest unit, are 16, 16, 25, 4, and 20.

NOTES.—1. The standard Avoirdupois pound is the weight of 27.7015 cubic inches of distilled water.

2. By the old method of weighing, adopted from the English system, 112 pounds were reckoned for a hundredweight; but now, the laws of most of the States, as well as general usage, fix the hundredweight at 100 pounds.

3. A ton of coal at the mines, is reckoned at 2240 lb., but at the yards, at 2000 lb.

EXAMPLES.

1. How many pounds are there in 15 T. 8 cwt. 3 qr. 15 lb.?

OPERATION.

15 T. 8 cwt. 3 qr. 15 lb.
20
308 cwt.
4
1235 qr.
25
6180 5 lb. added.
2471 1 ten added.
30890 lb.

2. In 30890 pounds, how many tons?

OPERATION.

25) 30890
4) 1235 qr. 15 lb.
20) 308 cwt. 3 qr.
15 T. 8 cwt.

Ans. 15 T. 8 cwt. 3 qr. 15 lb.

3. In 5 T. 8 cwt. 3 qr. 24 lb. 13 oz. 14 dr., how many drams?
4. In 28 T. 4 cwt. 1 qr. 21 lb., how many ounces?
5. In 2790366 drams, how many tons?
6. In 903136 ounces, how many tons?
7. In 3124446 drams, how many tons?
8. In 93 T. 13 cwt. 3 qr. 8 lb., how many ounces?
9. In 108910592 drams, how many tons?

10. What will be the cost of 11 T. 17 cwt. 3 qr. 24 lb. of hay, at half a cent a pound? How much would that be a ton?

11. What is the cost of 2 T. 13 cwt. 3 qr. 21 lb. of beef, at 8 cents a pound? How much would that be a ton?

II. TROY WEIGHT.

202. Gold, silver, jewels, and liquids are weighed by Troy Weight.

Table.

24 Grains (*gr.*)	. make	1 Pennyweight, marked	*pwt.*
20 Pennyweights	. . .	1 Ounce, ...	*oz.*
12 Ounces	.	1 Pound, ...	*lb.*

Table Reversed.

$$\begin{array}{ccccccc} & & & & \text{pwt.} & & \text{gr.} \\ & & \text{oz.} & & 1 & = & 24. \\ \text{lb.} & & 1 & = & 20 & = & 480. \\ 1 & = & 12 & = & 240 & = & 5760. \end{array}$$

SCALE.—The units of the Scale, beginning with the lowest unit, are 24, 20, and 12.

NOTE.—The standard Troy pound, is the weight of 22.794377 cubic inches of distilled water. It is less than the pound Avoirdupois.

---

201. For what is Avoirdupois Weight used? How is the Table to be read? How can you determine, from the second Table, the value of any unit in units of the lower denominations? Name the scales.

NOTES.—1. What is the standard Avoirdupois pound?

2. What is a hundredweight by the English method? What is a hundredweight by the United States method?

EXAMPLES.

1. How many grains are there in 16 lb. 11 oz. 15 pwt. 17 gr.?

OPERATION.

```
 16 lb. 11 oz. 15 pwt. 17 gr.
 12
----
 203 ounces.
  20
----
4075 pennyweights.
  24
----
97817 grains.
```

2. In 97817 grains, how many pounds?

OPERATION.

```
24 ) 97817
     -----
  20 ) 4075  pwt.   17 gr.
       ----
    12 ) 203  oz.   15 pwt.
         ---
          13  lb.   11 oz.
```

Ans. 16 lb. 11 oz. 15 pwt. 17 gr.

3. In 25 lb. 9 oz. 20 gr., how many grains?
4. In 6490 grains, how many pounds?
5. In 148340 grains, how many pounds?
6. In 117 lb. 9 oz. 15 pwt. 18 gr., how many grains?
7. In 8794 pwt., how many pounds?
8. In 6 lb. 9 oz. 21 gr., how many grains?
9. In 1 lb. 1 oz. 10 pwt. 16 gr., how many grains?
10. A jewel weighing 2 oz. 14 pwt. 18 gr., is sold for half a dollar a grain: what is its value?

III. APOTHECARIES' WEIGHT.

203. This weight is used by apothecaries and physicians in mixing their medicines. But medicines are generally sold, in the quantity, by avoirdupois weight.

TABLE.

20	Grains (*gr.*) . .	make 1	Scruple, . . marked	℈.
3	Scruples	1	Dram, ...	ʒ.
8	Drams	1	Ounce, ...	℥.
12	Ounces	1	Pound, ...	℔.

---

202. What articles are weighed by Troy Weight? What are its denominations? Repeat the Table. What are the units of the scale? What is the standard Troy pound?

Table Reversed.

℔	℥	ʒ	℈	gr.
			1 =	20.
		1 =	3 =	60.
	1 =	8 =	24 =	480.
1 =	12 =	96 =	288 =	5760.

Scale.—The units of the Scale, beginning with the lowest unit, are 20, 3, 8, and 12.

Note.—The pound and ounce are the same as the pound and ounce in Troy weight.

Examples.

1. How many grains in 9 ℔. 8 ℥ 6 ʒ 2 ℈ 12 gr.?

OPERATION.

9 ℔. 8 ℥ 6 ʒ 2 ℈ 12 gr.
12
116 ounces.
8
934 drams.
3
2804 scruples.
20
56092 grains.

2. In 56092 grains, how many pounds?

OPERATION.

20) 56092
3) 2804 ℈ 12 gr.
8) 934 ʒ 2 ℈
12) 116 ℥ 6 ʒ
9 ℔. 8 ℥

Ans. 9 ℔. 8 ℥ 6 ʒ 2 ℈ 12 gr.

3. In 27 ℔. 9 ℥ 6 ʒ 1 ℈, how many scruples?
4. In 94 ℔. 11 ℥ 1 ʒ, how many drams?
5. In 8011 scruples, how many pounds?
6. In 9113 drams, how many pounds?
7. How many grains in 12 ℔. 9 ℥ 7 ʒ 2 ℈ 18 gr.?
8. In 73918 grains, how many pounds?

VII. UNITS OF TIME.

204. Time is a part of duration. The time in which the earth revolves on its axis is called a *day*. The time in which it goes round the sun is 365 days and 6 hours, nearly, and is called a *solar year.*

Time is divided into parts, according to the following

TABLE.

60 Seconds (*sec.*)	make	1 Minute,	marked	*m.*
60 Minutes		1 Hour,		*hr.*
24 Hours		1 Day,		*da.*
7 Days		1 Week,		*wk.*
52 Weeks (nearly)		1 Year,		*yr.*
365 Days		1 Common Year,		*yr.*
366 Days		1 Leap Year,		*yr.*
12 Calendar Months,		1 Year,		*yr.*
100 Years		1 Century,		*C.*

Table Reversed.

yr.	m.	wk.	da.	hr.	m.	sec.
					1 =	60.
				1 =	60 =	360.
			1 =	24 =	1440 =	86400.
		1 =	7 =	168 =	10080 =	604800.
1 =	12		365 =	8760 =	525600 =	31536000.
			366 =	8784 =	527040 =	31622400.

SCALE.—The units of the Scale, beginning with the lowest unit, are 60, 60, 24, 7, 52, and 12.

CALENDAR YEAR.

WINTER,	1st	Month,	January,	has	31 days.
	2d	"	February,	"	28 or 29 days.
SPRING,	3d	"	March,	"	31 days.
	4th	"	April,	"	30 days.
	5th	"	May,	"	31 days.
SUMMER,	6th	"	June,	"	30 days.
	7th	"	July,	"	31 days.
	8th	"	August,	"	31 days.
AUTUMN,	9th	"	September,	"	30 days.
	10th	"	October,	"	31 days.
	11th	"	November,	"	30 days.
WINTER,	12th	"	December,	"	31 days.
					365 days in a year.

NOTES.—1. The years are numbered from the beginning of the Christian Era. The year is divided into 12 calendar months, numbered from January: the days are numbered from the beginning of the month: hours, from 12 at night and 12 at noon.

2. The length of the solar year, is 365 da. 5 hr. 48 m. 48 sec., nearly; but it is reckoned at 365 days 6 hours.

3. Since the length of the year is computed at 365 days and 6 hours, the odd 6 hours, by accumulating for 4 years, make 1 day, so that every fourth year contains 366 days. This is called, Bissextile or Leap Year. The leap years are exactly divisible by 4: 1876, 1880, 1884, 1888, will be leap years.

4. The additional day, when it occurs, is added to the month of February, so that this month has 29 days in the leap year.

Thirty days hath September,
April, June, and November;
All the rest have thirty-one,
Excepting February, twenty-eight alone.

EXAMPLES.

1. How many seconds in 365 da. 6 hr.?

OPERATION.

365 da. 6 hr.
24
1466
730
8766
60
525960 × 60 = 31557600 sec.

2. How many days, &c., in 31557600 seconds?

OPERATION.

60) 31557600
20) 525960
24) 8766
365 6 hr.

Ans. 365 da. 6 hr.

3. If the length of the year were 365 da. 23 hr. 57 m. 39 sec., how many seconds would there be in 12 years?

---

204. What are the denominations of Time? How long is a year? How many days in a common year? How many days in a leap year? How many calendar months in a year? Name them, and the number of days in each. How many days has February in the leap year? How do you remember which of the months have 30 days, and which 31?

4. In 126230400 seconds, how many common years?
5. In 756952018 seconds, how many common years?
6. In 285290205 seconds, how many years of 365 da. 6 hr. each?
7. How many hours in any year from the 31st day of March to the 1st day of January following, neither day named being counted?

VIII. CIRCULAR MEASURE.

205. **Circular Measure**, is used in estimating latitude and longitude, and also in measuring the motions of the heavenly bodies.

The circumference of every circle is supposed to be divided into 360 equal parts, called *degrees*. Each degree is divided into 60 minutes, and each minute into 60 seconds.

Table.

60 Seconds (″)	make	1 Minute,	marked ′.
60 Minutes		1 Degree,	°.
15 Degrees		1 ~~Hour~~ Angle,	*hr. an.*
30 Degrees		1 Sign,	*s.*
12 Signs, or 360 Degrees		1 Circle,	*c.*

Table Reversed.

c.	s.	hr. an.	°	′	″
				1 =	60.
			1 =	60 =	3600.
		1 =	15 =	900 =	54000.
	1 =	2 =	30 =	1800 =	108000.
1 =	12 =	24 =	360 =	21600 =	1296000.

Examples.

1. In 5 s. 29° 25′, how many minutes?
2. In 2 circles, how many seconds?
3. In 27894 seconds, how many degrees?
4. In 32295 minutes, how many circles?
5. In 3 circles 16° 20′, how many seconds?
6. In 8 s. 16° 25″, how many seconds?
7. In 8589 seconds, how many degrees?

MISCELLANEOUS TABLES.

COUNTING.

12 Units, or things,	make	1 Dozen.
12 Dozen		1 Gross.
12 Gross		1 Great Gross.
20 Things		1 Score.

LENGTH.

18 Inches	1 Cubit.
22 Inches, nearly,	1 Sacred Cubit.

WEIGHT.

100 Pounds	1 Quintal of fish.
196 Pounds	1 Barrel of flour.
200 Pounds	1 Barrel of pork.
14 Pounds of Iron or Lead	1 Stone.
21½ Stones	1 Pig.
8 Pigs	1 Fother.

PAPER.

24 Sheets	1 Quire.
20 Quires	1 Ream.
2 Reams	1 Bundle.
2 Bundles	1 Bale.

BOOKS.

The terms, *folio, quarto, octavo, duodecimo,* &c., indicate the number of leaves into which a sheet of paper is folded.

A sheet folded in 2 leaves, is called,	a folio.
A sheet folded in 4 leaves,	a quarto, or 4to.
A sheet folded in 8 leaves,	an octavo, or 8vo.
A sheet folded in 12 leaves,	a 12mo.
A sheet folded in 16 leaves,	a 16mo.
A sheet folded in 18 leaves,	an 18mo.
A sheet folded in 24 leaves,	a 24mo.
A sheet folded in 32 leaves,	a 32mo.

---

205. For what is Circular Measure used? How is every circle supposed to be divided? Repeat the Table.

MISCELLANEOUS EXAMPLES.

1. How many square inches are there in a floor that is 24 feet long and 18 feet wide?

2. A ceiling is 24 ft. long, and 18 ft. wide: what would be the cost of plastering it, at 50 cents a square yard?

3. How many solid feet in a pile of wood 28 feet long, 4 feet wide, and 6 feet in height? How many cords?

4. A block of marble is 9 feet long, 2½ feet wide, 1¼ ft. thick. how many cubic feet does it contain?

5. A room is 16 feet long, 11 feet wide, and 10 feet high: how many gallons of air does it contain?

6. What is the cost of a pile of wood that is 82 feet long, 18 feet high, and 4 feet thick, at $3.75 per cord?

7. A pile of bricks, solid, is 14 feet long, 8 feet wide, and 12 feet high: how many bricks are there in the pile, a brick being 8 inches long, 4 inches wide, and 2 inches thick?

8. How many bottles, holding 1½ pints, would be required to contain a hogshead of wine?

9. How many pails of water, each holding 3 gallons 2 quarts, can be drawn out of a cistern containing 12 hogsheads 24 gallons 2 quarts?

10. In 372¾ bushels, how many pints?

11. A grocer bought 2 bushels of peanuts at $3 a bushel, and sold them at 4 cents a pint: did he make or lose, and how much?

12. In 4044896 square inches of land, how many acres, and what is its value, at $75.18¾ per acre?

13. How many acres are there in 250 city lots of ground, each of which is 25 feet by 100?

14. In 6 years (of 52¼ weeks each), 32 wk. 6 da. 17 hr., how many hours?

15. In 811480″, how many signs?

16. In 2654208 cubic inches, how many cords?

17. In 18 tons of round timber, how many cubic inches?

18. In 84 chaldrons of coal, how many bags containing 3½ pecks?

19. In 302 els English, how many yards?

20. In 24 hhd. 18 gal. 2 qt. of molasses, how many gills?

21. In 76 A. 1 R. 8 P., how many square inches?

22. In 445577 feet, how many miles?

23. In 37444325 square inches, how many acres?

24. How many times will a wheel, 16 feet and 6 inches in circumference, turn round in a distance of 84 miles?

25. What will 28 rods 129 square feet of land cost, at $15 a square foot?

26. What will be the cost of a pile of wood, 36 feet long, 6 feet high, and 4 feet wide, at 50 cents a cord foot?

27. A man has a journey to perform of 288 miles. He travels the distance in 12 days, traveling 6 hours each day: at what rate does he travel per hour?

28. How many yards of carpeting, 1 yard wide, will carpet a room 18 feet by 20?

29. If the number of inhabitants in the United States were 24 millions, how long would it take a person to count them, counting at the rate of 100 a minute?

30. A merchant wishes to bottle a cask of wine containing 126 gallons, in bottles containing 1¾ pint each: how many bottles are necessary?

31. From a cube of wood, 4 feet each way, how many small cubes, of 1 inch each way, can be sawed from it, allowing no waste in sawing?

32. A merchant wishes to ship 285 bushels of flax-seed in casks containing 7 bushels 2 pecks each: what number of casks are required?

33. The square measure of a floor is 648 sq. ft., and its width is 18 ft.: what is the length?

34. The square measure of a board is 9 sq. feet, and its length is 12 feet: what is its width?

35. A cellar, 34 feet by 25 feet, and 8 feet deep, is to be dug. Supposing that 3 cart-loads of earth make 1 solid yard, how many loads of earth would be carted?

36. How many bricks will pave a sidewalk 25 ft. by 10, the dimensions of a brick being 8 in., 4 in., and 2 in.?

DENOMINATE FRACTIONS.

206. A DENOMINATE FRACTION is one in which the unit of the fraction is a denominate number. Thus, $\frac{5}{9}$ of a yard is a denominate fraction.

207. REDUCTION of denominate fractions is the operation of changing a fraction from one unit to another, without altering its value.

CASE I.

208. To change from a greater unit to a less.

1. In $\frac{5}{9}$ of a yard, how many inches?

ANALYSIS.—Since yards are reduced to feet by multiplying by 3, and feet are reduced to inches by multiplying by 12, if $\frac{5}{9}$ be multiplied by 3 and 12, the product will be inches: Hence,

OPERATION.

$$\frac{5}{\underset{\cancel{3}}{\cancel{9}}}\times\cancel{3}\times\overset{4}{\cancel{12}} = 20 \text{ in.}$$

RULE.—*Multiply the fraction by the units of the scale till you reach the required unit.*

EXAMPLES.

2. Reduce $\frac{7}{12}$ of a £ to the fraction of a penny.
3. Reduce $\frac{3}{4800}$ of a £ to the fraction of a farthing.
4. Reduce $\frac{3}{76}$ of an Ell Eng. to the fraction of a nail.
5. Reduce $\frac{3}{648}$ of a hogshead to the fraction of a quart.
6. Reduce $\frac{4}{320}$ of a bushel to the fraction of a pint.
7. Reduce $\frac{1}{8640}$ of a pound Troy to the fraction of a grain.
8. Reduce $\frac{14}{25600}$ of a cwt. to the fraction of an ounce.
9. Reduce .125 cwt. to the decimal of an ounce.
10. Reduce .3125 of a mile to the decimal of an inch.
11. Change .1875 of a day to minutes.
12. Reduce .29763 of a degree to the decimal of a second.
13. Reduce .1723 lb. to the decimal of a grain.
14. Reduce 2.333 £ to pence.

CASE II.

209. **To change from a less unit to a greater.**

1. In 20 inches, how many yards.

Analysis.—Inches are reduced to yards, by dividing by 12 and 3 in succession. To divide by 12 and 3, is the same as multiplying by $\frac{1}{12}$ and $\frac{1}{3}$; therefore, $\frac{20}{1}\times\frac{1}{12}\times\frac{1}{3}=\frac{5}{9}$ yd.

OPERATION.

$$\frac{\overset{5}{\cancel{20}}}{1}\times\frac{1}{\underset{3}{\cancel{12}}}\times\frac{1}{3}=\frac{5}{9}\text{ yd.}$$

Rule.—*Divide the fraction by the units of the scale, in succession, till the required unit is reached.*

Examples.

2. Reduce $\frac{4}{5}$ of a gallon to the fraction of a hogshead.
3. Reduce $\frac{5}{9}$ of a nail to the fraction of a yard.
4. Reduce $\frac{1}{3}$ of $\frac{4}{5}$ of a foot to the fraction of a mile.
5. Reduce $\frac{7}{8}$ of $3\frac{1}{3}$ pwt. to the fraction of a pound Troy.
6. Reduce .3125 pt. to the decimal of a gallon.

Note.—Divide by the units of the scale, and observe the rule for pointing in division of decimals.

7. Change .89725 oz. to the fraction of a cwt.
8. Change .9825 of a penny to the decimal of a pound.
9. Reduce 6.875 seconds to the decimal of a day.
10. What decimal of a yard are 27.9175 nails?
11. To what decimal of a mile are 262.318125 feet equal?
12. Reduce .009375 pt. to the decimal of a bushel.
13. Reduce 7 drams to the decimal of a lb. avoirdupois.
14. Reduce .056 of a perch to the decimal of an acre.
15. Reduce 14 minutes to the decimal of a day.
16. Reduce 21 pints to the decimal of a peck.
17. Reduce .375678 feet to the decimal of a mile.
18. Reduce .5 quarts to the decimal of a barrel.

---

206. What is a denominate fraction?
207. What is Reduction of Denominate Fractions?
208. What is Case I.? Give the rule.
209. How do you change from a less unit to a greater?

CASE III.

210. To find the value of a common fraction in integers of lower denominations.

1. What is the value of $\frac{4}{5}$ of a pound Troy?

OPERATION.

```
Numer.      4
           12        oz.  pwt.
Denom. 5 ) 48 ( 9    12, Ans.
           45
           --
            3
           20
           --
       5 ) 60
           60
```

Analysis.—$\frac{4}{5}$ of a pound, reduced to the fraction of an ounce, is $\frac{4}{5} \times 12 = \frac{48}{5}$ of an ounce, which is equal to $9\frac{3}{5}$ ounces: $\frac{3}{5}$ of an ounce, reduced to the fraction of a pennyweight, is $\frac{3}{5} \times 20 = \frac{60}{5}$ of a pwt., or 12 pwt.

Rule.

Multiply the numerator of the fraction by the units of the scale, and divide the product by the denominator; if there is a remainder, treat it in the same way, till the required denomination is reached. The quotients of the several operations will form the answer.

Examples.

1. What is the value of $\frac{7}{8}$ of a tun of wine?
2. What is the value of $\frac{9}{10}$ of a yard?
3. What is the value of $\frac{4}{5}$ of a month?
4. What is the value of $\frac{3}{8}$ of a chaldron?
5. What is the value of $\frac{7}{9}$ of a mile?
6. What is the value of $\frac{5}{32}$ of a ton?
7. What is the value of $\frac{6}{7}$ of 3 days?
8. What is the value of $\frac{1}{2}$ of $\frac{3}{4}$ of $6\frac{2}{3}$ bushels of grain?
9. What is the value of $\frac{4}{5}$ hhd.?
10. What is the value of $\frac{7}{15}$ of a cwt.?
11. What is the value of $\frac{1}{30}$ of a hogshead of wine?
12. What is the value of $\frac{7}{13}$ of an acre of land?
13. What is the value of $\frac{15}{19}$ tons?
14. What is the value of $\frac{15}{16}$ of a common year?

CASE IV.

211. **To fine the value of a decimal in integers of lower denominations.**

1. Find the value of .890625 of a bushel.

Analysis.—Multiplying the decimal by 4 (since 4 pecks make a bushel), we have 3.5625 pecks. Multiplying the new decimal by 8 (since 8 quarts make a peck), we have 4.5 quarts. Then, multiplying this last decimal by 2 (since 2 pints make a quart), and we have 1 pint.

Ans. 3 pk. 4 qt. 1 pt.

OPERATION.

$$\begin{array}{rl} .890625 & \text{bu.} \\ 4 & \\ \hline 3.562500 & \text{pk.} \\ 8 & \\ \hline 4.500000 & \text{qt.} \\ 2 & \\ \hline 1.000000 & \text{pt.} \end{array}$$

Rule.

I. *Multiply the decimal by the units of the scale, pointing off as in multiplication of decimal fractions:*

II. *Multiply the decimal part of the product as before, and continue the operations to the lowest denomination: the integers at the left, form the answer.*

Examples.

1. What is the value of .002084 lb. Troy?
2. What is the value of .625 of a cwt.?
3. What is the value of .625 of a gallon?
4. What is the value of £.3375?
5. What is the value of .3375 of a ton?
6. What is the value of .05 of an acre?
7. What is the value of .875 of a pipe of wine?
8. What is the value of .125 of a hogshead of wine?
9. What is the value of .375 of a year of 365 days?

---

210. How do you find the value of a common fraction in integers of lower denominations?

211. How do you find the value of a decimal in integers of lower denominations?

10. What is the value of .085 of a £?
11. What is the value of .86 of a cwt.?
12. What is the value of ⅜ of .86 cwt.?
13. What is the value of .82 of a day?
14. What is the value of 1.089 miles?
15. What is the value of .09375 of a pound Avoirdupois?
16. What is the value of .28493 of a year of 365 days?
17. What is the value of £1.046?
18. What is the value of £1.88?

CASE V.

212. **To reduce a compound number to a common fraction of a given denomination.**

1. Reduce 9 oz. 12 pwt. to the fraction of a pound Troy.

OPERATION.

oz.	pwt.	1 lb.
9	12	12
20		12
192	pwt.	20
		240 pwt.

$\frac{192}{240} = \frac{4}{5}$ lb. *Ans.*

Analysis.—In 9 oz. 12 pwt. there are 192 pwt. In 1 lb. there are 240 pwt. Therefore, the part of a pound is expressed by $\frac{192}{240}$: Hence,

Rule.

Reduce the compound number to the lowest denomination named in it, and divide the result by the number of units of that denomination which make 1 of the given denomination.

Examples.

1. What part of a tun of wine is 3 hhd. 31 gal. 2 qt.?
2. Reduce 3 gal. 2 qt. to the fraction of a hogshead.
3. Reduce 2 fur. 36 rd. 2 yd. to the fraction of a mile.
4. What part of a £ is 5s. 7½d.?
5. What part of a pound Troy is 10 oz. 13 pwt. 8 gr.?
6. 11 cwt. 0 qr. 12 lb. 7 oz. 1⅗ dr., is what part of a ton?

---

212. What is Case V.? Give the rule.

7. Reduce 2 R. 32 P. 8 yd. to the fraction of an acre.
8. Reduce 12s. 9d. 1⅓ far. to the fraction of a guinea.
9. What part of a cwt. is 9 tenths of a pound?
10. What part of an Ell English is 3 qr. 3 na. 1¼ in.?
11. Reduce 3° 15′ 18¾″ to the fraction of a sign.
12. Reduce 3⅞ inches to the fraction of a hand.
13. Reduce 5 yd. 2 ft. 9 inches to the fraction of a mile.

CASE VI.

213. To reduce a compound number to a decimal of a given denomination.

1. Reduce £1 4s. 9¾d. to the decimal of a £.

ANALYSIS.—Reduce the ¾d. to a decimal, and annex the result to the 9d., and we have 9.75d. Dividing 9.75d. by 12 (since 12 pence =1s.), and annexing the quotient to the 4s., we have 4.8125s. Then dividing by 20 (since 20s.=£1), and annexing the quotient to the £1, we have £1.240625:

OPERATION.

¾d. = .75d.
9¾d. = 9.75d.
12) 9.75d.
20) 4.8125s.

Ans. £1 4s. 9¾d. = 1.240625£.

RULE.

I. *If the lowest denomination contains a fraction, reduce it to a decimal, and prefix the integral part:*

II. *Then divide by the units of the scale, and annex the quotient as a decimal to the next higher denomination, and so on until the decimal is reduced to the required denomination.*

EXAMPLES.

1. Reduce 4 wk. 6 da. 5 hr. 30 m. 45 s. to the decimal of a week.

2. Reduce 2 lb. 5 oz. 12 pwt. 16 gr. to the decimal of a pound.

3. Reduce 3 feet 9 inches to the decimal of yards.

---

213. What is Case VI.? What is the rule?

4. Reduce 1 lb. 12 dr., avoirdupois, to the decimal of pounds.
5. Reduce 5 leagues 2 furlongs to the decimal of leagues.
6. Reduce 4 bu. 3 pk. 1 pt. to the decimal of bushels.
7. Reduce 5 oz. 13 pwt. 12 gr. to the decimal of a pound.
8. Reduce 15 cwt. 3 qr. 2½ lb. to the decimal of a ton.
9. Reduce 5 A. 3 R. 21 sq. rd. to the decimal of acres.
10. Reduce 11 pounds to the decimal of a ton.
11. Reduce 3 da. 12⅜ sec. to the decimal of a week.
12. Reduce 14 bu. 3⅜ qt. to the decimal of a chaldron.
13. Reduce 7 mi. 7 fur. 1 rd. to the decimal of miles.
14. Reduce 15s. 6d. 3.375 far. to the decimal of a pound.
15. Reduce 4° 36′.8125 to the decimal of a sign.

ADDITION.

214. ADDITION OF COMPOUND NUMBERS is the operation of finding a number equal to two or more given numbers.

1. How many pounds, shillings, and pence are there in £4 8s. 9d., £27 14s. 11d., and £156 17s. 10d.?

OPERATION.

£	s.	d.
4	8	9
27	14	11
156	17	10
£189	1s.	6d.

ANALYSIS.—Having written the numbers, add the column of pence; then 30 pence are equal to 2 shillings and 6 pence: write down the 6, carrying the 2 to the shillings. Find the sum of the shillings, which is 41; that is, 2 pounds and 1 shilling over. Write down 1s.; then, carrying the 2 to the column of pounds, we find their sum to be £189 1s. 6d.

NOTE.—In simple numbers, the number of units of the scale, at any place, is 10. Hence, we carry 1 for every 10. In denominate numbers, the scales vary. The number of units, in passing from pence to shillings, is 12; hence, we carry one for every 12. In passing from shillings to pounds, it is 20; hence, we carry one for every 20. In passing from one denomination to another, we *divide the sum of each column by the units of the scale, and add the quotient to the next column.* Hence,

RULE.

I. *Write the numbers to be added, so that units of the same name shall stand in the same column:*

II. *Beginning with the lowest denomination, add as in simple numbers; divide the sum of each column by the units of the scale, at that place, and add the quotient to the next column.*

PROOF.—The same as in simple numbers.

EXAMPLES.

(1.)

£	s.	d.
173	13	5
87	17	$7\frac{3}{4}$
75	18	$7\frac{1}{2}$
25	17	$8\frac{1}{4}$
10	10	$10\frac{1}{2}$
373	18	3

(2.)

lb.	oz.	pwt.	gr.
171	6	13	$14\frac{1}{3}$
391	11	9	12
230	6	6	13
94	7	3	$18\frac{2}{7}$
42	10	15	20

(3.)

℔	℥	ʒ	℈	gr.
24	7	2	1	16
17	11	7	2	19
36	6	5	0	17
15	9	7	1	13
9	3	4	1	9

(4.)

T.	cwt.	qr.	lb.	oz.
15	12	1	10	$10\frac{1}{2}$
71	8	2	6	0
83	19	3	15	15
36	7	0	20	$14\frac{3}{8}$
47	11	2	2	11

(5.)

tun.	pi.	hhd.	gal.	qt.
14	2	1	27	3
15	1	2	25	2
4	2	1	27	11
5	0	1	62	3
7	1	2	21	2

(6.)

ch.	bu.	pk.	qt.	pt.
27	25	3	7	1
59	21	2	6	3
2	1	2	7	11
5	9	1	8	2
44	7	3	5	1

---

214. What is Addition of Compound Numbers? How do you set down the numbers for addition? How do you add? What is the rule for addition? How do you prove addition?

(7.)

yr.	mo.	wk.	da.	hr.
4	11	3	6	$20\frac{5}{6}$
3	10	2	5	$21\frac{2}{5}$
5	8	1	4	$19\frac{4}{15}$
101	9	3	7	23
55	8	4	6	$17\frac{1}{20}$

(8.)

s.	°	′	″
5	17	36	29
7	25	41	21
8	15	16	09
3	12	08	10
4	17	50	40

Applications.

1. Add 46 lb. 9 oz. 15 pwt. 16 gr., 87 lb. 10 oz. 6 pwt. 14 gr., 100 lb. 10 oz. 10 pwt. 10 gr., and 56 lb. 3 pwt. 6 gr. together.

(2.)

L.	mi.	fur.	rd.	yd.	ft.
16	2	7	39	9	$2\frac{1}{9}$
327	1	2	20	7	$1\frac{3}{4}$
87	0	1	15	6	$1\frac{1}{3}$
1	1	1	1	2	$2\frac{5}{6}$

(3.)

E. Fl.	qr.	na.
126	4	4
65	3	1
72	1	3
157	2	3

(4.)

cu. yd.	cu. ft.	cu. in.
65	25	1129
37	26	132
50	1	1064
22	19	17

(5.)

M.	A.	R.	P.	sq. yd.
2	60	3	37	25
6	375	2	25	21
7	450	1	31	20
11	30	0	25	19

6. What is the weight of forty-six pounds, eight ounces, thirteen pennyweights, fourteen grains; ninety-seven pounds, three ounces; and one hundred pounds, five ounces, ten pennyweights, and thirteen grains?

7. Add the following together: 29 T. 16 cwt. 1 qr. 14 lb. 12 oz. 9 dr., 18 cwt. 3 qr. 1 lb., 50 T. 3 qr. 4 oz., and 2 T. 1 qr. 14 dr.

8. What is the weight of 39 T. 10 cwt. 2 qr. 2 lb. 15 oz. 12 dr., 17 cwt. 6 lb., 12 cwt. 3 qr., and 2 qr. 8 lb. 9 dr.?

9. What is the sum of the following: 314 A. 2 R. 39 P. 200 sq. ft. 136 sq. in., 16 A. 1 R. 20 P. 10 sq. ft., 3 R. 36 P., and 4 A. 1 R. 16 P.?

10. Find the contents of 64 T. 33 ft. 800 cu. in., 9 T. 1200 cu. in., 25 ft. 700 cu. in., and 95 T. 31 ft. 1500 cu. in. of round timber.

11. Add together, 96 bu. 3 pk. 2 qt. 1 pt., 46 bu. 3 pk. 1 qt. 1 pt., 2 pk. 1 qt. 1 pt., and 23 bu. 3 pk. 4 qt. 1 pt.

12. What is the area of the four following pieces of land: the first containing 20 A. 3 R. 15 P. 250 sq. ft. 116 sq. in.; the second, 19 A. 1 R. 39 P.; the third, 2 R. 10 P. 60 sq. ft.; and the fourth, 5 A. 6 P. 50 sq. in.?

13. A farmer raised from one field, 37 bu. 1 pk. 3 qt. of wheat; from a second, 41 bu. 2 pk. 5 qt. of barley; from a third, 35 bu. 1 pk. 3 qt. of rye; from a fourth, 43 bu. 3 pk. 1 qt. of oats: how much grain did he raise in all?

14. A grocer received an invoice of 4 hhd. of sugar: the first weighed 11 cwt. 15 lb.; the second, 12 cwt. 3 qr. 15 lb.; the third, 9 cwt. 1 qr. 16 lb.; the fourth, 12 cwt. 1 qr.: how much did the four weigh?

15. A lady purchased 32 yd. 3 qr. of sheeting; 31 yd. 1 qr. of shirting; 14 yd. 2 qr. of linen; and 6 yd. 2 qr. of cambric: what was the whole number of yards purchased?

16. Purchased a silver teapot, weighing 23 oz. 17 pwt. 11 gr.; a sugar-bowl, weighing 8 oz. 13 pwt. 19 gr.; a cream pitcher, weighing 5 oz. 11 gr.: what was the weight of the whole?

17. A stage goes one day, 87 mi. 6 fur. 24 rd.; the next, 75 mi. 3 fur. 17 rd.; the third, 80 mi. 7 fur. 10 rd.; the fourth, 78 mi. 5 fur.: how far does it go in the four days?

18. Bought three pieces of land: the first contained 17 acres 1 R. 35 P.; the second, 36 acres 2 R. 21 P.; and the third, 46 acres 0 R. 37 P.: how much land did I purchase?

ADDITION OF FRACTIONS.

215. 1. Add £$\frac{2}{3}$ to $\frac{5}{6}$s.

£$\frac{2}{3} = \frac{2}{3}$ of $\frac{20}{1} = \frac{40}{3}$ shilling.

Then, $\frac{40}{3} + \frac{5}{6} = \frac{240}{18} + \frac{15}{18} = \frac{255}{18}$s. $= \frac{85}{6}$s. = 14s. 2d.

Or, $\frac{5}{6}$s. may be reduced to the fraction of a £: thus,

$\frac{5}{6}$s. = $\frac{5}{6}$ of $\frac{1}{20}$£ = $\frac{5}{120}$ of a £ = $\frac{1}{24}$ of a £.

Then, $\frac{2}{3} + \frac{1}{24} = \frac{48}{72} + \frac{3}{72} = \frac{51}{72}$ of a £,
which, being reduced, gives 14s. 2d. *Ans.*

2. Add $\frac{3}{5}$ of a year, $\frac{1}{3}$ of a week, and $\frac{1}{8}$ of a day.

$\frac{3}{5}$ of a year = $\frac{3}{5}$ of $\frac{365}{1}$ days	=	31 wk.	2 da.	
$\frac{1}{3}$ of a week = $\frac{1}{3}$ of 7 days	=	. .	2 da.	8 hr.
$\frac{1}{8}$ of a day	=	. . . .		3 hr.
	Ans.	31 wk.	4 da.	11 hr.

Rule.

Reduce the fractions to the same unit, and then add as in simple fractions:

Or: *Reduce the fractions, separately, to integers of lower denominations, and then add as in denominate numbers.*

Examples.

1. Add $\frac{4}{7}$ of a ton and $\frac{9}{10}$ of a cwt.
2. Add $\frac{2}{3}$ of a yard, $\frac{3}{4}$ of a foot, and $\frac{7}{8}$ of a mile.
3. Add $1\frac{1}{4}$ miles, $\frac{7}{10}$ of a furlong, and 30 rods.
4. Add $\frac{3}{4}$ cwt., $\frac{42}{2}$ lb., 13 oz., $\frac{1}{2}$ cwt., and 6 lb.
5. Add $5\frac{5}{6}$ days and $52\frac{5}{19}$ minutes.
6. Add £$\frac{5}{8}$, 3.75s., and .975d.
7. Add .2965 T., .8725 cwt., .63725 qr., and .1625 lb.

---

215. What is the rule for the addition of denominate fractions?

8. A tailor bought 3 pieces of cloth, containing respectively, $18\frac{3}{4}$ yards, $21\frac{2}{3}$ Ells Flemish, and $16\frac{4}{5}$ Ells English: how many yards in all?

9. Mr. Merchant bought of farmer Jones, $22\frac{1}{4}$ bushels of wheat at one time, $19\frac{5}{12}$ bushels at another, and $33\frac{5}{8}$ at another: how much did he buy in all?

10. Mr. Warren pursued a bear for three successive days; the first day he traveled $28\frac{3}{7}$ miles; the second, $33\frac{1}{14}$ miles; the third, $29\frac{1}{21}$ miles, when he overtook him: how far had he traveled?

11. Bought 3 kinds of cloth: the first contained $\frac{1}{2}$ of 3 of $\frac{5}{2}$ of $\frac{2}{6}$ yards; the second, $\frac{1}{4}$ of $\frac{4}{5}$ of 5 yards; and the third, $\frac{1}{6}$ of $\frac{3}{2}$ of $\frac{1}{3}$ yards: how much in them all?

12. Add $1\frac{1}{7}$ cwt., $17\frac{2}{3}$ lb., and $7\frac{4}{5}$ oz.

◆ ◆ ◆

SUBTRACTION.

216. SUBTRACTION OF COMPOUND NUMBERS is the operation of finding the difference between two compound numbers.

1. What is the difference between £27 16s. 8d. and £19 17s. 9d.?

OPERATION.

	20	12
£27	16s.	8d.
19	17	9
7	18	11

ANALYSIS.—We cannot take 9d. from 8d.; we therefore add to the upper number as many units as are contained in the scale, and at the same time add 1, mentally, to the next higher denomination of the subtrahend. We then say, 9 from 20, leaves 11. Then, as we cannot subtract 18 from 16, we add 20, and say, 18 from 36, leaves 18. Now, as we have taken 1 pound = 20 shillings, from the pounds, and added it to the shillings, there are but 26 pounds left. We may then say, 19 from 26, leaves 7, or 20 from 27, leaves 7. The latter is the easiest in practice. The first step is called *borrowing:* the second, *carrying:* Hence,

RULE.

I. *Set down the less number under the greater, placing units of the same value in the same column:*

II. *Begin with the lowest denomination, and subtract as in simple numbers, borrowing and carrying when necessary, according to the scale.*

PROOF.—The same as in simple numbers.

EXAMPLES.

(1.)

	A.	R.	P.
From .	18	3	28
Take .	15	2	30
Remainder,			

(2.)

	T.	cwt.	qr.	lb.
From	4	12	3	20
Take	2	18	3	1

(3.)

	lb.	oz.	pwt.	gr.
From .	273	0	0	0
Take .	97	10	18	21
Remainder,				

(4.)

	lb.	oz.	pwt.	gr
From	18	9	10	0
Take	9	10	15	20

(5.)

	T.	cwt.	qr.	lb.	oz.
From .	7	14	1	3	6
Take .	2	6	3	4	11
Remainder,					

(6.)

	cwt.	qr.	lb.	oz.	dr.
From	14	2	12	10	8
Take	6	3	16	15	3

(7.)

	tun.	hhd.	gal.	qt.	pt.
From	151	3	50	3	2
Take	27	2	54	3	2
Rem.,					

(8.)

	yr.	wk.	da.	hr.	min.	sec.
From	95	25	4	20	45	50
Take	80	30	6	23	46	56

(9.)

	mi.	fur.	rd.	yd.	ft.
From	84	7	13	2	1
Take	69	7	25	4	$2\frac{3}{4}$

(10.) 9

	A.	R.	P.	sq. yd.	sq ft.
From	145	3	35	19	$6\frac{1}{3}$
Take	98	3	39	25	$8\frac{[illegible]}{9}$

---

216. What is the rule for Subtraction of Compound Numbers?

(11.)

	£	s.	d.
From	25	17	4½
Take	19	19	9¾

(12.)

	s.	°	′	″
From	11	24	19	32⅓
Take	9	29	59	59¾

13. From 38 mo. 2 wk. 3 da. 7 hr. 10 m., take 10 mo. 3 wk. 2 da. 10 hr. 50 m.

14. From 176 yr. 8 mo. 4 wk. 4 da., take 91 yr. 9 mo. 2 wk. 6 da.

15. From £3, take 3s.

16. From 2 lb., take 20 gr. Troy.

17. From 8 ℔, take 1 ℔ 1 ℥ 2 ʒ 2 ℈.

18. From 9 T., take 1 T. 1 cwt. 2 qr. 20 lb. 15 oz. 14 dr.

19. From 3 miles, take 3 fur. 19 rd.

20. I purchased 167 lb. 8 oz. 16 pwt. 10 gr. of silver, and sold 98 lb. 10 oz. 12 pwt. 19 gr.: how much had I left?

21. I bought 19 T. 11 cwt. 2 qr. 2 lb. 12 oz. 12 dr. of old iron, and sold 17 T. 13 cwt. 2 qr. 19 lb. 14 oz. 10 dr.: what had I left?

22. I purchased 101 ℔ 11 ℥ 7 ʒ 2 ℈ 19 gr. of medicine, and sold 17 ℔ 2 ℥ 3 ʒ 1 ℈ 5 gr.: how much remained unsold?

23. From 46 yd. 1 qr. 3 na., take 42 yd. 3 qr. 1 na. 2 in.

24. Bought 7 cords of wood, and 2 cords 78 feet having been stolen, how much remains?

TIME BETWEEN DATES.

217. **To find the time between any two dates.**

1. What time elapsed between July 5th, 1848, and August 8th, 1850?

NOTE.—In the first date, the number of the year is 1848; the number of the month, 7, and the number of the day, 5. In the second date, the number of the year is 1850, the number of the month, 8, and the number of the day, 8. Hence, to find the time between two dates:

OPERATION.

yr.	mo.	da.
1850	8	8
1848	7	5
2	1	3

RULE.

Write the numbers of the earlier date under those of the later, and subtract according to the preceding Rule.

NOTES.—1. In finding the difference between dates, as in casting interest, the month is regarded as the twelfth part of a year, and as containing 30 days.

2. The civil day begins and ends at 12 o'clock at night.

2. What is the difference of time between March 2d, 1847, and July 4th, 1856?

3. What is the difference of time between April 28th, 1834, and February 3d, 1856?

4. What time elapsed between November 29th, 1836, and January 2d, 1854?

5. What time elapsed between November 8th, at 11 o'clock, A. M., 1847, and December 16th, at 4 o'clock, P. M., 1850?

ANALYSIS.—The hours are numbered from 12 at night, when the civil day begins. The *numbers* of the years, months, days, and hours, are used.

OPERATION.

yr.	mo.	da.	hr.
1850	12	16	16
1847	11	8	11
3	1	8	5

6. What time elapsed between October 9th, at 11 P. M., 1840, and February 6th, at 9 P. M., 1853?

7. Mr. Johnson was born September 6th, 1771, at 9 o'clock, A. M., and his first child November 5th, 1801, at 9 o'clock, P. M.: what was the difference of their ages?

8. The revolution commenced April 19th, 1775, and a general peace took place January 20, 1783 how long did the war continue?

9. America was discovered by Columbus, October 11, 1492: what was the length of time to July 25, 1862?

---

217. Give the rule for finding the difference between two dates. How is the month reckoned? At what time does a civil day begin?

SUBTRACTION OF FRACTIONS.

1. From $\frac{1}{2}$ of a £, take $\frac{1}{3}$ of a shilling.

$\frac{1}{3}$ of a shilling $= \frac{1}{3}$ of $\frac{1}{20}$ of a £ $= \frac{1}{60}$ of a £.

Then, $\frac{1}{2}$ £ $- \frac{1}{60}$ £ $= \frac{30}{60} - \frac{1}{60} = \frac{29}{60}$ of a £ $=$ 9s. 8d.

2. From $1\frac{3}{4}$ lb. Troy weight, take $\frac{1}{6}$ oz.

	lb.	oz.	pwt.	gr.
$1\frac{3}{4}$ lb. $= \frac{7}{4}$ lb of $\frac{12}{1}$ oz. $=$	1	9		
$\frac{1}{6}$ oz. $= \frac{1}{6}$ of $\frac{20}{1}$ of $\frac{24}{1}$ gr. $=$ 80 gr. $=$	0	0	3	8
Ans.	1	8	16	16.

Hence, the following

RULE.

Reduce the fractions to the same unit, and then subtract as in simple fractions:

Or: *Reduce the fractions, separately, to integers of lower denominations, and then subtract as in compound numbers.*

EXAMPLES.

1. From $\frac{3}{8}$ oz., take $\frac{7}{8}$ pwt.
2. From £$1\frac{1}{2}$, take $\frac{3}{4}$ of a shilling.
3. From $1\frac{1}{5}$ oz., take $\frac{7}{8}$ pwt.
4. From $\frac{1}{2}$ of a day, take $\frac{2}{3}$ of a second.
5. From $\frac{2}{3}$ of a rod, take $\frac{3}{7}$ of an inch.
6. From $\frac{4}{15}$ of a hogshead, take $\frac{6}{7}$ of a quart.
7. From $\frac{3}{5}$ oz., take $\frac{7}{8}$ pwt.
8. From $4\frac{3}{7}$ cwt., take $4\frac{9}{10}$ lb.
9. From $8\frac{6}{7}$ cwt., take $4\frac{9}{10}$ lb.
10. From $3\frac{1}{2}$ lb. Troy weight, take $\frac{1}{6}$ oz.
11. From $1\frac{1}{3}$ rods, take $\frac{3}{7}$ of an inch.
12. From $\frac{49}{72}$℔, take $\frac{7}{16}$ ℥.
13. From .69875 of a tun, take .386125 hhd.
14. From 2.9675 wk., subtract 5.96974 days.
15. From $\frac{6}{7}$ cwt. of sugar, there was taken .2125 qr.: what was the remainder worth, at 6 cents a pound?

MULTIPLICATION.

218. **Multiplication of Compound Numbers** is the operation of taking a compound number as many times as there are units in the multiplier.

A tailor has 5 pieces of cloth, each containing 6 yd. 2 qr. 3 na.: how many yards are there in all?

Analysis.—In all the pieces there are 5 times as much as there is in 1 piece. If, in 1 piece, each denomination be taken 5 times, the result will be 5 times as great as the multiplicand. Taking each denomination 5 times, we have 30 yd. 10 qr. 15 na.

OPERATION.

yd.	qr.	na.
6	2	3
		5
30	10	15
33	1	3

But, instead of writing the separate products, we begin with the lowest denomination, and say, 5 times 3 na. are 15 na.; divide by 4, the units of the scale, write down the remainder, 3 na., and reserve the quotient, 3 qr., for the next product. Then say, 5 times 2 qr. are 10 qr., to which add the 3 qr., making 13 qr. Then divide by 4, write down the remainder, 1, and reserve the quotient, 3, for the next product. Then say, 5 times 6 are 30, and 3 to carry, are 33 yards: Hence,

Rule.

I. *Write down the denominate number, and set the multiplier under the lowest denomination:*

II. *Multiply as in simple numbers, and in passing from one denomination to another, divide by the units of the scale, set down the remainder, and carry the quotient to the next product.*

Proof.—The same as in simple numbers.

Examples.

(1.)

£	s.	d.	far.
17	15	9	3
			6
106	14	10	2

(2.)

T.	cwt.	qr.	lb.	oz.
	10	0	2	12
				7
3	10	0	19	4

(3.)

m.	fur.	rd.	yd.	ft.
9	3	20	3	2
				6

(4.)

s.	°	′	″
9	9	27	35
			3

(5.)

yr.	mo.	da.	hr.
6	5	15	18
			5

(6.)

T.	cwt.	qr.	lb.	oz.	dr.
6	12	3	20	12	9
					8

7. How much sugar in 12 barrels, each containing 3 cwt. 3 qr. 2 lb.

OPERATION.

T.	cwt.	qr.	lb.
	3	3	2
			3
	11	1	6
			4
2	5	0	24

ANALYSIS.—The multiplicand is 3 cwt. 3 qr. 2 lb.; and the multiplier 12, a composite number: we therefore multiply by 3 and 4, in succession.

8. A farmer has 11 bags of corn, each containing 2 bu. 1 pk. 3 qt.: how much corn in all the bags?

9. In 7 loads of wood, each containing 1 cord and 2 cord feet, how many cords?

10. A bond was given 21st of May, 1825, and was taken up the 12th of March, 1831: what will be the product, if the time which elapsed from the date of the bond till the time it was taken up be multiplied by 3?

11. What is the weight of 1 dozen silver spoons, each weighing 3 oz. 6 pwt.?

12. What is the weight of 7 tierces of rice, each weighing 5 cwt. 2 qr. 16 lb.?

13. Bought 4 packages of medicine, each containing 3 ℔ 4 ℥ 6 ʒ 1 ℈ 16 gr.: what is the weight of all?

14. How far will a man travel in 5 days, at the rate of 24 mi. 4 fur. 4¾ rd., per day?

---

218. What is Multiplication of Compound Numbers? What is the rule?

15. How much land is there in 9 fields, each field containing 12 A. 1 R. 25 P.?

16. How many yards in 9 pieces, each 29 yd. 2 qr. 3 na.?

17. If a vessel sails 5 L. 2 mi. 6 fur. 36 rd. in one day, how far will it sail in 8 days?

18. How much water will be contained in 96 hogsheads, each containing 62 gal. 1 qt. 1 pt. 1 gi.?

19. If one spoon weighs 3 oz. 5 pwt. 15 gr., what is the weight of 120 spoons?

20. If a man travels 24 mi. 7 fur. 4 rd. in one day, how far will he go in one month of 30 days?

21. If the earth revolves 0° 15′ of space per minute of time, how far does it revolve per hour?

22. Bought 90 hhd. of sugar, each weighing 12 cwt. 2 qr. 11 lb.: what was the weight of the whole?

23. What is the cost of 18 sheep, at 5s. 9¼d. apiece?

24. How much molasses is contained in 25 hhd., each hogshead having 61 gal. 1 qt. 1 pt.?

25. How many yards of cloth in 36 pieces, each piece containing 25 yd. 3 qr.?

DIVISION.

219. Division of Compound Numbers is the operation of finding how many times one number is contained in another, when one or both are compound.

1. Divide £25 15s. 4d. by 8.

Analysis.—We first say, 8 into 25, 3 times and £1 or 20s. over. Then, after adding the 15s., we say, 8 into 35, 4 times and 3s. over. Then, reducing the 3s. to pence, and adding in the 4d., we say, 8 into 40, 5 times.

OPERATION.

8) £25 15s. 4d.

£3 4s. 5d.

2. Divide 36 bu. 3 pk. 7 qt. by 7.

Analysis.—In this example, we find that 7 is contained in 36 bushels, 5 times and 1 bushel over. Reducing this to pecks, and adding 3 pecks, gives 7 pecks, which contains 7, 1 time and no remainder. Multiplying 0 by 8 quarts, and adding, gives 7 quarts to be divided by 7: Hence, when the divisor is an abstract number, we have the following

OPERATION.

7) 36 bu. 3 pk. 7 qt. (5 bu.
35
1
4
7) 7 pk. (1 pk.
7
0
8
7) 7 (1 qt.
7

Ans. 5 bu. 1 pk. 1 qt.

RULE.

I. *Begin with the highest denomination and divide as in simple numbers:*

II. *Reduce the remainder, if any, to the next lower denomination and add in the units of that denomination, for a new dividend:*

III. *Proceed in the same manner, through all the denominations.*

Proof.—By multiplication, as in simple numbers.

Notes.—1. If the divisor is a composite number, we may divide by the factors in succession, as in simple numbers.

2. Each quotient figure has the same unit as the dividend from which it was derived.

3. If the divisor is a denominate number, reduce it and the dividend to the same unit, and then divide as in simple numbers.

EXAMPLES.

(1.)

	T.	cwt.	qr.	lb.
7)	1	19	2	12
Quotient,		5	2	16

(2.)

	A.	R.	P.
9)	113	3	25
	12	2	2

(3.)

	L.	mi.	fur.	rd.
8)	47	1	7	8
Quotient,				

(4.)

	bu.	pk.	qt.
9)	25	3	4

5. Divide 17 cwt. 0 qr. 2 lb. 6 oz. by 7.

6. Divide 49 yd. 3 qr. 3 na. by 9.

7. If a man, lifting 8 times as much as a boy, can raise 201 lb. 12 oz., how much can the boy lift?

8. If a vessel sails 25° 42′ 40″ in 10 days, how far will she sail in one day?

9. Divide 9 hhd. 28 gal. 2 qt. by 12.

10. What is the quotient of 65 bu. 1 pk. 3 qt. divided by 12?

11. In 4 equal packages of medicine, there are 13 ℔ 7 ℥ 2 ʒ 1 ℈ 4 gr.: how much is there in each package?

12. In 9 fields there are 113 A. 3 R. 25 P. of land: if the fields contain an equal amount, how much is there in each field?

13. If 15 loads of hay contain 35 T. 5 cwt., what is the weight of each load?

14. In 25 hhd. of molasses, the leakage has reduced the whole amount to 1534 gal. 1 qt. 1 pt.: if the same quantity has leaked out of each hogshead, how much will each hogshead still contain?

15. Bought 65 yards of cloth, for which I paid £72 14s. 4½d.: what did it cost per yard?

16. £1138 12s. 4d. ÷ 53.

17. 27 bu. 7 qt. ÷ 84.

18. 70 T. 17 cwt. 7 lb. ÷ 79.

19. 114 hhd. 56 gal. 1 qt. ÷ 40.

20. If, in 30 days, a man travels 746 mi. 5 fur., traveling the same distance each day, what is the length of each day's journey?

21. Suppose a man had 98 lb. 2 oz. 19 pwt. 5 gr. of silver: how much must he give to each of 7 men, if he divides it equally among them?

22. When 175 gal. 2 qt. of beer are drank in 52 weeks, how much is consumed in one week?

---

219. What is Division of Compound Numbers? Give the rule for division. How do you prove division? How do you divide, when the divisor is a composite number? What will be the unit of each quotient figure?

APPLICATIONS IN THE FOREGOING RULES.

1. A farmer has 18 lots, each of which contains 41 A. 2 R. 11 P.; these are divided among his 7 children: how much does each child have?

2. A rich man divided 168 bu. 1 pk. 6 qt. of corn, among 35 poor men: how much did each receive?

3. There are three men, the sum of whose ages is 14 times 20 yr. 5 mo. 3 wk. 6 da.: if the ages are equal, what is the age of each?

4. In sixty-three barrels of sugar, there are 7 T. 16 cwt. 3 qr. 12 lb.: how much is there in each barrel?

5. One hundred and seventy-six men consumed, in a week, 13 cwt. 2 qr. 15 lb. 6 oz. of bread: how much did each man consume in 1 day?

6. If the earth revolves on its axis 15° in 1 hour, how far does it revolve in 1 minute?

7. A farmer has a granary containing 232 bushels 3 pecks 7 quarts of wheat, and he wishes to put it in 105 bags: how much must each bag contain?

8. If 59 casks contain 44 hhd. 53 gal. 2 qt. 1 pt. of wine, what are the contents of $\frac{1}{5}$ of a cask?

9. Bought 90 hhd. of sugar, each weighing 12 cwt. 2 qr. 14 lb.: what is the value of the sugar, at $6\frac{1}{4}$ cents per lb.?

10. If 90 hogsheads of sugar weigh 56 T. 14 cwt. 3 qr. 15 lb., what is the weight of 1 hogshead?

11. If a vessel sails 30 days, at the rate of 49 mi. 6 fur. 8 rd. per day, at what rate per day does another ship sail, that performs the same voyage in 12 days?

12. Divide £18 6s. 9d. by £4 9s. 3d.

NOTE.—Reduce both quantities to pence, and then divide.

13. A steamship, in crossing the Atlantic, has a distance of 3500 miles to go: if she sails 211 mi. 4 fur. 32 rd. a day, what distance, after 15 days, has she still to sail?

14. A printer uses one sheet of paper for every 16 pages of an octavo book: how much paper will be necessary to

print 500 copies of a book containing 336 pages, allowing 2 quires of waste paper in each ream, the waste paper being estimated only on the entire reams?

15. How many barrels of sugar, containing 2 cwt. 1 qr. 15 lb., can be filled from a hogshead of 1 T. 5 cwt. 2 qr. 20 lb.?

16. A man lends his neighbor £135 6s. 8d., and takes in part payment, 4 cows, at £5 8s. apiece, also a horse, worth £50: how much remained due?

17. If a man travels 24 mi. 7 fur. 30 rd. in a day, how long will it take him to travel 200 mi. 6 fur. 18 rd.?

18. Out of a pipe of wine, a merchant draws 12 bottles, each containing 1 pint 3 gills; he then fills six 5-gallon demijohns; then he draws off 3 dozen bottles, each containing 1 quart 2 gills: how much remained in the cask?

19. If a barrel of flour costs £1 4s. 9d., how many barrels can be bought for £275 10s. 6d.?

20. Suppose a man has 246 mi. 6 fur. 36 rd. to travel in 12 days: after traveling 9 days, how far has he yet to travel?

21. A vessel arrives in port, with a cargo of 50 hogsheads of sugar, each containing 1 T. 5 cwt. 3 qr.; 40 hogsheads, each containing 18 cwt. 2 qr. 12 lb., and 75 hhd., each containing 15 cwt. 3 qr. 18 lb.: 8 merchants buy the entire cargo: what amount of sugar belongs to each?

22. A ship, with a cargo of 250 bales of cotton, each weighing 12 cwt. 2 qr. 15 lb., was overtaken by a storm, and obliged to throw overboard 60 bales; the cotton was owned in equal shares, by 5 merchants: what was the loss to each, at 25 cents per lb., and what quantity did each receive?

23. How many pieces of cloth, each containing 35 yards, will clothe a company of 48 men, if it takes 5 yd. 3 qr. 2 na for each man?

24. A merchant bought 15 pieces of cloth, 3 of which contained each 34 yd. 3 qr.; 6 contained each 37 yd. 1 qr. 3 na., and the remainder, 40 yd. 2½ qr. each: allowing 2 yd. 3 qr. for waste, how many suits, each requiring 6 yd. 1 qr. 3 na., can be made from the cloth?

LONGITUDE AND TIME.

220. The Equatorial Circumference of the Earth is divided into 360°, which are called degrees of Longitude.

221. The Sun apparently goes round the earth once in 24 hours. This time is called a *day*. Hence, in 24 hours, the sun apparently passes over 360° of longitude; and in 1 hour, over $\frac{1}{24}$ of $360° = 15°$.

222. Since the sun, in passing over 15° of longitude, requires 1 hour, or 60 minutes of time, in 1 minute of time he will pass over $\frac{1}{60}$ of $15° = \frac{15}{60}° = \frac{1}{4}° = 15'$ of longitude; and in 1 second of time, over $\frac{1}{60}$ of $15' = \frac{15}{60}' = \frac{1}{4}' = 15''$ of longitude: Hence,

15° of longitude require . . 1 hour of time;
15′ " " " . . 1 minute of time;
15″ " " " . . 1 second of time.

Hence we see, that,

1. *If the longitude, expressed in degrees, minutes, and seconds, be divided by* $15 = 3 \times 5$, *the quotients will be hours, minutes, and seconds of time.*

2. *If time, expressed in hours, minutes, and seconds, be multiplied by* $15 = 3 \times 5$, *the product will be degrees, minutes, and seconds of longitude.*

223. When the sun is on the meridian of any place, it is 12 o'clock, or noon, at that place.

Now, as the sun apparently goes from east to west, at the instant of noon, at one place, it will be *past* noon for all

---

220. How is the circumference of the earth supposed to be divided?

221. How does the sun appear to move? What is a day? How far does the sun appear to move in 1 hour?

222. How do you reduce degrees of longitude to time? How do you reduce minutes of longitude to time? How do you reduce seconds to time? How do you reduce time to longitude?

places at the east of it, and *before* noon for all places at the west. Hence, if we find the difference of time between two places, and know the exact time at one of them, the corresponding time at the other will be found by *adding* this difference to the given time, if the place be *East*, or by subtracting it, if *West*.

224. The meridian of the Observatory of Greenwich, London, is the one from which longitude is reckoned; hence, the longitude of Greenwich is 0.

Longitude is estimated: West, 180°; and East, 180°.

1. Baltimore is in longitude 76° 37′ west, and New York in longitude 74° 01′ west. When it is 12 M. at Baltimore, what is the time at New York?

ANALYSIS.—The difference of longitude is 2° 36′, and, changed to time by dividing by 15 = 3 × 5, gives 10 m. 24 sec. for the difference of time; and, as New York is east of Baltimore, the time is later, and we add:

OPERATION.

$$15 = 3 \times 5 \qquad \begin{array}{r} 76^\circ\ 37' \\ 74\ \ \ 01 \\ \hline 3\,)\ 2^\circ\ 36' \\ \hline 5\,)\ 52\text{ m.} \\ \hline 10\text{ m. } 24\text{ sec.} \end{array}$$

$$12 + 10' + 24'' = 12\text{ hr. } 10\text{ m. } 24\text{ sec.}$$

2. The longitude of New York is 74° 1′ west, and that of Philadelphia 75° 10′ west: what is the time at Philadelphia when it is 12 M. at New York?

3. The longitude of Cincinnati, Ohio, is 84° 24′ west: what is the time at Cincinnati, when it is 12 M. at New York?

4. The longitude of New Orleans is 89° 2′ west: what time is it at New Orleans, when it is 12 M. at New York?

5. The longitude of St. Louis is 90° 15′ 10″ west: what is

---

223. What is the hour when the sun is on the meridian? When the sun is on the meridian of any place, how will the time be for all places East? How for all places West? If you have the difference of time, how do you find the time at either place?

224. From what meridian is longitude reckoned? What is the longitude of this meridian? How is longitude reckoned from it?

the time at St. Louis, when it is 3 h. 25 m., P. M., at New York?

6. The longitude of Boston is 71° 4′ west, and that of New Orleans 89° 2′ west: what is the time at New Orleans, when it is 7 o'clock 12 m., A. M., at Boston?

7. The longitude of Chicago, Illinois, is 87° 30′ west: what is the time at New York, when it is 12 M. at Chicago?

225. Knowing the difference of time of two places, to find their difference of longitude.

1. Louisville, in Kentucky, is in longitude 85° 30′ west, and it is 9 o'clock, A. M., at the City of Mexico, when it is 9 hr. 54 min. 20 sec., A. M., at Louisville: what is the longitude of the City of Mexico?

ANALYSIS.—The difference of time is first obtained, which is 54 min. 20 sec. Changing this into longitude by multiplying by 15, we have 13° 35′ for the difference of longitude. The earlier time being at the City of Mexico, it must lie to the west: hence, its longitude is found by adding the difference to the longitude of Louisville.

OPERATION.

hr.	m.	sec.
9	54	20
9	00	00
	54	20
		3
2	43	00
		5
13°	35′	00″
85	30	
99°	05′	*Ans.*

2. Cincinnati is in longitude 84° 21′ west, and it is 10 o'clock, A. M., at Cincinnati, when it is 21 min. 56 sec. past 10 at Buffalo: what is the longitude of Buffalo?

3. By the chronometer, it is 5 hr. 6 min. 28 sec., P. M., at Greenwich, London, when it is 12 M. at Baltimore; Greenwich is in 0° longitude: what is that of Baltimore?

4. By the chronometer, it is 4 hr. 56 min. 4$\frac{2}{15}$ sec., P. M., at Greenwich, when it is 12 M. at New York: what is the longitude of New York?

5. A captain, at sea, finds by his chronometer, that it is 2 hr. 15 min. 30 sec., P. M., at Greenwich, when it is 12 M. on board his vessel: in what longitude is the vessel?

DUODECIMALS.

226. If the linear unit, 1 foot, be divided into 12 equal parts, each part is called an *inch*, or *prime*, and marked, ′. If a square foot be divided into 12 equal parts, each part is also called a *prime*, and marked, ′. If a cubic foot be divided into 12 equal parts, each part is likewise called a *prime*, and marked, ′. Hence, there are *three different* units that are divided; viz., 1 foot in length, 1 square foot, and 1 cubic foot. If, in each class, the primes be again divided according to the scale of twelve; the quotients are called *seconds*, and marked, ″. If the seconds be divided, in like manner, the quotients are called thirds, and marked, ‴; and so on, for higher divisions.

Hence: Duodecimals are denominate fractions in which the primary units, 1 foot in length, 1 square foot, and 1 cubic foot, are divided according to the scale of twelve.

Notes.—1. Duodecimals are chiefly used in measuring *surfaces* and *solids*.

2. The marks, ′, ″, ‴, etc., which denote the fractional units, are called, *indices*.

Table.

12^{iv} make $1''' = \frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12} = \frac{1}{1728}$ of a foot.
$12'''$ " $1'' = \frac{1}{12}$ of $\frac{1}{12} = \frac{1}{144}$ of a foot.
$12''$ " $1' = \frac{1}{12} = \frac{1}{12}$ of a foot.
$12'$ " 1 foot = 1 foot.

Note.—The units, a foot, in these fractions may be the foot in length, the square foot, or the cubic foot; and similar fractions always express parts of these units.

Scale.—Uniform and equal to 12.

---

226. How many primary units are there which are divided into equal parts? What are they? According to what scale are the divisions made? What are the parts called in the first division? What in the second? What in the third? What in the fourth? What are duodecimals?

ADDITION AND SUBTRACTION.

227. The units of Duodecimals are reduced, added, and subtracted like those of other denominate numbers.

Examples.

1. In 185′, how many feet?
2. In 250″, how many feet and inches?
3. In 4367‴, how many feet?
4. What is the sum of 3 ft. 6′ 3″ 2‴, and 2 ft. 1′ 10″ 11‴?
5. What is the sum of 8 ft. 9′ 7″, and 6 ft. 7′ 3″ 4‴?
6. What is the difference between 9 ft. 3′ 5″ 6‴, and 7 ft. 3′ 6″ 7‴?
7. What is the difference between 40 ft. 6′ 6″, and 29 ft. 7‴?
8. What is the difference between 12 ft. 7′ 9″ 6‴, and 4 ft. 9′ 7″ 9‴?
9. Reduce 6′ 8″ to the fraction of a foot.
10. Reduce 9′ 10″ 8‴ to the fraction of a foot.
11. Reduce 4′ 5″ 3‴ to the decimal of a foot.
12. Reduce 7″ 6‴ to the decimal of a foot.

MULTIPLICATION OF DUODECIMALS.

228. **Multiplication of Duodecimals** is an abbreviated method of finding the measure of surfaces or solids.

Two dimensions, multiplied together, produce square measure; and three dimensions, multiplied together, produce cubic or solid measure.

---

227. How do you add and subtract Duodecimals?

228. What is Multiplication of Duodecimals? What do two dimensions, multiplied together, produce? Three dimensions? Feet multiplied by feet, give what? Primes by primes? Primes by seconds? Seconds by seconds? Primes by feet? What index must a product have? Give the rule.

1. Feet multiplied by feet, give square feet.
2. Square feet × feet, give cubic feet.
3. Feet × primes, give primes of surface.
4. Primes × primes, give seconds.
5. Primes × seconds, give thirds.
6. Seconds × seconds, give fourths.

Since the parts of a foot are marked by indices, the foregoing principles give rise to the following law:

The index of any product is equal to the sum of the indices of the factors.

1. Multiply 6 ft. 7′ 8″ by 2 ft. 9′.

Analysis.—Multiply by 9′. Since 8″$=\frac{8}{144}$ ft., and 9′$=\frac{9}{12}$ ft., 8″ × 9′$=\frac{8}{144}\times\frac{9}{12}=\frac{72}{1728}$ sq. ft., or 72 thirds. Since 12 thirds make 1 second, 72‴=72÷12=6″; therefore we put down 0‴, and carry 6″. 7′$=\frac{7}{12}$, and 7′ × 9′$=\frac{7}{12}\times\frac{9}{12}=\frac{63}{144}$ sq. ft. or 63 seconds; 63″ + 6″ = 69″; 69″÷12= 5′ 9″; we put down 9″ and carry 5′. 6 ft. × 9′$=\frac{6}{1}\times\frac{9}{12}=\frac{54}{12}$ sq. ft., or 54 primes; 54′+5′=59′; 59′÷12=4 sq. ft. 11′; which we set down, and then multiply by 2 feet. 8″ × 2 feet =16″; 16″÷12=1′ 4″; we put 4″ under the seconds, and carry 1′: 7′×2 feet=14′; 14′+1′=15′; 15′÷12=1 sq. ft. 3′; we put 3′ under primes, and carry 1 sq. ft.: 6 ft. × 2 ft.=12 sq. ft.; 12 sq. ft. +1 sq. ft.=13 sq. ft.; we set the feet under the feet and add. The result is 18 sq. ft. 3′ 1″: Hence the following

OPERATION.

6 ft.	7′	8″	
2 ft.	9′		
4 ft.	11′	9″	0‴
13	3′	4″	
18 ft.	3′	1″	0‴

Rule. I. *Write the multiplier under the multiplicand, so that units of the same order shall fall in the same column:*

II. *Begin with the lowest unit of the multiplier and the lowest of the multiplicand, and make the index of each product equal to the sum of the indices of the factors, and reduce each product to units of the next higher order—writing down the remainder. When there are three factors, multiply the product of either two by the third.*

EXAMPLES.

1. Multiply 9 ft. 4 in. by 8 ft. 3 in.

2. How many cords and cord feet in a pile of wood 24 feet long, 4 feet wide, and 3 feet 6 inches high?

3. Multiply 9 ft. 2 in. by 9 ft. 6 in. 8

7 4. How many square feet are there in a board 17 feet 6 inches in length, and 1 foot 7 inches in width?

5. Multiply 24 feet 10 inches by 6 feet 8 inches.

6. What is the number of cubic feet in a granite pillar 3 feet 9 inches in width, 2 feet 3 inches in thickness, and 12 feet 6 inches in length?

7. Multiply 70 feet 9 inches by 12 feet 3 inches.

8. There is a certain pile of wood, measuring 24 feet in length, 16 feet 9 inches high, and 12 feet 6 inches in width. How many cords are there in the pile?

9. How many square yards in the walls of a room, 14 feet 8 inches long, 11 feet 6 inches wide, and 7 feet 11 inches high?

10. If a load of wood be 8 feet long, 3 feet 9 inches wide, and 6 feet 6 inches high, how much does it contain?

11. How many cubic yards of earth were dug from a cellar which measured 42 feet 10 inches long, 12 feet 6 inches wide, and 8 feet deep?

12. What will it cost to plaster a room 20 feet 6′ long, 16 feet wide, 9 feet 6′ high, at 18 cents per square yard; and making allowance for a door that is 6 feet 6 in. long, by 3 feet 3′ wide?

13. How many feet of boards, 1 inch thick, can be cut from a plank 18 feet 9 in. long, 1 foot 6 in. wide and 3 in. thick, if there is no waste in sawing?

14. What will be the cost of building a stone wall, 45 ft. 6 in. long, 1 ft. 6′ thick, and 37 ft. 9′ high, at \$3.91¼ per cubic yard?

15. How many loads of earth must be taken out in digging a cellar that is to be 45 ft. 6 in. long, 25 ft. wide, and 10 ft. 9 in. deep, allowing 1 cubic yard of earth to 2 loads?

RATIO AND PROPORTION.

229. A RATIO is the quotient obtained by dividing one number by another.

230. The TERMS of a ratio are the divisor and dividend: hence, every ratio has two terms.

231. The divisor is called the ANTECEDENT.

232. The dividend is called the CONSEQUENT.

233. The ratio of one number to another is expressed in two ways:

1st. By a colon; thus, 3 : 12; and is read, 3 is to 12, or 12 divided by 3;

2d. In a fractional form; as, $\frac{12}{3}$, or 12 divided by 3.

234. The terms of a ratio, taken together, are called a COUPLET.

235. A SIMPLE RATIO is when both terms of a couplet are simple numbers. Thus, 6 : 18, is a simple ratio.

236. A COMPOUND RATIO, is one which arises from the multiplication of two simple ratios: thus, in the simple ratios,

$$3 : 7, \quad \text{and} \quad 6 : 8,$$

if we multiply the corresponding terms together, we have,

$$3 \times 6 \ : \ 7 \times 8,$$

which is compounded of the ratio of 3 to 7, and of 6 to 8.

The factors, 3 and 6, are called ELEMENTS of the first term; and the factors, 7 and 8, are ELEMENTS of the second term. The elements of a term are generally written in a column, thus,

$\left.\begin{matrix}3\\6\end{matrix}\right\} : \left.\begin{matrix}7\\8\end{matrix}\right\}$; and read, 3 multiplied by 6, to 7 multiplied by 8.

---

229. What is a ratio?—230. What are the terms of a ratio? How many terms has every ratio?—231. What is the divisor called?

232. What is the dividend called?

233. In how many ways is a ratio expressed? What are they?

234. What are the terms of a ratio, taken together, called?

235. What is a simple ratio?—236. What is a compound ratio?

237. When the antecedent is less than the consequent, the ratio shows *how many times* the consequent is as great as the antecedent.

238. When the antecedent is greater than the consequent, the ratio shows *what part* the consequent is of the antecedent.

NOTES.—1. Only numbers having the same unit value, can be compared with each other: hence, all numbers compared, must be reduced to the same unit.

2. The ratio, is always an abstract number.

239. To measure a number, is to find how many times it contains another number of the same kind, which is called, the *standard.* The unit 1, is the simplest standard of measure, and by this, all numbers, whether integral or fractional, are finally measured.

In every ratio, the antecedent is the standard.

EXAMPLES.

1. What is the ratio of 3 feet to 6 feet?
2. What is the ratio of 10 dollars to 40 dollars?
3. What is the ratio of the number 9 to 18?
4. What is the compound ratio of 3 × 9 to 9 × 9?
5. What is the compound ratio of 3 × 4 to 12 × 12?
6. What is the compound ratio of 5 × 3 × 2 to 6 × 10 × 3?
7. What part of 9 is 2?
8. What part of 16 is 4?
9. What part of 100 is 20?
10. What part of 300 is 200?
11. What part of 144 is 36?
12. 3 is what part of 12?
13. 5 is what part of 20?
14. 8 is what part of 56?
15. 7 is what part of 8?
16. 12 is what part of 132?

---

237. What does the ratio show, when the antecedent is less than the consequent?

238. What does the ratio show, when the antecedent is the greater?

239. What is the operation of measuring a number? What is the measure called? What is the simplest standard for all numbers? What is the standard in any ratio?

NOTE.—The standard is generally preceded by the word *of*, and in comparing numbers, may be named second, as in examples 12, 13, 14, 15, and 16; but it must always be used as a divisor, and should be placed first in the statement.

17. What part of $\frac{5}{6}$ is $\frac{1}{4}$?

18. $\frac{1}{2}$ of $\frac{3}{7}$ is what part of $\frac{9}{11}$?

19. $4\frac{1}{3}$ is what part of $9\frac{1}{5}$?

20. $\frac{3}{7}$ is what part of $4\frac{1}{5}$?

21. 2.75 is what part of 6.975?

22. $5\frac{1}{8}$ is what part of 7.1875?

23. What is the ratio of 2 T. 3 cwt. 2 qr. to 1 T. 11 cwt. 3 qr. 16 lb.?

24. What is the ratio of 1 mi. 6 fur. 8 rd. to 10 mi. 1 fur. 16 rd. 1 yd. 2 ft.?

25. The ratio of two numbers is 3, and the antecedent 16: what is the consequent?

ANALYSIS.—Since the ratio is equal to the consequent divided by the antecedent, it follows,

$$\overset{\text{Ratio}}{3} = \frac{\text{consequent}}{\text{antecedent}}$$

1st. *That the consequent is equal to the antecedent multiplied by the ratio:*

2d. *That the antecedent is equal to the consequent divided by the ratio.*

26. The ratio of two numbers is 6, and the antecedent 12: what is the consequent?

27. The ratio of two numbers is 9, and the consequent 108: what is the antecedent?

28. The ratio of two numbers is 5, and the consequent 125: what is the antecedent?

29. The ratio of two numbers is $\frac{3}{5}$, and the antecedent $\frac{1}{3}$: what is the consequent?

30. The ratio of two numbers is $\frac{5}{7}$, and the consequent $\frac{4}{9}$: what is the antecedent?

31. The ratio of two numbers is 6, and the consequent 12: what is the antecedent?

32. The antecedent is $\frac{1}{2}$, and the consequent $\frac{1}{3}$: what is the ratio?

33. The antecedent is 3 × 6 × 9, and the consequent 1 × 5 × 4 × 2: what is the ratio?

SIMPLE PROPORTION.

240. A SIMPLE PROPORTION is the comparison of the terms of two equal simple ratios.

Thus, the ratio of 3 : 6, is 2; and the ratio of 8 : 16, is 2; and we compare the terms by writing a double colon between the couplets; thus,

3 : 6 :: 8 : 16;

which is read, 3 is to 6, as 8 to 16.

Hence, every proportion has two couplets and four terms.

NOTE.—When the ratio of the first couplet is greater than 1, the second term is greater than the first, and the fourth term greater than the third. When the ratio is less than 1, the second term is less than the first, and the fourth term less than the third.

241. The first and fourth terms of a proportion are called the *extremes:* the second and third terms, the *means.* Thus, in the proportion,

3 : 12 :: 6 : 24,

3 and 24 are the *extremes,* and 12 and 6 the *means.*

242. Since the ratio in the first couplet is equal to that in the second, we have,

$$\frac{12}{3} = \frac{24}{6};$$

and we shall have, by reducing to a common denominator,

$$\frac{12 \times 6}{3 \times 6} = \frac{24 \times 3}{6 \times 3}.$$

---

240. What is a Simple Proportion? How is it written?

NOTE.—When the ratio in the first couplet is greater than 1, what follows?

241. What are the first and fourth terms of a proportion called? The second and third?

242. What is the product of the extremes equal to?

Since the fractions are equal, and have the same denominators, their numerators must be equal, viz.:

$$12 \times 6 = 24 \times 3;\ \text{that is,}$$

In any proportion, the product of the extremes is equal to the product of the means.

243. Since, in any proportion, the product of the extremes is equal to the product of the means, it follows that,

1st. *Either extreme is equal to the product of the means divided by the other extreme.*

2d. *Either mean is equal to the product of the extremes divided by the other mean.*

Note.—We shall denote the required term of a proportion by the letter x.

Examples.

1. In the proportion,

$$6 : 12 :: 24 : x,$$

find the value of the fourth term:

$$x = \frac{12 \times 24}{6} = 48.$$

Find the value of the required term, in the following proportions:

2. $9 : x :: 15 : 45.$
3. $8 : 10 :: x : 50.$
4. $9 : 54 :: x : 30.$
5. \$15 : \$3 :: x : 4 yards.
6. $\frac{1}{7} : \frac{4}{9} :: x : \frac{5}{12}.$

7. To what number has 5 the same ratio as exists between 2 and 4?

8. To what number has one-half the same ratio as exists between 3 and 21?

9. To what number has 5 the same ratio as exists between 6 and 18?

---

243. What is either extreme equal to? Either mean? By what is the required term of a proportion designated?

COMPOUND PROPORTION.

244. A COMPOUND PROPORTION is the comparison of the terms of two equal ratios, when one or both are compound:

$$\text{Thus,}\quad \left.\begin{matrix}2\\6\end{matrix}\right\} : \left.\begin{matrix}3\\8\end{matrix}\right\} :: \left.\begin{matrix}5\\6\end{matrix}\right\} : \left.\begin{matrix}15\\4\end{matrix}\right\}$$

is a compound proportion, in which factors 2 and 6, 3 and 8, 5 and 6, 15 and 4, are the elements of the terms. (Art. 236.)

Any compound proportion may be reduced to a simple one, by multiplying the elements of each term together: thus, by multiplying the elements, we have

$$12 : 24 :: 30 : 60.$$

Hence, in any compound proportion,

The product of the extremes is equal to the product of the means: therefore, in a compound proportion, we can find the required term, as in Art. 243.

What are the required terms in the following proportions:

1. $5\times10 : 3\times7 :: 48 : x.$
2. $2\times4 : 16\times6 :: 9 : x.$

If all the parts of a compound proportion are known, except one element, as in the proportion

$$\left.\begin{matrix}2\\6\end{matrix}\right\} : \left.\begin{matrix}3\\8\end{matrix}\right\} :: \left.\begin{matrix}5\\6\end{matrix}\right\} : \left.\begin{matrix}15\\x\end{matrix}\right\}$$

that *element is equal to the product of the means divided by the product of the elements of the first term and the known elements of the fourth term:* thus,

$$x = \frac{3\times8\times5\times6}{2\times6\times15} = 4.$$

3. What is the required element in the proportion,

$$\left.\begin{matrix}3\\9\\2\end{matrix}\right\} : \left.\begin{matrix}5\\7\\8\end{matrix}\right\} :: \left.\begin{matrix}3\\4\\9\end{matrix}\right\} : \left.\begin{matrix}2\\5\\x\end{matrix}\right\}$$

---

244. What is a compound proportion?

RULE OF THREE.

245. The RULE OF THREE is the process of finding, from three given numbers, a fourth, to which one of the given numbers shall have the same ratio as exists between the other two.

The Single Rule of Three embraces all the cases of Simple Ratios.

1. If 3 yards of cloth cost \$12, what will 6 yards cost?

ANALYSIS.—The quantity, 3 yards, bears the same ratio to the quantity, 6 yards, as \$12, the cost of 3 yards, to x dollars, the cost of 6 yards: the 4th term is found by multiplying the second and third terms together, and dividing the product by the first (Art. 243).

OPERATION.

$$3 : 6 :: \$12 : x.$$

$$x = \frac{\overset{2}{\cancel{6}} \times 12}{\cancel{3}} = \$24.$$

2. If 6 barrels of flour will last a family of 5 persons, 8 months, how long will they last a family of 10 persons?

ANALYSIS.—Write the required term (x), in the 4th place, and the term 8, having the same unit value, in the third place: then consider whether the third term is greater, or less, than the 4th: when greater, place the greater of the remaining terms in the first place, and when less, place the less term there: then find the value of the 4th term.

OPERATION.

$$10 : 5 :: 8 : x.$$

$$x = \frac{5 \times 8}{10} = 4.$$

In this example, it is plain, that the same provisions (6 barrels of flour) will not last ten persons as long as it did 5; hence, the 3d term will be greater than the 4th, which requires the 1st term to be greater than the 2d; hence, we write 10 in the first place, and 5 in the second.

In the first example, it is plain, that the 3d term, \$12, will be less than the 4th; therefore, the less of the remaining terms, 3, is written in the first place. Hence, the following

---

245. What is the Rule of Three? What is the Single Rule of Three? Give the rule.

Rule.

I. *Write the required term* (*x*) *in the 4th place, and the terms having like units in the 3d place:*

II. *Then consider, from the nature of the question, whether the 3d term is greater or less than the 4th: when greater, place the greater of the remaining terms in the first place, and when it is less, place the less term there, and the remaining term in the second place:*

III. *Then multiply the second and third terms together, and divide their product by the first.*

Notes.—1. If the first and second terms have different units, they must be reduced to the same unit.

2. If the third term is a compound denominate number, it must be reduced to its smallest unit.

3. The preparation of the terms, and writing them in their proper places, is called, the *Statement.*

Examples.

1. If I can walk 84 miles in 3 days, how far can I walk in 11 days?

2. If 4 hats cost $12, what will be the cost of 55 hats, at the same rate?

3. If a certain quantity of food will subsist a family of 12 persons, 48 days, how long will the same food subsist a family of 8 persons?

4. If 40 yards of cloth cost $170, what will 325 yards cost, at the same rate?

5. If 240 sheep produce 660 pounds of wool, how many pounds will be obtained from 1200 sheep?

6. If 30 barrels of flour will subsist 100 men for 40 days, how long will it subsist 25 men?

7. If 2 gallons of molasses cost 65 cents, what will 3 hogsheads cost?

8. If a man travels at the rate of 210 miles in 6 days, how far will he travel in a year, supposing him not to travel on Sundays?

9. If 90 bushels of oats will feed 40 horses for six days, how many horses would consume the same in 12 days?

10. If 4 yards of cloth cost \$13, what will be the cost of 3 pieces, each containing 25 yards?

11. If 48 yards of cloth cost \$67.25, what will 144 yards cost, at the same rate?

12. If 3 common steps, or paces, are equal to 2 yards, how many yards are there in 160 paces?

13. If 750 men require 22500 rations of bread for a month, how many rations will a garrison of 1200 men require?

14. A certain work can be done in 12 days, by working 4 hours a day: how many days would it require the same number of men to do the same work, if they worked 6 hours a day?

15. A pasture of a certain extent, supplies 30 horses for 18 days: how long will the same pasture supply 20 horses?

16. If $14\frac{1}{2}$ yards of cloth cost $\$19\frac{1}{2}$, how much will $19\frac{7}{8}$ yards cost?

NOTE.—Make the statement, and then change the mixed to improper fractions; and afterwards, multiply the 2d and 3d terms together, and divide by the 1st.

$$\overset{\text{yds.}}{14\tfrac{1}{2}} : \overset{\text{yds.}}{19\tfrac{7}{8}} :: \overset{\$}{\$19\tfrac{1}{2}} : x.$$

$$19\tfrac{1}{2} = \tfrac{39}{2};$$

$$19\tfrac{7}{8} = \tfrac{159}{8};$$

$$14\tfrac{1}{2} = \tfrac{29}{2}.$$

$$\tfrac{39}{2} \times \tfrac{159}{8} = \tfrac{6201}{16};$$

$$\tfrac{6201}{16} \div \tfrac{29}{2} = \frac{6201}{\underset{8}{\cancel{16}}} \times \frac{\cancel{2}}{29} = \tfrac{6201}{232} = 26\tfrac{169}{232}.$$

$$\text{Or, } \tfrac{39}{2} \times \tfrac{159}{8} \div \tfrac{29}{2} = \frac{39}{\cancel{2}} \times \frac{159}{8} \times \frac{\cancel{2}}{29} = \tfrac{6201}{232} = 26\tfrac{169}{232}.$$

17. If 2 lb. of beef cost $\frac{1}{8}$ of a dollar, what will 30 lb. cost?

18. If 6 men can dig a ditch in 40 days, what time will 30 men require to dig the same?

19. If $1\frac{4}{11}$ bushels of wheat cost $\$2\frac{5}{8}$, how much will 60 bushels cost?

20. If $4\frac{1}{2}$ yd. of cloth cost \$9.75, what will $13\frac{1}{2}$ yd. cost?

21. If $\frac{2}{5}$ of a yard of cloth costs $\frac{7}{8}$ of a dollar, what will $2\frac{1}{2}$ yards cost?

22. If $\frac{3}{16}$ of a ship costs £273 2s. 6d., what will $\frac{5}{32}$ of her cost?

23. If a post, 8 feet high, casts a shadow 12 feet in length, what must be the height of a tree that casts a shadow 122 feet in length, at the same time of day?

24. If a man performs a journey in $22\frac{1}{2}$ days, when the days are 12 hours long, how many days will it take him to perform the same journey, when the days are 15 hours long?

25. If 7 cwt. 1 qr. of sugar cost \$64.96, what will be the cost of 4 cwt. 2 qr.?

26. A merchant, failing in trade, pays 65 cents for every dollar which he owes; he owes A \$2750, and B \$1975: how much does he pay each?

27. If a person drinks 20 bottles of wine per month, when it costs 2s. per bottle, how much must he drink without increasing the expense, when it costs 2s. 6d. per bottle?

28. If 6 sheep cost \$15, and a lamb costs one-third as much as a sheep, what will 27 lambs cost?

29. If $4\frac{1}{7}$ gallons of molasses cost \$$2\frac{5}{8}$, how much is it per quart?

30. A man receives $\frac{3}{5}$ of his income, and finds it equal to \$3724.16: how much is his whole income?

31. A cistern, containing 200 gallons, is filled by a pipe which discharges 3 gallons in 5 minutes; but the cistern has a leak, which empties at the rate of 1 gallon in 5 minutes: if the water begins to run in when the cistern is empty, how long will it run before filling the cistern?

32. If 9 men, in 18 days, will cut 150 acres of grass, how many men will cut the same in 27 days?

33. If a garrison of 536 men have provisions for 326 days, how long will those provisions last, if the garrison be increased to 1304 men?

34. If 4 barrels of flour cost \$$34\frac{2}{3}$, how much can be bought for \$$175\frac{1}{6}$?

35. If 2½ gallons of molasses cost 65 cents, what will 3⅛ hogsheads cost?

36. What is the cost of 6 bushels of coal, at the rate of £1 14s. 6d. a chaldron?

37. What quantity of corn can I buy for 90 guineas, at the rate of 5 shillings a bushel?

38. A merchant, failing in trade, owes $3500, and his effects are sold for $2100: how much does B receive, to whom he owes $420?

39. If 3 yards of broadcloth cost as much as 4 yards of cassimere, how much cassimere can be bought for 18 yards of broadcloth?

40. What length must be cut off from a board that is 9 inches wide, to make a square foot, that is, as much as is contained in 12 inches in length and 12 in breadth?

41. If 7 hats cost as much as 25 pair of gloves, worth 84 cents a pair, how many hats can be purchased for $216?

42. How many barrels of apples can be bought for $114.33, if 7 barrels cost $21.63?

43. If 27 pounds of butter will buy 15 pounds of sugar, how much butter will buy 36 pounds of sugar?

44. If 42½ tons of coal cost $206.21, what will be the cost of 2½ tons?

45. If a certain sum of money will buy 40 bushels of oats, at 45 cents a bushel, how many bushels of barley will the same money buy, at 72 cents a bushel?

46. If 40 gallons run into a cistern, holding 700 gallons, in an hour, and 15 run out, in what time will it be filled?

47. A piece of land of a certain length, and 12½ rods in width, contains 1¼ acres: how much would there be in a piece of the same length, 26⅔ rods wide?

48. If 13 men can be boarded 1 week for $39.585, what will it cost to board 3 men and 6 women the same time, the women being boarded at half price?

49. What will 75 bushels of wheat cost, if 4 bushels 3 pecks cost $10.687?

DOUBLE RULE OF THREE.

1. If a family of 6 persons expend $300 in 8 months, how much will serve a family of 15 persons for 20 months?

STATEMENT.

$$\left.\begin{matrix}6\\8\end{matrix}\right\} : \left.\begin{matrix}15\\20\end{matrix}\right\} :: \$300 : x.$$

$$x = \frac{15 \times 20 \times 300}{6 \times 8} = \$1875.$$

Analysis.—Write the required term in the 4th place, and $300, having the same unit, in the 3d place.

Then, since a family of 6 persons, in 8 months (being equal to 1 person for 48 months), will require *less* than 15 persons for 20 months (equal 1 person for 300 months), the 3d term will be *less* than the 4th, and, consequently, the *first term of the proportion will be less than the second:* hence, 6×8 is written in the first place, and 15×20 in the second: then find the value of x, as in Art. 244.

2. If 16 men, in 6 days of 12 hours each, can do a certain piece of work, in how many days will 24 men, working 8 hours a day, do the same work?

STATEMENT.

$$1 : 1 :: \left.\begin{matrix}16\\6\\12\end{matrix}\right\} : \left\{\begin{matrix}24\\8\\x\end{matrix}\right.$$

$$\frac{16 \times 6 \times 12}{24 \times 8} = x = 3 \text{ days.}$$

Analysis.—Write the term containing the unknown element in the 4th place, and the term having like units with it in the 3d place. Then, since the work done in both cases is the same, it may be denoted by 1; hence, the statement; after which find the value of x, as in Art. 244.

3. If 32 men build a wall 36 feet long, 8 feet high, and 4 feet thick, in 4 days, working 12 hours a day; how long a wall, that is 6 feet high and 3 feet thick, can 48 men build in 36 days, working 9 hours a day?

STATEMENT.

$$\left.\begin{matrix}32\\4\\12\end{matrix}\right\} : \left.\begin{matrix}48\\36\\9\end{matrix}\right\} :: \left.\begin{matrix}36\\8\\4\end{matrix}\right\} : \left.\begin{matrix}x\\6\\3\end{matrix}\right\}$$

$$x = \frac{48 \times 36 \times 9 \times 36 \times 8 \times 4}{32 \times 4 \times 12 \times 6 \times 3} = 648 \text{ feet.}$$

Rule.—I. *Denote the required term, or element, by x:*

II. *Write the required term, or the term containing the required element, in the fourth place, and the term having like units with it, in the third place:*

III. *Then analyze the question and see whether the third term is greater or less than the fourth:*

IV. *When greater, write the greater of the two remaining terms in the first place, and when less, write the less term there, and the remaining term in the second place; then find the required element, as in* Art. 244.

Note.—This rule gives, when quantity and cost are considered:

quantity : quantity : : cost : cost.

When labor and time are considered:

labor : labor : : time : time.

When labor and work done are considered:

labor : labor : : work done : work done.

Examples.

1. If I pay $24 for the transportation of 96 barrels of flour 200 miles, what must I pay for the transportation of 480 barrels 75 miles?

96 : 480 :: 24
75

2. If 12 ounces of wool be sufficient to make 1½ yards of cloth 6 quarters wide, what number of pounds will be required to make 450 yards of flannel 4 quarters wide?

3. What will be the wages of 9 men for 11 days, if the wages of 6 men for 14 days be $84?

4. How long would 406 bushels of oats last 7 horses, if 154 bushels serve 14 horses 44 days?

154 : 406 :: 44 : x
7 : 14

5. If a man travels 217 miles in 7 days, traveling 6 hours a day, how far would he travel in 9 days, if he traveled 11 hours a day?

7

6. How long will it take 5 men to earn $11250, if 25 men can earn $6250 in 2 years?

7. If 15 weavers, by working 10 hours a day for 10 days, can make 250 yards of cloth, how many must work 9 hours a day for 15 days, to make 607½ yards?

9 : 10
[illegible] : 607½ } :: 15 : x
15 : 10

25 : 30 :: 100 : x
20 : 12 ::

8. A regiment of 100 men drank in 1 day 20 dollars' worth of wine, at 30 cents a bottle: how many men, having a like allowance, will require 12 dollars' worth, at 25 cents a bottle?

9. If a footman travels 341 miles in 7⅓ days, traveling 12½ hours each day, in how many days, traveling 10⅓ hours a day, will he travel 155 miles?

341 : 155 :: 7 1/3 : x
10 1/3 : 12 1/2 ::

10. If 25 persons consume 300 bushels of corn in 1 year, how much will 139 persons consume in 8 months, at the same rate?

25 : 139 :: 300 : x
12 : 8 ::

x 11. How much hay will 32 horses eat in 120 days, if 96 horses eat 3¾ tons in 7½ weeks?

x 12. If $2.45 will pay for painting a surface 21 feet long and 13⅓ feet wide, what length of surface that is 10⅝ feet wide, can be painted for $31.72?

13. How many pounds of thread will it require to make 60 yards of 3 quarters wide, if 7 pounds make 14 yards 6 quarters wide?

14 : 60 :: 7 : x
6 : 3 ::

14. If 500 copies of a book, containing 210 pages, require 12 reams of paper, how much paper will be required to print 1200 copies of a book of 280 pages?

500 : 1200 :: 12 : x
210 : 280 ::

15. If the transportation of 9 T. 15 cwt. 20. lb. for 260 miles, costs $76.50, what will be the cost of transporting 25 T. 16 cwt. 3 qr. for 189 miles, at the same rate?

16. If a cistern, 17½ feet long, 10½ feet wide, and 13 feet deep, holds 546 barrels of water, how many barrels will a cistern 12 feet long, 10 feet wide, and 7 feet deep, contain?

17½ : 12
10½ : 10
13 : 7

17. A contractor agreed to build 24 miles of railroad in 8 months, and for this purpose, employed 150 men; at the end of 5 months, but 10 miles of the road were built: how many more men must be employed, to finish the road in the time agreed upon?

18. If 336 men, in 5 days of 10 hours each, can dig a trench of 5 degrees of hardness, 70 yards long, 3 wide, and 2 deep, what length of trench of 6 degrees of hardness, 5 yards wide and 3 yards deep, may be dug by 240 men, in 9 days of 12 hours each?

PARTNERSHIP.

247. A PARTNERSHIP is an association of two or more persons, under an agreement to share the profits and losses of business. The persons thus associated, are called, *Partners.*

248. CAPITAL, or STOCK, is the amount of money or property contributed by the partners, and used in the business.

249. DIVIDEND is the gain or profit, divided to each partner.

250. Loss, is the opposite of Gain or Profit.

251. When the capital of each partner is employed for the same time.

Since the Capital or Stock produces the gain or profit, each man's share should be proportional to his amount of Stock: Hence, we have,

Whole Stock : each man's Stock :: Whole Profit : each man's Profit.

EXAMPLES.

1. A and B buy certain goods, amounting to \$160, of which A pays \$90, and B \$70; they gain \$32, by the sale of them: what is the share of each ?

OPERATION.

$$160 \ : \ 90 \ :: \ 32 \ : \ x = \frac{\overset{9}{\cancel{90}} \times \overset{2}{\cancel{32}}}{\underset{16}{\cancel{160}}} = \$18, \text{ A's share.}$$

$$160 \ : \ 70 \ :: \ 32 \ : \ x = \frac{\overset{7}{\cancel{70}} \times \overset{2}{\cancel{32}}}{\underset{16}{\cancel{160}}} = \$14, \text{ B's share.}$$

---

247. What is a Partnership? What are partners?

248. What is capital, or stock?—249. What is dividend?

250. What is loss?

251. What produces the gain or profit? What is the rule for finding each man's share?

2. A and B have a joint stock of $2100, of which A owns $1800 and B $300; they gain in a year, $1000: what is each one's share of the profits?

3. A, B, and C fit out a ship for Liverpool. A contributes $3200, B $5000, and C $4500; the profits of the voyage amount to $1905: what is the portion of each?

4. A, B, and C agree to build a railroad, and contribute $418000 of capital, of which B pays 2 dollars, and C 3 dollars, as often as A pays 1 dollar; they lose $2400 by the operation: what is the loss of each?

5. Three drovers hire a pasture for 6 weeks, at an expense of $275; the first puts in 300 cattle, the second 450, and the third 500: what ought each to pay?

6. Two merchants enter into partnership. One puts in $5000, and the other $2000. The partner that puts in the less sum, is to receive $300 extra, before division, for his superior knowledge. They gain $4725: what is the share of each?

7. A, B, and C make up a capital of $20000; B and C each contribute twice as much as A; but A is to receive one-third of the profits for extra services; at the end of the year, they have gained $4000: what is each to receive?

8. Three merchants own a ship, in the following proportions;—$\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{6}$. The ship required repairs, to the amount of $1350: what was each one's share of the expense?

252. When the capital is employed for unequal times.

When the partners employ their capital for unequal periods of time, the profit of each will depend on the two elements, *Capital* and *Time*, and will be proportional to their product: Hence,

Multiply each man's stock by the time he continued it in trade: then say,

As the sum of the products : each product,
: : the whole gain or loss : each man's share.

---

252. What are the elements of profit, when the capital is employed for unequal times? What is the rule for finding the profit of each partner?

EXAMPLES.

1. A and B entered into partnership. A put in \$840 for 4 months, and B, \$650 for 6 months; they gained \$363: what is each one's share?

OPERATION.

$$\begin{array}{l} \text{A. } \$840 \times 4 = 3360 \\ \quad\;\; 650 \times 6 = \underline{3900} \\ \qquad\qquad\quad\; 7260 : \left.\begin{array}{l}3360\\3900\end{array}\right\} :: 363 : \left.\begin{array}{l}\$168, \text{ A's.}\\ \$195, \text{ B's.}\end{array}\right\} \end{array}$$

2. A puts in trade \$550 for 7 months, and B puts in \$1625 for 8 months; they make a profit of \$337: what is the share of each?

3. A and B hire a pasture, for which they agree to pay \$92.50; A pastures 12 horses for 9 weeks, and B, 11 horses for 7 weeks: what portion must each pay?

4. Four traders form a company. A puts in \$400 for 5 months; B, \$600 for 7 months; C, \$960 for 8 months; D, \$1200 for 9 months. In the course of trade, they lost \$750: how much falls to the share of each?

5. A, B, C contribute to a capital of \$15000, in the following manner: every time A puts in 3 dollars, B puts in \$5, and C \$7. A's capital remains in trade 1 year, B's $1\frac{2}{5}$ years, and C's $2\frac{3}{7}$ years; at the end of the time, there is a profit of \$15000: what is the share of each?

6. A commenced business January 1st, with a capital of \$3400. April 1st, he took B into partnership, with a capital of \$2600; at the expiration of the year, they had gained \$750: what is each one's share of the gain?

7. James Fuller, John Brown, and William Dexter formed a partnership, under the firm of Fuller, Brown & Co., with a capital of \$20000; of which Fuller furnished \$6000, Brown \$5000, and Dexter \$9000. At the expiration of 4 months, Fuller furnished \$2000 more; at the expiration of 6 months, Brown furnished \$2500 more; and at the end of a year, Dexter withdrew \$2000. At the expiration of one year and a half, they found their profits amounted to \$5400: what was each partner's share?

PERCENTAGE.

253. PER CENT. means, by the hundred. Thus, 1 per cent. of a number, is one-hundredth of it; 2 per cent., two-hundredths; 3 per cent., 3 hundredths, &c.

254. The RATE PER CENT. is the number of hundredths taken; thus, if 1 hundredth is taken, the rate is 1 per cent.; if 2 hundredths, the rate is 2 per cent.; if 3 hundredths, 3 per cent., &c.

255. The BASE of percentage, is the number on which the percentage is computed; and the result of the computation is called, the PERCENTAGE.

256. The rate per cent. is generally expressed decimally; thus,

1 per cent. of a number, is $\frac{1}{100}$ of it $=$ .01 of it.
3 per cent. of a number, is $\frac{3}{100}$ of it $=$ .03 of it.
50 per cent. of a number, is $\frac{50}{100}$ of it $=$ .50 of it.
100 per cent. of a number, is $\frac{100}{100}$ of it $=$ 1 time it.
116 per cent. of a number, is $\frac{116}{100}$ of it $=$ 1.16 of it.
200 per cent. of a number, is $\frac{200}{100}$ of it $=$ 2 times it.
$\frac{1}{2}$ per cent. of a number, is $\frac{\frac{1}{2}}{100}$ of it $=$ .005 of it.
$\frac{3}{4}$ per cent. of a number, is $\frac{\frac{3}{4}}{100}$ of it $=$ .0075 of it.
.7 per cent. of a number, is $\frac{.7}{100}$ of it $=$ .007 of it.
.45 per cent. of a number, is $\frac{.45}{100}$ of it $=$ .0045 of it.
.5$\frac{1}{2}$ per cent. of a number, is $\frac{.5\frac{1}{2}}{100}$ of it $=$ .0055 of it.

Write, decimally, 2 per cent.; 8$\frac{1}{2}$ per cent.; 6$\frac{3}{4}$ per cent., $\frac{3}{5}$ per cent.; $\frac{7}{8}$ per cent.; 117 per cent.; 205 per cent.; .1 per cent.; 275.25 per cent.

---

253. What is the meaning of per cent.?
254. What is the rate per cent.?
255. What is the base of percentage? What is Percentage?
256. How is the rate per cent. generally expressed?

257. Having given the base and rate, to find the percentage.

1. What is the percentage of $320, the rate being 5 per cent.?

Analysis.—The base is $320, and the rate being 5 per cent., is expressed decimally by .05. We are then to take .05 of the base; this we do, by multiplying $320 by .05.

OPERATION.
320
.05
$16.00, *Ans.*

Hence, to find the percentage of a number,

Rule.—*Multiply the number by the rate, expressed decimally, and the product will be the percentage.*

Examples.

1. What is the percentage of $657, the rate being 4⅓ per cent.?

Note.—When the rate cannot be reduced to an exact decimal, it is most convenient to multiply by the fraction, and then by that part of the rate which is expressed in exact decimals.

OPERATION.
657
.04⅓
219 = ⅓ per cent.
2628 = 4 per cent.
$28.47 = 4⅓ per cent.

Find the percentage of the following numbers:

2. 2½ per cent. of 650 dollars.
3. 3 per cent. of 650 yards.
4. 4½ per cent. of 875 cwt.
5. 6⅓ per cent. of $37.50.
6. 5¾ per cent. of 2704 miles.
7. ½ per cent. of 1000 oxen.
8. 2⅜ per cent. of $376.
9. 5⅔ per cent. of $327.33.
10. 66⅔ per cent. of 420 cows.
11. 105 per cent. of 850⅜ T.
12. 116 per cent. of $875\frac{2}{16}$ lb.
13. 241 per cent. of $875.12½.
14. 3.7½ per cent. of $200.
15. .33⅓ per cent. of $687.24.
16. .87½ per cent. of $400.
17. .0⅞ per cent. of $225.40.

18. A has $852 deposited in the bank, and wishes to draw out 5 per cent. of it: how much must he draw for?

---

257. When the base and rate are given, how do you find the percentage?

19. A merchant had 1200 barrels of flour; he shipped 64 per cent. of it, and sold the remainder: how much did he sell?

20. A merchant bought 1200 hogsheads of molasses. On getting it into his store, he found it short 3½ per cent.: how many hogsheads were wanting?

21. What is the difference between 5½ per cent. of $800, and 6½ per cent. of $1050?

22. Two men had each $240. One of them spends 14 per cent., and the other 18½ per cent.: how many dollars more did one spend than the other?

23. A man has a capital of $12500; he puts 15 per cent. of it in State stocks, 33⅓ per cent. in railroad stocks, and 25 per cent. in bonds and mortgages: what per cent. has he left, and what is its value?

24. A farmer raises 850 bushels of wheat: he agrees to sell 18 per cent. of it, at $1.25 a bushel; 50 per cent. of it, at $1.50 a bushel; and the remainder, at $1.75 a bushel: how much does he receive in all?

258. **To find the per cent. which one number is of another.**

1. What per cent. of $16, is $4?

ANALYSIS.—Since the percentage is equal to the base multiplied by the rate, the rate will be equal to the percentage divided by the base. In this example, the percentage is $4, and the base is $16; hence, the rate is equal to $4 ÷ $16 = 25 per cent.: Hence,

OPERATION.

$4 \div 16 = .25$, or,
25 per cent.

RULE.—*Divide the percentage by the base, and the quotient, in decimals, will express the rate.*

EXAMPLES.

1. What per cent. of 20 dollars is 5 dollars?
2. Forty dollars is what per cent. of eighty dollars?
3. What per cent. of 200 dollars is 80 dollars?

---

258. How do you find what per cent. one number is of another?

4. Ninety bushels of wheat is what per cent. of 1800 bu.?

5. Nine yards of cloth is what per cent. of 870 yards?

6. Forty-eight head of cattle are what per cent. of a drove of 1600?

7. What per cent. of \$75.25, is \$8.62$\frac{1}{2}$?

8. $\frac{5}{6}$ is what per cent. of $\frac{3}{4}$?

9. What per cent. of 16.1875 yd. is 37$\frac{5}{8}$ yd.?

10. .875 is what per cent. of .125?

11. 15 is what per cent. of 65?

12. 27 is what per cent. of 35?

13. A man has \$550, and purchases goods to the amount of \$82.75: what per cent. of his money does he expend?

14. A merchant goes to New York, with \$1500; he first lays out 20 per cent., after which he expends \$660: what per cent. was his last purchase of the money that remained after his first?

15. Out of a cask containing 300 gallons, 60 gallons are drawn: what per cent. is this?

16. The population of a town, in a certain year, was 5682; 5 years afterward, it was 7296: what was the per cent. of increase during the interval?

17. A man purchased a farm of 75 acres, at \$42.40 an acre; he afterward sold the same farm for \$3577.50: what was his gain per cent. on the purchase-money?

259. Having given the percentage and rate, to find the base.

1. \$750 is 18 per cent. of what number of dollars?

ANALYSIS.—Since the percentage is equal to the product of the base by the rate, the base is equal to the percentage divided by the rate: Hence,

OPERATION.

$750 \div .18 = \$4166\frac{2}{3}$.

RULE.—*Divide the given percentage by the rate, expressed decimally, and the quotient will be the base.*

---

259. When the percentage and rate are given, how do you find the base?

Examples.

1. 960 is 25 per cent. of what number?

2. 74 is 62½ per cent. of what number?

3. 450 is 112 per cent. of what number?

4. Of what number is 66, ⅔ per cent.?

5. Of what number is 1.75, 37½ per cent.?

6. ⅚ is ⅞ per cent. of what number?

7. In a school, 77 pupils are present, which is 87½ per cent. of the number on roll: what is the number on roll?

8. Suppose the population of a town, in 1855, to have been 15624, which was 56 per cent. of its population in 1860: what was the population in 1860?

9. In a mixture of wine and water, there are 16 gallons of water, which is 80 per cent. of the whole: what is the amount of the mixture?

10. A and B are in partnership; A receives of the profits, $370, which was 18½ per cent. of the whole profit: what was the total profit, and what was B's share?

COMMISSION.

260. Commission is an allowance made to an agent for the transaction of business. It is reckoned at a certain rate per cent. on the amount of money employed.

261. To find the percentage of Commission, when the rate and base are known.

1. What is the commission on $4396, at 6 per cent.?

Analysis.—The base and rate being given, we find the percentage by multiplying $4396 by .06 (Art. 257).

OPERATION.

$$\begin{array}{r} \$4396 \\ .06 \\ \hline \end{array}$$

Hence the $263.76 = Commissic

Rule.

Multiply the amount employed by the rate, in decimals and the product is the commission.

EXAMPLES.

1. A land agent sells a farm for $27560: what is the amount of his commission, at 5 per cent.?

2. A commission merchant, in New York, received from St. Louis a quantity of flour, which he sold for $5695; what is the amount of his commission, at 9½ per cent.?

3. A house agent collects rents to the amount of $1756.75: what is his commission, at 3 per cent., and what amount does he pay over to the landlord?

4. A factor sells 60 bales of cotton at $425 per bale, and is to receive 2½ per cent. commission: how much must he pay over to his principal?

5. A commission merchant sells goods to the amount of $8750, on which he is to be allowed 2 per cent.; but in consideration of paying the money over before it is due, he is to receive 1½ per cent. additional: how much must he pay over to his principal?

6. A drover agreed to take a drove of cattle to New York, and sell them on a commission of 5 per cent. on the estimated value, $4250, or on any higher sum for which he might sell: he sold them at an advance on their estimated value of 10 per cent.: what commission did he receive?

7. What does a commission merchant, who charges 5 per cent. commission and 2½ per cent. guarantee, receive on a bill of goods for $2765.50?

NOTE.—Guarantee is indemnity against risk.

8. A real-estate agent purchased a house for $3650, charging 2½ per cent. commission. In the course of a few days, the value of the property advanced 15 per cent., and he was then directed to sell the property. What was the amount of his commission, for purchase and sale, the rate being the same in both cases?

---

260. What is Commission?

261. How do you find the commission, when the base and rate are known?

9. I directed my agent to purchase 25 lots of ground, at $650 per lot, and to pay the expense of examining titles, which was $5 per lot: what did the lots cost me, if the commission was 4 per cent.?

10. A commission merchant was allowed 5 barrels of flour commission for every 60 barrels that he sold: what was his rate of commission, and how much did he receive on 570 barrels?

11. An agent was employed to sell a ship, whose price was $30000: if he sold it at that or any higher price, he was to receive $850: he sold it at the price named: what was the rate per cent. of commission?

262. **To find the commission and base, when the rate and the sum of the base and commission are known.**

1. Merchant A sent to B, a commission merchant, $3825, to be invested in the purchase of flour; B is to receive 2 per cent. on the amount paid for the flour: what was the value of the flour, and what was the commission?

ANALYSIS.—Since the broker receives 2 per cent., it will require $1.02 to purchase 1 dollar's worth of flour; hence, there will be as many dollars' worth purchased as $1.02 is contained times in $3825; that is, $3750 worth. The commission will be 2 per cent. of $3750, or $3825 − 3750 = $75: Hence,

OPERATION.

```
1.02 ) 3825.00 ( $3750 Ans.
       306
       ---
        765
        714
        ---
         510
         510
```

RULE.

I. *Divide the given amount by 1 plus the rate of commission, expressed decimally, and the quotient will be the base of percentage:*

II. *Subtract the base from the given amount, and the remainder will be the commission.*

---

262. How do you find the commission and base when the rate and sum of the commission and base are known?

Examples.

1. A grocer received $750, to expend in the purchase of flour. Allowing 7½ per cent. commission, what did he pay for the flour?

2. A merchant at Chicago sends to his agent in New York $5413.05, with directions to buy coffee, and to charge a commission on the money expended of 3½ per cent.: what was the amount of commission?

3. What value of stock can be purchased for $20119, if a commission of 5½ per cent. be allowed on its cost?

4. How many barrels of flour, at $7 a barrel, can be bought for $2657.20, if 4 per cent. commission be allowed on the money paid?

5. A merchant in New York receives from Boston $25000, to be expended in flour; he was allowed a commission of 2¼ per cent. on the money paid: what was the amount of commission?

6. A merchant in New York sends $12600 to a commission merchant in Chicago for the purchase of flour; the latter charges 5 per cent. commission: what amount was expended in buying the flour, and what was the commission?

7. A commission merchant in New York received $5000 from Cincinnati for purchasing dry goods; he charged a commission of 3 per cent. on what he paid: what commission was received?

8. In a given time, a sum of money, placed at interest, had increased 17 per cent., and amounted to $5679.45: what was the sum of money at interest?

9. A merchant is directed to expend $5642.48 in the purchase of cloth; he is allowed a commission of 2½ per cent. on what he pays for the cloth, and charges in addition 1½ per cent. for storage: how much did he lay out in cloth?

10. A merchant in New Orleans received from New York $21630, with orders to invest in cotton, allowing a commission of 2½ per cent.; marine insurance, 1¼ per cent.; cartage and freight, 1¼ per cent.: what amount was laid out in buying cotton, and how much was bought, at 15 cents per lb.?

PROFIT AND LOSS.

263. PROFIT AND LOSS are commercial terms, indicating gain or loss in business transactions. The gain or loss is always estimated on the cost price.

264. To find the gain or loss, when the cost and selling price are given.

1. Bought a piece of cloth, containing 75 yards, at $5.25 per yard, and sold it at $5.75 per yard: how much was gained in the trade?

ANALYSIS.—Subtract the entire cost from the entire selling price, and the remainder will be the gain; or multiply the gain on 1 yard by the number of yards: Hence,

OPERATION.

$5.25 × 75 = $393.75
5.75 × 75 = $431.25
431.25 — 393.75 = 37.50 gain.
Or,
5.75 — 5.25 = .50 gain on 1 yd.
.50 × 75 = $37.50 = whole gain.

RULE.—*Find the difference between the cost and selling price, and it will be the gain or loss.*

EXAMPLES.

1. A merchant bought a horse for $175.50, and sold it for $215¾: what was the gain?

2. A merchant bought a ship for $7500, and paid $1900 for repairs; he then sold it for $12000: what was the gain?

3. Bought a hogshead of brandy at $1.25 per gallon, and sold it for $78: was there a loss or gain?

265. To determine the selling price, when the cost and gain or loss are known.

1. Bought a piece of calico, containing 56 yards, at 27 cents a yard: what must it be sold for a yard, to gain $2.24?

---

263. What do the terms Profit and Loss indicate? On what is the gain or loss estimated?

Analysis.—First find the cost, then add the profit, and divide the sum by the number of yards.

OPERATION.

56 yards, at 27 cents = $15.12
Profit 2.24
It must sell for . . . $17.36
56) 17.36
31 cts. a yd.

Rule.—*Add the gain to the cost, or deduct the loss, and the sum or difference will be the selling price.*

Examples.

1. If a hogshead of wine cost $159: for what must it be sold, per pint, that there may be a gain of 50 cents per gallon?

2. A house cost $4750: for what must it be sold, that the owner may realize $600, after paying his agent a commission of $65?

3. A merchant, in selling 500 bushels of corn, which cost $236, lost $45: what did he obtain per bushel?

4. For what must a farm, which cost $7960, be sold, that the owner may realize $1800, after paying to his agent a commission of $50?

266. **To find the gain or loss, when the cost and rate per cent. of gain or loss are known. (Rule, Art. 257.)**

Examples.

1. What is the gain on $3750, at 6 per cent.?

2. What would be the gain in the sale of a house for $12750, at 18¾ per cent.?

3. A gentleman lost, in the sale of bank-stock that cost $24760, 6 per cent.: what was the loss?

---

264. How do you find the gain or loss, when the cost and selling price are given?

265. How do you find the selling price, when you know the cost and loss or gain?

266. How do you find the gain or loss, when the cost and rate are known?

4. Bought a piece of cassimere, containing 28 yards, at 1¼ dollar a yard; but finding it damaged, am willing to sell it at a loss of 15 per cent.: how much must be asked per yard?

5. A merchant purchased 3275 bushels of wheat, for which he paid $3517.10; but finding it damaged, is willing to lose 10 per cent.: what must it sell for per bushel?

6. Bought 50 gallons of molasses, at 75 cents a gallon, 10 gallons of which leaked out. At what price, per gallon, must the remainder be sold, that I may clear 10 per cent. on the cost of the whole?

7. A merchant buys 158 yards of calico, for which he pays 20 cents per yard; one-half is so damaged that he is obliged to sell it at a loss of 6 per cent.; the remainder he sells at an advance of 19 per cent.: how much did he gain?

267. The cost and selling price being known, to determine the rate per cent of gain or loss.

1. If I buy coffee at 16 cents, and sell it at 20 cents a pound, how much do I make per cent.?

ANALYSIS.—The gain is 4 cents. The gain divided by the cost, gives the rate (Art. 258).

OPERATION.

$20 - 16 = 4.$

$4 \div 16 = .25.$

EXAMPLES.

1. A man bought a house and lot for $1850.50, and sold them for $1517.41: how much per cent. did he lose?

2. A merchant bought 650 pounds of cheese at 10 cents per pound, and sold it at 12 cents per pound: how much did he gain on the whole, and how much per cent. on the money laid out?

3. If I sell a piano, which cost $275, for $315, what was the rate per cent. of gain?

4. A herd of cattle was bought in Kentucky, at an expense of $3750; the cost of transportation was $250; it was sold in New York, for $5725: what was the rate per cent. of gain, after paying the expense?

---

267. How do you find the rate, when the cost and selling price are known?

268. **To find the cost, when the selling price and rate per cent. of gain or loss, are known.**

1. I sold a parcel of goods for $195.50, on which I made 15 per cent.: what did they cost me?

Analysis.—1 dollar of the cost plus 15 per cent., will be what that which cost $1 sold for, viz., $1.15: hence, there will be as many dollars of cost, as $1.15 is contained times in what the goods brought.

OPERATION.

1.15) 195.50

$170 *Ans.*

2. If I sell a parcel of goods for $170, by which I lose 15 per cent., what did they cost?

Analysis.—1 dollar of the cost less 15 per cent., will be what that which cost 1 dollar sold for, viz., $0.85: hence, there were as many dollars of cost, as .85 is contained times in what the goods brought.

Hence, to find the cost,

OPERATION.

.85) 170

$200 *Ans.*

Rule.—*Divide the amount received by 1 plus the per cent. when there is a gain, and by 1 minus the per cent. when there is a loss, and the quotient will be the cost.*

Examples.

1. A carriage was sold for $350, by which a gain of 25 per cent. was made: what was the cost?

2. If I sell a parcel of goods for $170, by which I lose 15 per cent., what did they cost?

3. A cargo of wheat was sold for $12500, by which a gain of 25 per cent. was made: what was the amount of net gain, after paying $150 for freight, and $75 for other charges?

4. A commission merchant sold a lot of iron, which had been consigned to him, for $25600, by which a gain of 31 per cent. on the invoice was made: allowing him 5 per cent. commission, what was the net gain?

---

268. How do you find the cost, when the selling price and rate are known?

INSURANCE.

269. INSURANCE is an obligation, generally in writing, by which individuals or companies bind themselves to indemnify the owners of certain property, such as ships, goods, houses, &c., from loss or hazard.

270. The BASE of insurance, is the amount for which the property is insured.

271. The POLICY is the written agreement made by the parties.

272. PREMIUM is the amount paid by him who owns the property, to those who insure it.

273. To find the premium, when the base and rate are known.

RULE.—Same as in Art. 257.

EXAMPLES.

1. What is the premium for the insurance of a house valued at $8754, against loss by fire, for one year, at ½ per cent.?

2. What would be the premium for insuring a ship and cargo, valued at $37500, from New York to Liverpool, at 3½ per cent.?

3. What would be the insurance on a ship valued at $47520, at ½ per cent.? Also, at ⅓ per cent.?

4. A merchant wishes to insure on a vessel and cargo at sea, valued at $28800: what will be the premium, at 1¾ per cent.?

---

269. What is Insurance?—270. What is the base of insurance?
271. What is the policy?—272. What is the premium?
273. How do you find the premium, when the base and rate are known?

5. A merchant owns three-fourths of a ship valued at $24000, and insures his interest at 2½ per cent.: what does he pay for his policy?

6. A merchant learns that his vessel and cargo, valued at $36000, have been injured to the amount of $12000; he effects an insurance on the remainder, at 5½ per cent.: what premium does he pay?

7. My furniture, worth $3440, is insured at 2⅜ per cent.; my house, worth $10000, at 1¼ per cent.; and my barn, horses, and carriages, worth $1500, at 3¼ per cent.: what is the whole amount of my premium?

8. A merchant imported 250 pieces of broadcloth, each piece containing 36½ yards, at $3.25 a yard; he paid 4½ per cent. insurance on the selling price, $4.50 a yard: if the goods were destroyed by fire, and he got the amount of insurance, how much did he make?

9. A vessel and cargo, worth $65000, are damaged to the amount of 20 per cent., and there is an insurance of 50 per cent. on the loss: how much will the owner receive?

10. If I insure my house for $10000, at ¼ of 1 per cent., my barn for $2500, at ⅜ per cent., and a soap factory, worth $15000, at 1⅓ per cent., what is the entire amount of the insurance?

11. A cargo of goods is insured, for the voyage, at ¾ of 1 per cent. The cargo is valued at $36000, and is damaged to the one-twelfth its value. How much must the company pay?

12. Mr. James imported six packages of silks, invoiced at $2500 a package, and had them insured at 1½ per cent. It was found that they were damaged to the amount of 15 per cent. of their value. What was the amount of insurance?

13. What is the amount of premium of insurance on a ship worth $30000, at 1⅓ per cent., and on the cargo, valued at $36000, at 1¼ per cent.?

INTEREST.

274. **Interest** is a payment for the use of money.

275. **Principal** is the money on which interest is paid.

276. The **Rate** of interest, is the per cent. paid for 1 year.

277. **Amount** is the sum of the principal and the interest.

Interest is always reckoned at a certain rate, by the year, or *per annum.*

In interest, by general custom, a year consists of 12 months, each having 30 days; hence, in a year, for computing interest, there are 360 days.

In almost every country and State, the rate of interest is fixed by law, and is called, the *Legal Rate.* This rate differs in different States and countries.

Any rate above the legal rate, is *usury,* which is forbidden by law.

CASE I.

278. **To find the interest on any principal for one or more years.**

1. What is the interest of $1960, for 4 years, at 7 per cent.?

Analysis.—The principal is the base, and the interest is the percentage, which is found by multiplying the principal by the rate; therefore, $137.20 is the interest for 1 year, and this interest multiplied by 4, gives the interest for 4 years: Hence,

OPERATION.

$1960	
.07,	rate.
137.20,	int. for 1 yr.
4,	No. of years.
$548.80	

Rule.—*Multiply the principal by the rate, expressed decimally, and the product by the number of years.*

---

274. What is Interest?—275. What is principal?—276. What is rate of interest?—277. What is amount? What does per annum mean? What is legal interest?

278. How do you find the interest of any principal, for any number of years? Give the analysis.

EXAMPLES.

1. What is the interest of $650, for one year, at 6 per cent.?
2. What is the interest of $950, for 4 years, at 7 per cent.?
3. What is the amount of $3675 in 3 years, at 7 per cent.?
4. What is the amount of $459 in 5 years, at 8 per cent.?
5. What is the interest of $211.26, for 1 year, at 4½ per ct.?
6. What is the interest of $1576.91, for 3 yr., at 7 per ct.?
7. What is the amount of $957.08 in 6 years, at 3½ per ct.?
8. What is the interest of $375.45, for 7 years, at ⅞ per ct.?
9. What is the amount of $4049.87 in 2 years, at 5 per ct.?
10. What is the amount of $16199.48 in 16 yr., at 5½ per ct.?

NOTE.—When there are years and months, and the months are aliquot parts of a year, *multiply the interest for 1 year by the years and months reduced to the fraction of a year.*

11. What is the interest of $326.50, for 4 years and 2 months, at 7 per cent.?

12. What is the interest of $437.21, for 9 years and 3 months, at 3 per cent.?

13. What is the amount of $1119.48, after 2 years and 6 months, at 7 per cent.?

14. What is the amount of $179.25, after 3 years and 4 months, at 7 per cent.?

15. What is the amount of $1046.24, after 3 months, at 5½ per cent.?

16. What is the amount of $6704.25, after 1 year and 4 months, at 6½ per cent.?

17. What is the interest of $3750.87, for 2 years and 9 months, at 8 per cent.?

CASE II.

279. **To find the interest on a given principal for any rate and time.**

1. What is the interest of $876.48, at 6 per cent., for 4 years 9 months and 14 days?

---

279. How do you find the interest for any time, at any rate?

Analysis.—The interest for 1 year is the product of the principal and the rate. If the interest for 1 year be divided by 12, the quotient will be the interest for 1 month; if the interest for 1 month be divided by 30, the quotient will be the interest for 1 day.

The interest for 4 years is 4 times the interest for 1 year; the interest for 9 months, 9 times the interest for 1 month; and the interest for 14 days, 14 times the interest for 1 day.

OPERATION.

$876.48
.06
12) 52.5888 = int. for 1 yr. 52.5888 × 4 = $210.3552 4 yr.
30) 4.3824 = int. for 1 mo. 4.3824 × 9 = $ 39.4416 9 mo.
.14608 = int. for 1 da. .14608 × 14 = $ 2.0451 14 da.
Total interest, $251.8419 +

Rule.

I. *Find the interest for 1 year:*

II. *Divide this interest by 12, and the quotient will be the interest for 1 month:*

III. *Divide the interest for 1 month by 30, and the quotient will be the interest for 1 day:*

IV. *Multiply the interest for one year by the number of years, the interest for 1 month by the number of months, and the interest for 1 day by the number of days, and the sum of the products will be the required interest.*

Second Rule.

280. There is another rule resulting from the last analysis, which is a good general rule for computing interest.

I. *Find the interest for 1 year, and divide it by 12: the quotient will be the interest for 1 month:*

II. *Multiply the interest for 1 month by the time expressed in months and tenths of a month, and the product will be the required interest.*

Note.—Since a month is reckoned at 30 days, the quotient of any number of days, divided by 3, will be tenths of a month.

1. What is the interest of \$327.50, for 3 years 7 months and 13 days, at 7 per cent.?

OPERATION.

3 years = 36 mos.
7 mos.
13 days = .4$\frac{1}{3}$ mos.
Time = 43.4$\frac{1}{3}$ mos.

\$327.50
.07
12) 22.9250 =int. for 1 year.
1.9104+ =int. for 1 month.
43.4$\frac{1}{3}$ =time in months.
.6368
76416
57312
76416
\$82.97504 *Ans.*

NOTE.—The method employed, and the number of decimal places used, in computing interest, may affect the mills, and, *possibly*, the last figure in cents. It is best to use 5 places of decimals.

EXAMPLES.

1. What is the interest of \$132.26, for 1 year 4 months and 10 days, at 6 per cent. per annum?

2. What is the interest of \$25.50, for 1 year 9 months and 12 days, at 6 per cent.?

3. What is the interest of \$1728.60, at 7 per cent., for 2 years 6 months and 21 days?

4. What is the interest of \$288.30, at 7 per cent., for 1 year 8 months and 27 days?

5. What is the interest of \$576.60, at 6 per cent., for 10 months and 18 days?

6. What is the interest of \$854.42, at 6 per cent., for 3 months and 9 days?

7. What is the interest of \$1123.20, at 6 per cent., for 11 months and 6 days?

8. What is the interest of \$2306.54, at 5 per cent., for 7 months and 28 days?

9. What is the interest of \$4272.10, at 5 per cent., for 10 months and 28 days?

---

280. How do you find the interest for years, months and days, by the second method?

10. What is the interest of $1620, at 4 per cent., for 5 years and 24 days?

11. What is the interest of $2430.72, at 4 per cent., for 10 years and 4 months?

12. What is the interest of $3689.45, at 7 per cent., for 4 years and 7 months?

13. What is the interest of $2945.96, at 7 per cent., for 7 years and 3 days?

14. What is the interest, at 8 per cent., of $675.89, for 3 years 6 months and 6 days?

15. What is the interest of $648.54, for 7 years 6 months, at 4½ per cent.?

16. What is the interest of $1297.10, for 8 years 5 months, at 5½ per cent.?

17. What is the interest of $864.768, for 9 months 25 days, at 6¼ per cent.?

18. What is the interest of $2594.20, for 10 months and 9 days, at 7½ per cent.?

19. What is the amount of $2376.84, for 3 years 9 months and 12 days, at 8⅓ per cent.?

20. What is the amount of $5148.40, for 7 years 11 months and 23 days, at 9½ per cent.?

21. What is the amount of $3565.20, for 3 years 9 months, at 10½ per cent.?

22. A person bought a bill of goods amounting to $750, but delayed its payment for 6 months and 13 days after it was due: allowing 7 per cent. interest, what amount should he pay?

23. A merchant bought flour to the amount of $2560, and kept it in store 4 months 15 days before he sold it: he then sold the whole for $4250. Allowing interest at 7 per cent., what was the gain?

24. What amount is necessary to discharge a mortgage of $5675, the interest of which, at 6 per cent., has not been paid for 3 years 9 months 22 days?

25. A merchant sold flour which cost him $4912, at an advance of 30 per cent.; but he sold the flour on a credit

of 3 months, and had kept it in store 2 months 15 days: allowing 7 per cent. interest, what did he gain?

26. What is the amount of $256, for 10 months 15 days, at 7½ per cent.

27. What is the interest on a note of $264.42, given January 1st, 1852, and due Oct. 10th, 1855, at 4 per cent.?

28. Gave a note of $793.26, April 6th, 1850, on interest at 7 per cent.: what is due September 10th, 1852?

29. What amount is due on a note of hand given June 7th, 1850, for $512.50, at 6 per cent., to be paid Jan. 1st, 1851?

30. What is the interest on $1250.75, for 90 days, at 10 per cent.?

31. What is the amount of $7109, from Feb. 8th, 1848, to Dec. 7th, 1852, at 6¾ per cent.?

32. What will be due on a note of $213.27, on interest after 90 days, at 7 per cent., given May 19th, 1836, and payable October 16th, 1838?

33. What is the interest of $2132.70, from Nov. 17th, 1838, to Feb. 2d, 1839, at 7½ per cent.?

34. What is the interest of $38463, from April 27th, 1815, to Sept. 2d, 1824, at 8 per cent.?

35. What is the interest of $14231.50, from June 29th, 1840, to April 30th, 1845, at 8¼ per cent.?

36. What is the interest of $426.40, from Sept. 4th, 1843, to May 4th, 1849, at 9 per cent.?

37. What is the interest of $4320, from Dec. 1st, 1817, to Jan. 22d, 1818, at 9½ per cent.?

38. What is the amount of $397.16, from March 24th, 1834, to March 31st, 1835, at 10½ per cent.?

39. What is the amount of $164.60, from Sept. 27th, 1845, to March 24th, 1855, at 1½ per cent.?

CASE III.

281. When the principal is in pounds, shillings, and pence.

1. What is the interest, at 7 per cent., of £27 15s. 9d., for 2 years?

Analysis.—The interest on pounds and decimals of a pound is found in the same way as the interest on dollars and decimals of a dollar: after which the decimal part of the interest may be reduced to shillings and pence: Hence,

OPERATION.

£27 15s. 9d. = 27.7875
.07
1.945125
2
£3.890250
£.89025 = 17s. 9¾d.
Ans. £3 17s. 9¾d.

Rule.

I. *Reduce the shillings and pence to the decimal of a pound, and annex the result to the pounds:*

II. *Find the interest as though the sum were United States Money, after which reduce the decimal part to shillings and pence.*

Examples.

1. What is the interest of £67 19s. 6d., at 6 per cent., for 3 years 8 months and 16 days?

2. What is the interest of £127 15s. 4d., at 6 per cent., for 3 years and 3 months?

3. What is the interest of £107 16s. 10d., at 7 per cent., for 3 years, 6 months, and 6 days?

4. What will £279 13s. 8d. amount to, in 3 years and a half, at 5¼ per. cent. per annum?

◆ ◆ ◆

PARTIAL PAYMENTS.

282. A Partial Payment is a payment of a part of the amount due on a note or bond.

We shall give the rule established in New York (see Johnson's Chancery Reports, vol. i., page 17), for computing the interest on a bond or note, when partial payments have been

---

281. How do you find the interest, when the principal is in pounds, shillings, and pence?

made. The same rule is also adopted in Massachusetts, and in most of the other States.

RULE.

I. *Compute the interest on the principal to the time of the first payment, and if the payment exceeds this interest, add the interest to the principal, and from the sum subtract the payment: the remainder forms a new principal.*

II. *But if the payment is less than the interest, take no notice of it until other payments are made, which in all shall exceed the interest computed to the time of the last payment: then add the interest, so computed, to the principal, and from the sum subtract the sum of the payments: the remainder will form a new principal, on which interest is to be computed as before.*

NOTE.—In computing interest on notes, observe that the day on which a note is dated and the day on which it falls due, are not both reckoned in determining the time, *but one of them is always excluded.* Thus, a note dated on the first day of May, and falling due on the 16th of June, will bear interest but one month and 15 days.

EXAMPLES.

$349.998.

BUFFALO, May 1st, 1826.

1. For value received, I promise to pay James Wilson or order, three hundred and forty-nine dollars, ninety-nine cents, and eight mills, with interest at 6 per cent.

JAMES PAYWELL.

On this note were indorsed the following payments·

Dec. 25th, 1826,	received	$49.998
July 10th, 1827,	. . .	4.998
Sept. 1st, 1828,	. . .	15.008
June 14th, 1829,	. . .	99.999

What was due, April 15th, 1830?

---

282. What is a Partial Payment? What is the rule for computing interest, when there are partial payments?

Principal on interest, from May 1st, 1826, ...		$349.998
Interest to Dec. 25th, 1826, time of first payment, 7 months 24 days, ...		13.649+
Amount, ...		$363.647
Payment, Dec. 25th, exceeding interest then due, .		49.998
Remainder for a new principal, ...		$313.649
Interest of $313.649, from Dec. 25th, 1826, to June 14th, 1829, 2 years 5 months 19 days, ...		46.4721
Amount, ...		$360.1211
Payment, July 10th, 1827, less than interest then due, ...	$ 4.998	
Payment, Sept. 1st, 1828, ...	15.008	
Their sum, less than interest then due,	$20.006	
Payment, June 14th, 1829, ...	99.999	
Their sum exceeds the interest then due, ...		120.005
Remainder for a new principal, June 14th, 1829, . .		$240.1161
Interest of $240.116, from June 14th, 1829, to April 15th, 1830, 10 months 1 day, ...		12.0458
Total due, April 15th, 1830, . . .		$252.1619+

$3469.32.

New York, Feb. 6th, 1825.

2. For value received, I promise to pay William Jenks, or order, three thousand four hundred and sixty-nine dollars and thirty-two cents, with interest from date, at 6 per cent.

Bill Spendthrift.

On this note were indorsed the following payments:

May 16th, 1828, received	$ 545.76.
May 16th, 1830, . . .	1276.00.
Feb. 1st, 1831, . . .	2074.72.

What remained due, Aug. 11th, 1832?

3. A's note, of $635.84, was dated Sept. 5th, 1817; on which were indorsed the following payments, viz.: Nov. 13th, 1819, $416.08; May 10th, 1820, $152.00: what was due March 1st, 1821, the interest being 6 per cent.?

PROBLEMS IN INTEREST

283. In all questions of Interest, there are four things considered, viz.:

1st, The Principal; 2d, The Rate of Interest; 3d, The Time; and 4th, The Amount of Interest.

If three of these are known, the fourth can be found. By Art. 292, the interest is found by multiplying together the principal, rate, and time in years; therefore, the interest is the product of the principal, rate, and time. Either of these factors is found by dividing the product by the product of the other two: Hence, we have the following principles:

1st, *The principal is equal to interest divided by the rate and time:* 2d, *The rate is equal to interest divided by the principal and time:* 3d, *The time is equal to interest, divided by principal and rate.*

Examples.

1. The interest of a certain sum for 4 years, at 7 per cent., is $266: what is the principal?

OPERATION.

$.07 \times 4 = .28.$

$266 \div .28 = \$950 =$ principal.

Analysis.—The interest, $266, divided by the product of the rate and time, .07 × 4 = .28, will give the principal.

2. The interest of $3675, for 3 years, is $771.75: what is the rate?

3. The principal is $459, the interest $183.60, and the rate 8 per cent.: what is the time?

4. The interest of a certain sum for 3 years, at 6 per cent., is $40.50: what is the principal?

5. The principal is $918, the interest $269.28, and the rate 4 per cent.: what is the time?

---

283. How many things are considered in every question of interest? What are they? The interest is the product of what factors? How may one of these factors be found? What are the three principles?

6. What sum of money must be placed on interest at 7 per cent., for 3 yrs. 9 mos., that the interest may be $396?

7. In what time, at 7 per cent., will a mortgage of $8762.50, whose interest is unpaid, amount to $10000?

8. If, by purchasing a house for $5620, I have received, in 2 yrs. 3 mos. 15 days, $1800 rent: what rate of interest have I received?

9. A merchant, who had bought goods for $15960, sold them, at the end of 5 months 16 days, at an advance of 27 per cent.: what rate of interest did he receive?

10. What sum of money, at 6 per cent., will produce, in 2 yrs. 9 mos. 10 days, the same interest that $350 produces, at 8 per cent., in 3 yrs. 10 mos. 5 days?

11. In what time will $5000, at 7 per cent., produce the same interest that $9625 produces, at 6¼ per cent., in 4 yrs. 5 mos. 18 days?

◆◆◆

COMPOUND INTEREST.

284. COMPOUND INTEREST is the interest of the amount of the principal and its unpaid interest.

This interest may be computed annually, semi-annually, quarterly, monthly, or daily. In Savings Banks, the interest is generally computed semi-annually.

From the definition, we deduce the following

RULE.—*Compute the interest to the time at which it becomes due; then add it to the principal, and compute the interest on the amount as on a new principal: add the interest again to the principal, and compute the interest as before; do the same for all the times at which payments of interest become due; from the last result subtract the principal, and the remainder will be the compound interest.*

---

284. What of Compound Interest? How do you compute it?

Examples.

1. What will be the compound interest, at 7 per cent., of $3750, for 2 years, the interest being added yearly?

OPERATION.

3750	
.07	
262.50,	Interest for 1st year.
3750	
4012.50,	Principal for 2d year.
.07	
280.8750,	Interest for 2d year.
4012.50	
4293.375,	Amount at 2 years.
3750	1st Principal.
$543.375,	Compound interest.

Note.—When there are months and days in the time, find the amount for the *years*, and on this amount cast the interest for the months and days: this, added to the last amount, will be the required *amount* for the whole time.

2. If the interest be computed annually, what will be the compound interest on $100, for 3 years, at 6 per cent.?

3. What will be the compound interest on $295.37, at 6 per cent., for 2 years, the interest being added annually?

4. What will be the compound interest, at 5 per cent., of $1875, for 4 years?

5. What is the amount, at compound interest, of $250, for 2 years, at 8 per cent.?

6. What is the compound interest of $939.64, for 3 yr. 9 mo., at 7 per cent.?

7. What will $125.50 amount to, in 10 years, at 4 per cent. compound interest?

8. What will be the amount of $250, which has been in savings bank for 2 years, supposing interest computed semi-annually, at 6 per cent.? What the compound interest?

9. What will be the interest of $500, which has been in savings bank for 1 yr. 6 mo., supposing the interest to be computed semi-annually, at 5 per cent.?

Note.—The operation is rendered much shorter and easier, by taking the amount of 1 dollar for any time and rate, given in the following table, and multiplying it by the given principal; the product will be the required amount, from which subtract the given principal, and the result will be the compound interest.

The result may differ in the mills place, from that obtained by the other rule.

Table,

Showing the *amount* of $1 or £1, compound interest, from 1 year to 20, and at the rate of 3, 4, 5, 6, and 7 per cent.

Years.	3 per cent.	4 per cent.	5 per cent.	6 per cent.	7 per cent.
1	1.03000	1.04000	1.05000	1.06000	1.07000
2	1.06090	1.18160	1.10250	1.12360	1.14490
3	1.09272	1.12486	1.15762	1.19101	1.22504
4	1.12550	1.16985	1.21550	1.26247	1.31079
5	1.15927	1.21665	1.27628	1.33822	1.40255
6	1.19405	1.26531	1.34009	1.41851	1.50073
7	1.22987	1.31593	1.40710	1.50363	1.60578
8	1.26677	1.36856	1·47745	1.59384	1.71818
9	1.30477	1.42331	1.55132	1.68947	1.83845
10	1.34391	1.48028	1.62889	1.79084	1.96715
11	1.38423	1.53945	1.71033	1.89829	2.10485
12	1.42576	1.60103	1.79585	2.01219	2.25219
13	1.46853	1.66507	1.88564	2.13292	2.40984
14	1.51258	1.73167	1.97993	2.26090	2.57853
15	1.55796	1.80094	2.07892	2.39655	2.75903
16	1.60470	1.87298	2.18287	2.54035	2.95216
17	1.65284	1.94790	2.29201	2.69277	3.15881
18	1.70243	2.02581	2.40661	2.85433	3.37993
19	1.75350	2.10684	2.52695	3.02559	3.61652
20	1.80611	2.19112	2.65329	3.20713	3.86968

10. What is the amount of $96.50, for 8 years and 6 mo., interest being compounded annually, at 7 per cent.?

11. What is the compound interest of $300, for 5 years, 8 months, and 15 days, at 6 per cent.?

12. What is the compound interest of $1250, for 3 years, 3 months, and 24 days, at 7 per cent.?

STOCKS AND BROKERAGE.

285. A CORPORATION is a company of persons, authorized by law to do business together.

286. A CHARTER is the law which defines the rights, powers, and duties of a corporation.

287. CAPITAL or STOCK is the money paid in to carry on the business of the corporation.

288. STOCKHOLDERS are the individuals, who own the stock and compose the corporation.

289. SHARES are equal portions of the stock; and these are held, in larger or smaller numbers, by the stockholders, or shareholders.

290. CERTIFICATES OF STOCK, or SCRIP, are the written evidences of the ownership of stock.

BONDS are written evidences of money loaned, and are an obligation of payment within a time named in the bond. Bonds are for different sums; usually, for $1000.

291. UNITED STATES STOCKS, or STATE STOCKS, are the bonds of the United States, or of a State, bearing a fixed interest.

292. The NOMINAL, or PAR VALUE of a stock, is the number of dollars named in each share. Shares may be of any value: they are usually $100; sometimes $50 and sometimes $25. When not $100, the amount is named.

293. The MARKET VALUE of a stock, is what the stock brings, per share, when sold for cash. It may be at par, at a premium, or at a discount

---

285. What is a corporation?—286. What is a charter?—287. What is capital, or stock?—288. What are stockholders?—289. What are shares?—290. What are certificates of stock, or scrip? What are bonds?—291. What are United States or State Stocks?—292. What is the nominal or par value of a stock?—293. What is the market alue of a stock? When is a stock at par? When is a stock at a emium? When is a stock at a discount?

A Stock is at *par,* when its market value is just equal to the value named in the certificates.

A stock is at a *premium,* when the market value exceeds its par value.

A stock is at a *discount,* when its market value is less than its par value.

294. PREMIUM is the *Rate* per cent. which a stock sells for, above its par value.

295. DISCOUNT is the *Rate* per cent. which a stock sells for, below its par value.

296. A BROKER is an agent who buys and sells goods, stocks, bonds, and uncurrent money, on commission. The commission charged is called *Brokerage.*

297. To find the market value of any number of shares of stock, when at a premium or discount.

1. What is the value of 150 shares of Erie Stock, par 100, when selling at 16 per cent. discount?

ANALYSIS.—Since the stock is selling at 16% discount, the market rate would be 100% − 16% = 84% of the par value. $\$100 \times 150 = \$15000$ = Par : $15000 \times .84 = \$12600$ = market value. Hence,

RULE.—*Multiply the nominal or par value by the rate per cent. to be taken of the base, and the product will be the market value.*

2. What is the market value of 200 shares of stock, par 50, and selling at 20 per cent. premium? *Ans.* $12000.

3. How much must be paid for $25600 of stock, which is selling at $87\frac{5}{8}$ per cent.? *Ans.* $22432.

4. A broken bank has a circulation of $98000, and purchases the bills at 85 per cent.: how much is made by the operation? *Ans.* $14700.

---

294. What is premium?—295. What is discount?—296. What is a broker? What is brokerage?—297. How do you find the market value of any number of shares of stock?

5. A broker sells $50000 of stock on commission, premium 3½% and brokerage ⅜ per cent.: what is the brokerage? *Ans.* $187.50.

6. What must be paid for 175 shares of Hudson River Railroad Stock, par 100, which is selling at 9 per cent. discount, brokerage being at ¼ per cent.? *Ans.* $15968.75.

7. Mr. Jones directs a broker to purchase 250 shares of stocks, par 100, and at 8 per cent. premium: what is its cost, brokerage being at ⅝ per cent.? *Ans.* $27156.25.

8. What amount of bank bills would pay for $37500 of gold, when it is at 9½ per cent. premium? *Ans.* $41062.50.

9. A broker bought for another party 250 shares of Railroad stock, par 100, at a premium of 3½%. After a short time, he was directed to sell the same; which he did at a discount of 2¼%. What was the double brokerage at ⅜%, and what amount was lost by the transactions?

Ans. Brok. $187.50. Loss $1625.00.

298. **To find how much stock, at par, can be purchased for a given sum.**

1. What amount of stock, at par value, can be purchased for $12192, when it is at 5 per cent. discount, if 1 per cent. be charged for brokerage?

ANALYSIS.—Since the stock is at 5 per cent. discount, $1 of it would cost 95 cents; adding the brokerage, it will cost 96 cents; and therefore the par value will be as many times $1 as 96 cents is contained times in $12192.

OPERATION.

$.95 + .01 = .96$

$$.96\,\overline{)\,12192.00}$$

$12700

RULE.—*Divide the given sum by the market value of one dollar of the stock plus the brokerage.*

2. What amount of government stock can I buy for $15525, when it is selling at 3½ per cent. premium? *Ans.* $15000.

---

298. How do you find how much stock, at par, can be purchased for a given sum?

3. How many shares of stock, par \$25, can be bought for \$2730, when selling at 5% premium? *Ans.* 104.

4. A broker is authorized to expend \$20450 in purchasing N. Y. State Stocks, which are selling at 2 per cent. premium: what amount of stock does he buy, charging $\frac{1}{4}$ per cent. brokerage? *Ans.* \$20000.

5. Erie Railroad Stock is selling at 24% discount, and brokerage is charged at $\frac{5}{8}$ per cent.: how many shares can be bought for \$9195? *Ans.* 12000.

6. If gold is selling at a premium of 10%, and railroad stock at a discount of 4%: how many shares, par \$50, can be bought for \$4800 in gold? *Ans.* 110.

7. How many shares of bank stock, \$100 par, selling at 5% discount, can be bought for 250 shares of insurance stock, par \$100, selling at 14% premium? *Ans.* 300.

298*a*. To find the rate of premium or of discount on stock sold at any given price.

1. If 50 shares of Erie Stock, par \$100, are sold for \$2180, what is the rate of discount on the stock?

Analysis.—50 shares at 100 are equal to \$5000 par value. \$5000 less the price paid for them, equals the discount: and this discount, \$2820, divided by the par value or base, \$5000, gives the rate of discount 56$\frac{2}{5}$ per cent.

OPERATION.

$$\$100 \times 50 = \$5000 = \text{par};$$
$$5000 - 2180 = 2820, \text{ amount of discount};$$
$$2820 \div 5000 = 56\tfrac{2}{5} \text{ per cent.} = \text{rate}.$$

Rule.—*Find the amount of premium or discount, and divide it by the par value: the quotient will be the rate of premium or discount.*

2. If 60 shares of bank stock, par 50, be sold for \$4125, what is the rate of premium? *Ans.* 37$\frac{1}{2}$ per cent.

298*a*. How do you find the rate of premium or discount on stock sold at a given price?

3. If 90 shares of Central Railroad Stock, par 100, be given for 210 shares of Erie Stock, selling at 35, is Central selling at par, premium, or discount? what is the rate of premium or discount? *Ans.* $18\frac{1}{3}$ per cent. discount.

3. If gold is at a premium of 5%, and $5000 in gold pays for 60 shares of Harlem Railroad Stock, par 100: how is this stock selling, and what is the rate?

Ans. $12\frac{1}{2}$ per cent. discount.

5. A capitalist bought 500 shares of Pacific Mail Stock, par $100, when it was worth 140; after holding this stock 6 months, to escape further loss, he sold it at 60: what was his rate of loss, excluding interest and brokerage?

Ans. $57\frac{1}{7}$ per cent.

6. A merchant bought, through a broker, 75 shares of bank stock, par 100, the brokerage at $\frac{1}{2}$ per cent.; he paid the entire sum with 1000 barrels of flour at $8 per barrel: what was the rate of premium? *Ans.* $6\frac{1}{6}$ per cent.

7. If I give a house and lot worth $20000, for 175 shares New York Gas Co.'s Stock, par 100, what is the rate of premium? *Ans.* $14\frac{2}{7}$ per cent.

298*b*. The interest on bonds and stocks is always calculated on their par value, and is usually made payable at the rate and place named in the bond or certificate of stock. Bonds and stocks often take their names from the rate of interest which they bear. Thus, when the rate of interest is 5 per cent. the stock is called a five per cent. stock; and similarly for other rates.

298*c*. To find the rate of interest on an investment in stock paying a fixed rate of interest, when the stock is above or below par.

298*b*. On what is the interest on bonds and stocks calculated? From what do bonds and stocks often take their name?—298*c*. How do you find the rate of interest on an investment in stock?

1. If I buy 6 per cent. stock at 15% discount: what is the rate of interest on the investment?

Analysis.—The interest on \$100 for one year is $100 \times .06 = \$6.00$. The market value of \$100 of the stock is $\$100 - 15 = \$85$. Then \$6 is the percentage and \$85 the base; and $\$6 \div \$85 = .07\frac{1}{17}$ or $7\frac{1}{17}\% =$ rate.

OPERATION.

$$
\begin{aligned}
100 \times .06 &= 6.00 = \text{int. on \$100 for 1 yr.};\\
100 \times .15 &= 15.00 = \text{discount on \$100};\\
100 - 15 &= 85 \quad = \text{market value};\\
6 \div 85 &= .07\tfrac{2}{17} = \text{rate of int.}
\end{aligned}
$$

Hence,

Rule.—*Find the interest of \$100 of the stock at the given rate, and also its market value. Divide the interest by the market value, and the quotient will be the rate on the investment.*

2. If I buy 5% bonds of the United States at 95, what is the rate of interest on the investment? *Ans.* $5\frac{5}{19}$ per cent.

3. If a merchant invests in United States 6 per cents, payable in gold, when they are selling at 114, what rate does he receive on his investment, supposing gold to be at a premium of 12%? *Ans.* $5\frac{51}{57}$ per cent.

4. What is the difference in the rates of interest between an investment in a 7% stock, selling at a discount of 4%, and a State 6% stock, selling at a discount of 15%?
Ans. $\frac{95}{408}$ per cent.

5. In which would it be most profitable to invest—in a 5 per cent. stock, selling at 12% discount, or in a 7% stock, selling at a premium of 8%? *Ans.* 7% stock.

6. If an investment be made in Tennessee 6% bonds, par 100, selling at 26% discount, brokerage $\frac{3}{8}\%$, what rate of interest would be received? *Ans.* $8\frac{8}{119}$ per cent.

298*d*. To find how much a stock must be above or below par, to produce a given rate of Interest.

1. At what rate must a 5% stock be sold to produce 8% on the investment?

Analysis.—The interest on $100 of stock is equal to $5. What investment, at 8%, will produce the same interest? Here, it is plain, that $5 is the product of the rate and principal. Hence, if $5 be divided by the rate, .08, the quotient, $62½, will be the principal. Therefore 100 − 62½ = 37½, the discount.

OPERATION.

$$\$100 \times .05 = \$5.00 \text{ int. on stock;}$$
$$\$5 \div .08 = 62\tfrac{1}{2} = \text{market value of stock;}$$
$$100 - 62\tfrac{1}{2} = 37\tfrac{1}{2}\text{; rate of discount} = 37\tfrac{1}{2}\%.$$

Rule.—*Find the interest of $100 of the stock. Divide this interest by the rate of the investment, and the quotient will be the market value of the stock. The difference between 100 and this value, will indicate the rate of discount or premium.*

2. At what rate must I buy a six per cent. State stock, so as to yield 10% on the investment? *Ans.* 40% discount.

3. What is the premium or discount on an 8% stock, when the purchase yields 12% on the investment?
Ans. 33⅓% discount.

4. At what rate must I buy New York 6% State Bonds, in order to produce 7% on the investment?
Ans. $14\frac{2}{7}$% discount.

298*e*. To find the income derived from an investment in any given stock.

1. What income shall I derive, yearly, from an investment of $13200 in 7% stock, selling at 12% discount?

298*d*. How do you find how much a stock must be above or below par to produce a given rate of interest?—298*e*. How do you find the income derived from an investment in any given stock?

ANALYSIS.—Since the discount is 12%, the market value will be 88% of the par. Hence, $13200 ÷ .88 = $15000 = par value. The par value multiplied by the rate of interest gives the required amount. Hence,

OPERATION.
$13200÷.88=$15000 par.
15000 × .07=$1050 *Ans.*

RULE.—*Divide the sum to be invested by the rate of the market value, and the quotient will be the par value. Then multiply the par value by the rate of interest, and the product will be the income.*

2. If I invest $5200 in 6% stock, which is selling at 4% premium: what income shall I derive from the investment? *Ans.* $300.

3. A gentleman bequeathed to a benevolent institution $25000: of which $10300 was to be invested in 6% U. S. Stocks, at 103; $12240 in New York 7 per cents, selling at 102; and the remainder in Erie 7% mortgages, selling at 82: what income was received? *Ans.* $1650.

4. Which would yield the larger income—$11400 invested in 7% stock, at 95, or the same amount invested in 5% stock, selling at 57? *Ans.* 5% stock.

298*f*. To find what sum must be invested in any given stock to produce a given income.

1. What sum must I invest in 6% New York State Stocks, selling at 94, to realize an income of $2520?

ANALYSIS.—Since the stock is a 6% stock, $1 par will produce $.06; and $2520 will produce as many times one dollar as .06 is contained times in $2520, which is $42000. The stock is worth 94; hence, $42000 × .94 = $39480, the sum required.

OPERATION.
2520÷.06= 42000=par.
42000 × .94=$39480 *Ans.*

RULE.—*Divide the income to be realized by the interest of one dollar, and the quotient will be the par value of the*

298*f*. How do you find the sum which must be invested in a given stock to produce a given income?

stock. Multiply this par by the selling price of the stock, and the product will be the sum to be invested.

2. How much must I invest in United States 6% Currency Bonds, selling at 108, that I may receive an annual income of $3900? *Ans.* $70200.

3. How much must be invested in currency in U. S. 6% stock, worth 112, the interest being payable in gold, which is at a premium of 10 per cent., to produce an income of $1320? *Ans.* $22400.

NOTE.—The premium on gold must be added to the interest on $1, since the interest is payable in gold.

4. What must be paid for such an amount of Virginia 6% State stocks, selling at 80, as will produce an income of $5412? *Ans.* $72160.

5. What sum must be invested in 6% stocks, worth 95, to give an income of $4500? *Ans.* $71250.

DIVIDENDS.

298*g*. DIVIDEND is the share of the profits divided among the stockholders in proportion to the amount of stock held by each, and is generally estimated at a certain rate per cent. on the par value of the stock.

GROSS EARNINGS, or GROSS RECEIPTS, are all the moneys received from the regular business of the company.

NET EARNINGS are the gross earnings diminished by all the expenses of the business. All dividends are from the net earnings.

298*h*. To determine the rate of dividend.

1. The capital of a bank is $200000; its gross earnings are $30000, and its expenses are $12000: what is the rate of dividend?

298*g*. What is a dividend? What are gross earnings? What are net earnings?—298*h*. How do you determine the rate of dividend?

Analysis.—The gross earnings being $30000, and the expenses $12000, the net earnings are equal to the difference of $30000, and $12000, or $18000.

OPERATION.

$$30000 - 12000 = \$18000 \text{ net earnings};$$
$$18000 \div 200000 = .09 = 9\% \text{ rate.}$$

The base, $200000, and percentage, $18000, being known, the rate is found by dividing the percentage by the base. Hence,

Rule.—*Divide the net earnings by the capital, and the quotient will be the rate of dividend.*

2. The capital of an insurance company is $150000, its expenses $9500, and its gross earnings $35000: what was the rate of dividend? *Ans.* 17 per cent.

3. The capital of a bank is $500000, its gross earnings $25000, and its expenses $10500: what was the rate of dividend? *Ans.* $2\frac{9}{10}$ per cent.

4. The gross earnings of a railroad company whose capital was $3750000, was $675680, and its expenses $425680: what would be its rate of dividend if the entire net earnings were divided? *Ans.* $6\frac{2}{3}$ per cent.

5. The net earnings of a bank are $75000, and its capital is $375000: how much would a person owning $21000 of stock receive of the dividend? *Ans.* $4200.

Note.—Find the rate of dividend and multiply the par value by it, and the product will be the amount of dividend due to the stock.

6. A merchant owns 27 shares of railroad stock, par 100. The capital of the company is $6750000; its gross earnings were, last year, $1260000, and its expenses $774000: what was the rate of dividend, and what amount did the merchant receive on his shares?

Ans. $7\frac{1}{5}\%$ = Rate: $194.40 = Amount.

DISCOUNT.

299. DISCOUNT is an allowance made for the payment of money before it is due.

300. PRESENT VALUE of a debt, due at a future time, is such a sum as, being placed at interest until the debt becomes due, would increase to an amount equal to the debt.

301. FACE of a note, is the amount named in a note.

302. DISCOUNT on a note, is the difference between the face of a note and its present value.

1. I give my note to Mr. Wilson, for $107, payable in 1 year: what is the present value of the note, if the interest is 7 per cent.? What the discount?

ANALYSIS.—Since $1 in 1 year, at 7 per cent., will amount to $1.07, the present value of $1.07 is $1: hence,

$$1 : 1+.07 :: \text{present val. of any sum} : \text{face of note};$$

or, $$\text{Present val.} = \frac{\text{face of note}}{1+.07}; \text{ Hence,}$$

RULE.—*Divide the face of the note by 1 dollar plus the interest of 1 dollar for the given time, and the quotient will be the present value: take this sum from the face of the note, and the remainder will be the discount.*

EXAMPLES.

1. What is the present value of a note for $1828.75, due in 1 year, and bearing an interest of 4½ per cent.?

2. A note of $1651.50 is due in 11 months, but the person to whom it is payable, sells it with the discount off, at 6 per cent.: how much shall he receive?

NOTE.—When payments are to be made at different times, *find the present value of the sums separately, and their sum will be the present value of the note.*

3. What is the present value of a note for $10500, on which $900 are to be paid in 6 months, $2700 in one year, $3900 in eighteen months, and the residue at the expiration of 2 years, the rate of interest being 6 per cent. per annum?

4. What is the discount of $4500, one-half payable in six months, and the other half at the expiration of a year, at 7 per cent. per annum?

5. What is the present value of $5760, one-half payable in 3 months, one-third in 6 months, and the rest in 9 months, at 6 per cent. per annum?

6. Mr. A gives his note to B, for $720, one-half payable in 4 months, and the other half in 8 months: what is the present value of said note, discount at 5 per cent. per annum?

7. What is the difference between the interest and discount of $750, due nine months hence, at 7 per cent.?

8. What is the present value of $4000, payable in 9 months, discount 4½ per cent. per annum?

9. Mr. Johnson has a note against Mr. Williams, for $2146.50, dated August 17th, 1862, which becomes due Jan. 11th, 1863: if the note is discounted at 6 per cent., what ready money must be paid for it September 25th, 1862?

10. C owes D $3456, to be paid October 27th, 1862; C wishes to pay on the 24th of August, 1858, to which D consents: how much ought D to receive, interest at 6 per cent.?

11. What is the present value of a note of $4800, due 4 years hence, the interest being computed at 5 per cent. per annum?

12. A man having a horse for sale, offered it for $225 cash in hand, or $230 at 9 months; the buyer chose the latter: did the seller lose or make by his offer, supposing money to be worth 7 per cent.?

---

299. What is Discount?—300. What is present value of a debt?

301. What is face of a note?

302. What is discount on a note? What is the rule for finding the present value? What is the discount?

BANKING.

303. A BANK is a joint stock company, incorporated for the purpose of loaning money and issuing bills for currency.

304. BANK DISCOUNT is the deduction made by a bank, for the use of money payable at a future time.

305. A PROMISSORY NOTE is a written obligation to pay a specified sum at a future time, named in the note.

By the custom of banks, the interest is always paid in advance.

306. By mercantile usage, a note at bank does not legally fall due until 3 days after the expiration of the time named on its face; these days are called, *Days of Grace.*

The present value of a note, is sometimes called its Cash Value, or Proceeds, or Avails.

CASE I.

307. To find the bank discount and present worth.

RULE.—*Add the 3 days of grace to the time the Note has to run, and then calculate the interest at the given rate: the result is the bank discount, and the face of the note diminished by the discount, is the present worth.*

EXAMPLES.

1. What is the bank discount on a note for $350, payable 3 months after date, at 7 per cent. interest?

2. What is the bank discount of a note of $1000, payable in 60 days, at 6 per cent. interest?

3. A merchant sold a cargo of cotton for $15720, for which he receives a note at 6 months: how much money will he receive at a bank for this note, discounting it at 6 per cent. interest?

4. What is the bank discount on a note of $556.27, payable in 60 days, discounted at 6 per cent. interest?

5. A has a note against B, for $3456, payable in three months; he gets it discounted, at 7 per cent. interest: how much does he receive?

6. What is the bank discount on a note of $367.47, having 1 year, 1 month, and 13 days to run, as shown by the face of the note, discounted at 7 per cent.?

7. What is the bank discount, on a note at 5 months, for $672.50, dated August 7th, 1862, the interest being 6 per cent.?

8. Mr. Jones gave his note at bank, for 8 months, at 7 per cent., for $1670: what was the product?

NEW YORK, July 3d, 1860.

9. For value received, I promise to pay to John Jones, on the 20th of November next, six thousand five hundred and seventy-nine dollars and fifteen cents.

PHILIP MASON.

What will be the discount on this, if discounted on the 1st of August, at 6 per cent. per annum?

CASE II.

308. To make a note due at a future time, whose present value shall be a given amount.

1. For what sum must a note be drawn at 3 months, so that, when discounted at a bank at 6 per cent., the amount received shall be $500?

ANALYSIS.—If we find the interest on 1 dollar for the given time, and then subtract that interest from 1 dollar, the remainder

---

303. What is a bank?—304. What is bank discount?—305. What is a promissory note? How is the interest paid at a bank?

306. By mercantile usage, when does a note at bank fall due? What are these days called?

307. How do you find the discount and present worth?

308. How do you make a note payable at a future time, whose present value shall be a given amount?

will be the *present value* of 1 dollar, due at the expiration of that time. Then, the number of times which the present value of the note contains the present value of 1 dollar, will be the number of dollars for which the note must be drawn.

OPERATION.

Interest of $1 for the time, 3 mos. and 3 days = $0.0155, which taken from $1, gives present value of $1 = 0.9845; then, $500 ÷ 0.9845 = $507.872 + = face of note.

Proof.—Bank interest on $507.872, for 3 months, including 3 days of grace, at 6 per cent. = 7.872; which being taken from the face of the note, leaves $500 for its present value.

Hence, we have the following

Rule.

Divide the present value of the note by the present value of 1 dollar, reckoned for the same time and at the same rate of interest, and the quotient will be the face of the note.

Examples.

1. For what sum must a note be drawn, at 7 per cent., payable on its face in 1 year 6 months and 15 days, so that, when discounted at bank, it shall produce $307.27?

2. A note is to be drawn, having on its face 8 months and 12 days to run, and to bear an interest of 7 per cent., so that it will pay a debt of $5450: what is the amount?

3. What sum, 6 months and 9 days from July 18th, 1862, drawing an interest of 6 per cent., will pay a debt of $674.89 at bank, on the 1st of August, 1862?

4. Mr. Jones bought a bill of goods amounting to $1683.75, and paid the bill with a note running 6 months: for what amount must the note be drawn, that, when discounted, the merchant will receive exactly the amount of the bill, supposing interest to be at 7 per cent.?

5. Mr. Wilson is indebted at the bank in the sum of $367.464, which he wishes to pay by a note at 4 months, with interest at 7 per cent.: for what amount must the note be drawn?

EXCHANGE.

309. EXCHANGE is a process of remitting money from one place to another, by means of written orders.

310. A BILL OF EXCHANGE is a written order from one person to another, to pay to a third party a specific sum, at a given time.

311. The DRAWER, or MAKER, is the person who draws the bill.

312. The DRAWEE is the person on whom the bill is drawn.

313. The PAYEE is the person to whom the money is ordered to be paid; and he is the owner of the bill.

314. The PAYER, or REMITTER, is any person who purchases a bill of exchange, either of the drawer or payee.

315. An ACCEPTANCE is an agreement of the drawee, to pay the bill when it falls due, and is signified by writing, "Accepted," with his signature, on the face of the bill.

316. An INDORSEMENT of a bill, by the payee, is the writing of his name on the back of it. This transfers the bill to any person who may rightfully hold it. Or, if he writes on the back, "Pay to John James or order," then the bill is transferred to Mr. James.

317. DAYS OF GRACE are days granted to the person who pays a bill, after the time named in it has expired. In the United States and Great Britain, 3 days are generally allowed.

318. An INLAND BILL is when the drawer and drawee both reside in the same country.

319. The COURSE OF EXCHANGE is the difference between the face of a bill, and the price paid for it.

320. PAR OF EXCHANGE is when the face of a bill and the price paid, are the same.

321. Premium is when the price paid is greater than the face of the bill, and the exchange is then said to be, *above par.*

322. Discount is when the price paid is less than the face of the bill, and the exchange is then said to be, *below par.*

Rules of Articles 257, 262, and 308, apply to all cases of Exchange.

Examples.

1. A merchant at Chicago wishes to pay a bill in New York, amounting to $3675, and finds that exchange is 1¼ per cent. premium: what must he pay for his bill?

2. A merchant in Philadelphia wishes to remit to Charleston, $8756.50, and finds exchange to be 1 per cent. below par: what must he pay for the bill?

3. A merchant in Mobile wishes to pay in New York, $6584, and exchange is 2½ per cent. premium: how much must he pay for such a bill?

4. A merchant in Boston wishes to pay in New Orleans, $4653.75; exchange between Boston and New Orleans is 1½ per cent. below par: what must he pay for a bill?

5. A merchant in New York has $3690, which he wishes to remit to Cincinnati; the exchange is 1¼ per cent. below par: what will be the amount of his bill?

6. What must be paid for the following bill, when exchange is at 2⅛ per cent. premium?

$950.50. New York, May 9th, 1860.

Thirty days after date, pay to Samuel Lewis or order, Nine hundred and Fifty dollars and Fifty cents, and charge the same to my account. Daniel Shaw.

To Messrs. Jones & Willis.

7. If exchange is at 1½ per cent. premium, what bill will $3950 purchase?

8. If exchange is at 1¾ per cent. premium, what bill of exchange can be bought for $762, uncurrent funds, supposing a discount of ½ per cent. is charged on the funds?

FOREIGN BILLS.

323. A Foreign Bill of Exchange is one in which the drawer and drawee live in different countries.

England and France are the principal countries with which the United States have exchanges.

In all Bills of Exchange on England, the £ sterling is the unit or base, and is still reckoned at its former value of $\$4\frac{4}{9} = \$4.4444+$, instead of its present value, $4.8665.

Hence, ... £1 = $4.4444+
Add 9½ per cent, ...4222+
Gives the present value of £1, $4.8666.

Hence, to get the *true par value* of Exchange on England, we must add 9½ per cent. to the exchange computed on the *nominal base.*

324. To find the value of a bill of exchange in sterling money, in United States money.

1. A merchant in New York, wishes to remit to England, a bill of exchange for £125 15s. 6d.: how much must he pay for this bill, when exchange is at 9½ per cent. premium?

£125 15s. 6d. . . . . . . = £125.775
Add 9½ per cent. . . . . . 11.9486+
gives amount in £'s, ... £137.7236+

The pounds and decimals of a pound are reduced to dollars, by multiplying by 40 and dividing by 9,—giving, in this case, $612.105 (Art. 323).

Rule.—I. *Reduce the amount of the bill to pounds and decimals of a pound, and then add the premium of exchange:*

II. *Multiply the result by 40, and divide the product by 9: the quotient will be the answer in United States money.*

Examples.

1. A merchant shipped 100 bales of cotton to Liverpool, each weighing 450 pounds. They were sold at 7½d. per pound, and the freight and charges amounted to £187 10s. He

sold his bill of exchange at 9¾ per cent. premium: how much should he receive in United States money?

2. There were shipped from Norfolk, Va., to Liverpool, 85 hhd. of tobacco, each weighing 450 pounds. It was sold at Liverpool, for 12½d. per pound, and the expenses of freight and commissions were £92 1s. 8d. If exchange on New York is at a premium of 9¼ per cent., what should the owner receive for the bill of exchange, in United States money?

325. Exchange on France.

The *unit* or *base* of the French Currency, is the French franc, of the value of 18 cents 6 mills. The franc is divided into tenths, called *decimes*, corresponding to our *dimes*, and into *centimes*, corresponding to *cents*. Thus, 5.12 is read, 5 francs and 12 centimes.

All Bills of Exchange on France, are drawn in francs. Exchange is quoted in New York, at so many francs and centimes to the dollar.

1. What will be the value of a bill of exchange for 4536 francs, at 5.25 to the dollar?

Analysis.—Since 1 dollar will buy 5.25 francs, the bill will cost as many dollars as 5.25 is contained times in the amount of the bill: Hence,

OPERATION.

5.25) 4536 ($864, *Ans.*

Rule.—*Divide the amount of the bill by the value of $1 in francs: the quotient is the amount to be paid in dollars.*

Examples.

1. What will be the amount to be paid, United States money, for a bill of exchange on Paris, of 6530 francs, exchange being 5.14 francs per dollar?

2. What will be the amount to be paid, in United States money, for a bill of exchange on Paris, of 10262 francs, exchange being 5.09 francs per dollar?

3. What will be the value, in United States money, of a bill for 87595 francs, at 5.16 francs per dollar?

EQUATION OF PAYMENTS.

326. Equation of Payments is the operation of finding the time in which several amounts, due at different times, may be paid without loss of interest to either party. The time of payment thus found is called the *mean* time, or *equated* time.

327. When the times of payment are reckoned from the same date.

1. If I owe Mr. Wilson \$2, to be paid in 6 months, from July 1st; \$3, to be paid in 8 months; and \$1, to be paid in 12 months: what is the mean time of payment?

Analysis.—The interest on all the amounts, to their various times of payment, equals the interest of \$48 for one month; but \$48 is equal to the sum of all the products which arise from multiplying each amount by the time at which it becomes due. It will take \$6 (the sum of the payments) as many months, to produce the same interest, as \$6 is contained times in \$48, which is 8 times: therefore, the equated time is 8 months.

OPERATION.

Int. of	\$2	for 6 mos.	= int. of	\$12	for 1 mo.	2 ×	6 = 12.
"	\$3	" 8 "	= int. of	\$24	" "	3 ×	8 = 24.
"	\$1	" 12 "	= int. of	\$12	" "	1 ×	12 = 12.
	\$6			\$48		6	6) 48.

Rule.

Multiply each payment by the time before it becomes due, and divide the sum of the products by the sum of the payments: the quotient will be the mean time.

Examples.

1. A merchant owes \$600, to be paid in 12 months from January 1st; \$800, to be paid in 6 months, and \$900, to be paid in 9 months: what is the equated time of payment?

2. A owes B \$600; one-third is to be paid in 6 months from August 1st; one-fourth in 8 months, and the remainder in 12 months: what is the mean time of payment?

3. A merchant has due him \$300, to be paid in 60 days; \$500, to be paid in 120 days; and \$750, to be paid in 180 days: what is the equated time of payment?

328. When the times are reckoned from different dates.

1. I owe Mr. Wilson \$100, to be paid on the 15th of July; \$200, on the 15th of August; and \$300, on the 9th of September: what is the mean time of payment?

Analysis.—The earliest date named, or any day previous to it, may be taken as the date from which the times are reckoned. If the earliest payment is the date of reckoning, the first multiplier is 0, and consequently the first product is 0, but the payment must be added, in finding the sum of the payments.

OPERATION.

From 1st of July to 1st payment, 14 days.
" " " to 2d payment, 45 days.
" " " to 3d payment, 70 days.

$$\begin{array}{rcrcr} 100 & \times & 14 & = & 1400 \\ 200 & \times & 45 & = & 9000 \\ \underline{300} & \times & 70 & = & \underline{21000} \\ 600 & & 6|00 &) & \underline{314|00} \\ & & & & 52\frac{1}{3} \end{array}$$

Hence, the equated time is 52$\frac{1}{3}$ days from the 1st of July; that is, on the 23d day of August.

But if we estimate the time from the 15th of July, we shall have,

From July 15th to 1st payment, 0 days.
" " " to 2d payment, 31 days.
" " " to 3d payment, 56 days.

Then,

$$\begin{array}{rcrcr} 100 & \times & 0 & = & 000 \\ 200 & \times & 31 & = & 6200 \\ \underline{300} & \times & 56 & = & \underline{16800} \\ 600 & & 6|00 &) & \underline{230|00} \\ & & & & 38\frac{1}{3} \end{array}$$

Hence, the payment is due in $38\frac{1}{3}$ days from July 15th; or, on the 23d of August—the same as before.

Rule.—*Assume the earliest date as the point of reckoning. Find the number of days intervening between this date and that of each payment, and multiply each payment by its number of days: add the products, and divide the sum by the sum of the amounts, and the quotient will be the equated time in days. This number, reckoned from the earliest date, will give the equated time.*

Examples.

1. I owe $1000, to be paid on the 1st of January; $1500, on the 1st of February; $3000, on the 1st of March; and $4000, on the 15th of April: reckoning from the 1st of January, and calling February 28 days, on what day must the money be paid?

Note.—If one of the payments, as in the above example, is due on the day from which the equated time is reckoned, its corresponding product will be nothing, but the payment must still be added in finding the sum of the payments.

2. Mr. Jones agreed to give his note to Mr. Wilson, for goods to be delivered according to the following bills, with a credit of sixty days on each bill:

15th of January,	a bill of $3750
10th of February,	a bill of 3000.
6th of March,	a bill of 2400.
8th of June,	a bill of 2250.

At what time must the note be made payable?

3. A merchant bought several lots of goods, as follows:

A bill of $650,	June 6th.
A bill of 890,	July 8th.
A bill of 7940,	Aug. 1st.

Now, if the credit is 6 months, how many days from December 6th before the note becomes due? At what time?

4. Mr. Tappan rendered to Mr. Duck the following bill of goods sold him:

NEW YORK, May 6th, 1861.

G. DUCK, To A. TAPPAN, *Dr.*

1861.	Jan.	2d.	To merchandise on 4 mos.,	$1275
	Jan.	31st.	“ “ “	1600
	Feb.	5th.	“ “ “	595
	March	19th.	“ “ “	675

What is the equated time of payment of this bill?

5. A merchant owes the following bill:

1860.	May	6th.	To merchandise on 6 mos.,	$ 892
	June	19th.	“ “ “	1063
	July	4th.	“ “ “	2585

What is the equated time of payment from July 4th?

328. **To settle, by payment of cash, an account in which there are debtor and creditor items.**

BALANCE is a term which denotes the difference between the debtor and creditor sides of an account. There are three balances; viz., *merchandise balance, interest balance,* and *cash* or *net balance.*

1. MERCHANDISE BALANCE is the balance, in which interest on the items is not considered.

2. INTEREST BALANCE is the balance of the interest of the items of the two sides.

3. CASH OR NET BALANCE is the balance which arises after adding the merchandise and interest balances to the proper sides of the account.

In equating the cash balance, interest is allowed on each item, and the balance of interest becomes a new item, and must be added to its proper side of the account.

1. It is required to find the cash balance of the following account, on April 1st, 1861:

Dr. ROBERT CHEAP. *Cr.*

1861	Jan.	7	To Merchandise	$750	00	Feb.	9	By Cash	$560	00
	“	16	“ “	816	00	M'h	16	“ Merchandise	175	00
	Feb.	14	“ “	195	00	“	27	“ “	260	00
	Apr.	1	“ Interest Bal.	19	54					
						Apr.	1	“ Cash Bal.	785	54

Interest on each item is calculated from April 1st. We reckon the number of days from April 1st to each date, and these are used as multipliers.

Dr. items.	*Cr. items.*
$195 \times 46 = 8970$	$560 \times 51 = 28560$
$816 \times 75 = 61200$	$175 \times 16 = 2800$
$750 \times 84 = 63000$	$260 \times 5 = 1300$
133170	32660

The difference of the sums of these products is multiplied by the interest of $1 for one day, and this gives the interest balance. The difference of the products is $133170 - 32660 = 100510$, and the interest for 1 day is $.07 \div 360$; hence,

$$100510 \times \tfrac{.07}{360} = \tfrac{7035.70}{360} = \$19.54 = \text{balance of interest.}$$

This balance belongs to the debtor side, because the sum of the debtor products is the greater. It is, therefore, added to the debtor side, and the final balance is the cash or net balance.

Rule.—I. *Take the latest date of the account, or any later date, at which the balance is to be struck, as the point of reckoning, and find the days between this date and the date of each item; and consider these days as multipliers:*

II. *Multiply each item by its multiplier; then take the difference of the sums of these products, and multiply it by the interest of $1 for one day: the result will be the interest balance, which is to be added to the side having the greater sum:*

III. *Then find the difference of the sums in the two columns, and this will be the cash balance.*

2. What was the cash balance on Aug. 1st, 1862, of the following account?

Dr. Richard Moneypenny. *Cr.*

1862	May	16	To Cash	$716	00	May	1	By Merchandise	$975	00
	"	21	" Merchandise	595	00	"	12	" "	1640	00
	June	19	" "	1697	75	June	17	" Cash	500	00
	July	7	" "	950	00	July	1	" Mdse.	615	00
	"	13	" Cash	176	00					

ASSESSING TAXES.

329. A Tax is a certain sum required to be paid by the inhabitants of a town, county, or State, for the support of government. It is generally collected from each individual in proportion to the amount of his property.

Property is of two kinds, real and personal. Real property, or real estate, is fixed property, such as houses and lands. Personal property is movable property, such as money, furniture, &c.

In some States, however, every white male citizen over the age of twenty-one years is required to pay a certain tax. This tax is called a poll-tax; and each person so taxed is called a *poll.*

330. In assessing taxes, the first thing to be done is to make a complete inventory of all the property in the town on which the tax is to be laid. If there is a poll-tax, a full list of the polls must be made, and the number multiplied by the tax on each poll, and the product must be subtracted from the whole tax to be raised by the town; the remainder will be the amount to be raised on the property. This being done, *the whole tax to be raised must be divided by the amount of taxable property, and the quotient will be the rate per cent. of tax. Then this quotient must be multiplied by the inventory of each individual, and the product will be the tax on his property.*

Examples.

1. A certain town is to be taxed $4280; the property on which the tax is to be levied is valued at $1000000. Now, there are 200 polls, each taxed $1.40. The property of A is valued at $2800, and he pays 4 polls;

B's at $2400, pays 4 polls; E's at $7242, pays 4 polls.
C's at $2530, pays 2 " F's at $1651, pays 6 "
D's at $2250, pays 6 " G's at $1600.80, pays 4 "

What will be the tax on 1 dollar, and what will be A's tax, and also that of each on the list ?

First, $1.40 × 200 = $280, amount of poll-tax.
$4280 — $280 = $4000, amount to be levied on property.
Then, $4000 ÷ $1000000 = 4 mills on $1.

Now, to find the tax of each, as A's, for example:

A's inventory, ...	$2800
	.004
	11.20
4 polls, at $1.40 each, ...	5.60
A's whole tax, ...	$16.80

In the same manner the tax of each person in the township may be found.

Having found the per cent., or the amount to be raised on each dollar, form a table showing the amount which certain sums would produce at the same rate per cent. Thus, after having found, as in the last example, that 4 mills are to be raised on every dollar, we can, by multiplying in succession by the numbers 1, 2, 3, 4, 5, 6, 7, 8, &c., form the following

TABLE.

$		$	$		$	$		$
1	gives	0.004	20	gives	0.080	300	gives	1.200
2	"	0.008	30	"	0.120	400	"	1.600
3	"	0.012	40	"	0.160	500	"	2.000
4	"	0.016	50	"	0.200	600	"	2.400
5	"	0.020	60	"	0.240	700	"	2.800
6	"	0.024	70	"	0.280	800	"	3.200
7	"	0.028	80	"	0.320	900	"	3.600
8	"	0.032	90	"	0.360	1000	"	4.000
9	"	0.036	100	"	0.400	2000	"	8.000
10	"	0.040	200	"	0.800	3000	"	12.000

This table shows the amount to be raised on each sum in the column under $'s.

EXAMPLES.

1. Find the amount of B's tax from this table.

B's tax on $2000	is	$8.000
B's tax on $400	is	$1.600
B's tax on 4 polls, at $1.40,	is	$5.600
B's total tax . . .	is	$15.200

2. Find the amount of C's tax from the table.

C's tax on $2000	is	$8.000
C's tax on 500	is	$2.000
C's tax on 30	is	$0.120
C's tax on 2 polls	is	$2.800
C's total tax . .	is	$12.920

In a similar manner, we might find the taxes to be paid by D, E, &c.

3. If the people of a town vote to tax themselves $1500 to build a public hall, and the property of the town is valued at $300,000, what is D's tax, whose property is valued at $2450?

4. In a school district a school is supported by a tax on the property of the district, valued at $121340. A teacher is employed for 5 months, at $40 a month, and contingent expenses are $42.68: what will be a farmer's tax, whose property is valued at $3125?

◆◆◆

CUSTOM-HOUSE BUSINESS.

331. DUTIES are sums of money levied by government on goods imported from foreign countries.

332. A SPECIFIC DUTY is a certain sum on a particular kind of goods named.

333. An AD VALOREM DUTY is a certain per cent. on the cost of the goods in the country from which they are imported.

334. A PORT of ENTRY is a port designated by law, where goods from a foreign country may be landed.

335. TONNAGE is a tax levied on vessels, according to their size, for the privilege of entering a port.

336. A CUSTOM-HOUSE is an establishment created by government, at a port of entry, for the collection of duties.

337. REVENUE is the income of government, derived from all sources. These sources are, Duties, Tonnage, and Taxes.

338. ALLOWANCES are deductions made from the weights and measures of goods, on account of the bags, casks, and boxes which contain them.

339. GROSS WEIGHT is the whole weight of the goods, together with that of the casks, bags, and boxes which contain them.

NET WEIGHT is what remains after all deductions are made.

340. DRAFT was an allowance formerly made from the gross weight, but it is now abolished by law.

341. TARE is an allowance made for the weight of the boxes, barrels, or bags containing the commodity, and is of three kinds: 1st, Legal tare, or such as is established by law; 2d, Customary tare, or such as is established by the custom among merchants; and 3d, Actual tare, or such as is found by removing the goods and actually weighing the boxes or casks in which they are contained.

In estimating the allowance for tare on all chests, boxes, cases, casks, bags, or other envelope or covering, of all articles imported liable to pay duty, the Collector may estimate the tare according to the invoice, or may ascertain the actual tare.

342. CUSTOMARY TARE on liquors in casks, is sometimes allowed, on the supposition that the cask is not full, or what is called its *actual wants;* and then an allowance of 5 per cent. for leakage.

A tare of 10 per cent. is allowed on porter, ale, and beer, in bottles, on account of breakage, and 5 per cent. on all other liquors in bottles. At the custom-house, bottles of the common size are estimated to contain 2¼ gallons the dozen.

NOTE.—For Tables of Tare and Duty, see Ogden on the Tariff of 1842.

EXAMPLES.

1. What will be the duty on 125 cartons of ribbons, each containing 48 pieces, and each piece weighing 3 oz. net, and paying a duty of $2.50 per pound?

2. What will be the duty on 225 bags of coffee, each weighing gross 160 lb., invoiced at 6 cents per pound; 2 per cent. being the actual tare, and 20 per cent. the duty?

3. What duty must be paid on 275 dozen bottles of claret, estimated to contain 2¼ gallons per dozen, 5 per cent. being allowed for breakage, and the duty being 35 cents per gallon?

4. A merchant imports 175 cases of indigo, each case weighing 194 lb. gross; 15 per cent. is the actual tare, and the duty 5 cents per pound: what duty must he pay on the whole?

CURRENCY.

343. CURRENCY is what passes for money. There are three kinds of Currency: 1st, The coins of the country; 2d, Foreign coins, having a fixed value; 3d, Treasury Notes and Bank Notes.

NOTE.—The foreign coins most in use in this country, are the English shilling, valued at 24 cents 3 mills; the English sovereign, valued at $4.8665; the French franc, valued at 18 cents 6 mills; and the five-franc piece, valued at $0.93.

344. The Currency of the United States, is the decimal currency of dollars, cents, and mills.

In some of the States, the old local currency of pounds, shillings, and pence, is yet preserved.

The following table shows the value of a pound in dollars, in each currency as formerly used; the number of shillings and pence in a dollar, and the value of $\$1$ in pence:

In English Currency,	£1 = $\$4.8665$, by a recent law.	
In N. E., Inda., Ill., Missouri, Va., Ky., Tenn., Miss., & Texas	£1 = $\$3\frac{1}{3}$, and $\$1$ = 6s.	= 72d.
New York, Ohio, Michigan, and N. Carolina	£1 = $\$2\frac{1}{2}$, and $\$1$ = 8s.	= 96d.
In N. J., Pa., Del., and Maryland	£1 = $\$2\frac{2}{3}$, and $\$1$ = 7s. 6d.	= 90d.
In S. C. and Georgia,	£1 = $\$4\frac{2}{7}$, and $\$1$ = 4s. 8d.	= 56d.
In Canada and Nova Scotia	£1 = $\$4$, and $\$1$ = 5s.	= 60d.

The pound is always reckoned at 20 shillings, and the dollar at 100 cents..

ANALYSIS.

345. An ANALYSIS of a proposition, is an examination of its separate parts, and their connection with each other.

In analyzing, we reason from a *given number* to its *unit*, and then from this unit to the *required* number.

346. To find the cost of several things, when the price of 1 and number are known.

1. If 9 bushels of wheat cost 18 dollars, what will 27 bushels cost?

ANALYSIS.—One bushel costs $\frac{1}{9}$ as much as 9 bushels: $\frac{1}{9}$ of 18 dollars is 2 dollars. 27 bushels will cost 27 times as much as 1 bushel; $\$2 \times 27 = \$54$; therefore, 27 bu. will cost $54.

OPERATION.

$$\frac{1}{9} \text{ of } \$18 = \frac{18}{9} = \$2 = \text{cost of 1 bushel;}$$
$$\$2 \times 27 = \$54 = \text{cost of 27 bushels.}$$

NOTE.—The two expressions may be combined in one; thus,

$$\frac{1}{\cancel{9}} \text{ of } \frac{18}{1} \times \frac{\overset{3}{\cancel{27}}}{1} = \frac{54}{1} = \$54.$$

2. If 7 yards of cloth cost $49, what will 16 yards cost?

3. If 12 barrels of flour cost $72, what will 37 bbl. cost?

4. What will be the cost of 9½ yards of cloth, if 35 yd. cost $70?

5. How many sheep, at 4 dollars a head, must I give for 6 cows, worth 12 dollars apiece?

347. To find the cost of articles in dollars and cents when the price is given in shillings and pence.

6. What will 12 yards of cloth cost, at 5 shillings a yard, New York currency?

Analysis.—Since 1 yard costs 5 shillings, 12 yards will cost 12 times 5 shillings, or 60 shillings: and as 8 shillings make 1 dollar, New York currency, there will be as many dollars as 8 is contained times in 60, viz., 7½.

OPERATION.

$$\frac{\overset{3}{\cancel{12}} \times 5}{\underset{2}{\cancel{8}}} = \frac{15}{2} = \$7\tfrac{1}{2} = \$7.50.$$

7. What will 56 bushels of oats cost, at 3s. 6d. a bushel, New England currency?

Analysis.—3s. 6d. = 42d., and 72d. make 1 dollar: the cost in pence, will be 56 × 42, and this product divided by 72, will give the answer.

OPERATION.

$$\frac{56 \times 42}{72} = \frac{7 \times 42}{9} = \frac{7 \times 14}{3} = \frac{98}{3} = \$32.666+.$$

Rule.—*Multiply the commodity by the price and divide the product by the value of a dollar, expressed in the same unit.*

8. What will 18 yards of satinet cost, at 3s. 9d. a yard, Pennsylvania currency?

9. What will 7½ lb. of tea cost, at 6s. 8d. a pound, New England currency?

10. What will be the cost of 120 yards of cotton cloth, at 1s. 5d. a yard, Georgia currency?

11. What will be the cost, in New York currency?

12. What will be the cost, in New England currency?

13. What will be the cost of 75 bushels of potatoes, at 3s. 6d., New York currency?

14. What will it cost to build 148 feet of wall, at 1s. 8d. per foot, N. Y. currency?

15. What will a load of wheat, containing 46½ bushels, come to, at 10s. 8d. a bushel, N. Y. currency?

16. What will 7 yards of Irish linen cost, at 3s. 4d. a yard, Penn. currency?

17. How many pounds of butter, at 1s. 4d. a pound, must be given for 12 gallons of molasses, at 2s. 8d. a gallon?

18. What will be the cost of 12 cwt. of sugar, at 9d. per lb., N. Y. currency?

19. What will be the cost of 9 hogsheads of molasses, at 1s. 3d. per quart, N. E. currency?

20. How many days' work, at 7s. 6d. a day, must be given for 12 bushels of apples, at 3s. 9d. a bushel?

348. Analysis, when the numbers are fractional.

21. If 3¾ pounds of tea cost $3⅓, what will 9 pounds cost?

ANALYSIS.—3¾ lb. $=\frac{15}{4}$ lb., and $3⅓ $=\$\frac{10}{3}$: since $\frac{15}{4}$ lb. cost $\$\frac{10}{3}$, 1 lb. will cost $\$\frac{10}{3} \div \frac{15}{4} = \frac{10}{3} \times \frac{4}{15}$, and 9 lb. will cost 9 times as much, or $\frac{10}{3} \times \frac{4}{15} \times 9$.

OPERATION.

$$\frac{\overset{2}{\cancel{10}}}{\cancel{3}} \times \frac{4}{\underset{\cancel{3}}{\cancel{15}}} \times \overset{\cancel{3}}{\cancel{9}} = \$8.$$

22. If 5⅓ bushels of potatoes cost $2⅘, how much will 12½ bushels cost?

23. If 1 acre of land costs ⅙ of 3/7 of ⅘ of $50, what will 3½ acres cost?

24. If ½ of ¾ of a gallon of wine costs ⅝ of a dollar, what will 5½ gallons cost?

25. A person purchased 4/7 of a vessel, and divided it into 5 equal shares, and sold each of those shares for $1200: what was the value of the whole vessel?

26. What number is that, $\frac{6}{7}$ of which is 18?

Analysis.—Since $\frac{6}{7}$ of a number is 18, $\frac{1}{7}$ of it will be $\frac{1}{6}$ of 18, which is 3: if $\frac{1}{7}$ of a number is 3, the number is equal to $7 \times 3 = 21$.

27. 32 is $\frac{4}{9}$ of what number?

28. 63 is $\frac{7}{12}$ of what number?

29. A traveler, after going 196 miles, found that he had performed $\frac{7}{11}$ of his journey: how long was his journey, and how much had he yet to perform?

30. A father gave his younger son $420, which was $\frac{3}{4}$ of what he gave to his elder son; and 3 times the elder son's portion, was $\frac{1}{3}$ the value of the father's estate: what was the value of the estate?

349. Proportions of Numbers.

31. If 6 men can build a boat in 120 days, how long will it take 24 men to build it?

Analysis.—Six men, in 120 days, will do $120 \times 6 = 720$ days' work; hence, 1 man would do the work in 720 days; 24 men will do the work in $\frac{1}{24}$ of that time; therefore, $720 \div 24 = 30$ days.

OPERATION.

$$120 \times 6 = 720 \text{ days} = \text{time 1 man can do it.}$$
$$\frac{720}{24} = 30 \text{ days} = \text{time 24 men can do it.}$$

32. If 6 men can do a piece of work in 10 days, how long will it take 5 men to do it?

33. If 36 men can build a house in 16 days, how long will it take 12 men to build it?

34. If 3 pipes can empty a reservoir of water in 7 days, in what time can 8 pipes empty it?

35. If, by working 8 hours a day, a certain number of men can do a piece of work in 15 days, in what time could they do it, if they work 11 hours a day?

36. A piece of ground is 32 rods long, and 19 rods wide: what must be the width of another piece that shall have the same square measure, and whose length is 25 rods?

Analysis.—The square measure is found by multiplying the length and breadth; $32 \times 19 = 608$ sq. rods: the square measure of the second piece is the same, or 608 sq. rods, and its length is 25 rods; its width must be the quotient of 608 divided by 25, which is $24\frac{8}{25}$ rods.

OPERATION.

$$\left.\begin{array}{r} 32 \times 19 = 608 \text{ sq. rods} = \\ \text{area of 1st piece.} \\ \frac{608}{25} = 24\frac{8}{25} \text{ rods} = \text{width} \\ \text{of 2d piece.} \end{array}\right\} \text{Or, } \frac{32 \times 19}{25} = 24\tfrac{8}{25} \text{ rods.}$$

37. If a piece of cloth is 9 feet long and 3 feet wide, how long must be a piece of cloth that is $2\frac{3}{5}$ feet wide, to contain the same number of yards?

38. If it take 44 yards of carpeting, that is $1\frac{1}{4}$ yards wide, to cover a floor, how many yards, of $\frac{7}{8}$ of a yard wide, will it take to cover the same floor?

39. If a piece of wall-paper, 14 yards long and $1\frac{1}{2}$ feet wide, will cover a certain piece of wall, how long must another piece be, that is 2 feet wide, to cover the same wall?

40. If it takes 5.1 yards of cloth, 1.25 yards wide, to make a gentleman's cloak, how much serge, $\frac{5}{8}$ of a yard wide, will be required to line it?

41. If 6 men can build a wall 80 feet long, 6 feet wide, and 4 feet high, in 15 days, in what time can 18 men build one 240 feet long, 8 feet wide, and 6 feet high?

ANALYSIS.—The solid measure of the wall is found by multiplying the three dimensions together; $80 \times 6 \times 4 = 1920$ cu. ft. In 15 days, 1 man can do $\frac{1}{6}$ of $1920 = 320$ cu. ft.; and in 1 day, he will do $\frac{1}{15}$ of 320 cu. ft. $= \frac{320}{15}$. In 1 day, 18 men can do 18 times $\frac{320}{15} = 384$ cu. ft.; and it will take 18 men as many days to build the wall, as 384 cu. ft. is contained times in the solid measure of the second wall: $240 \times 8 \times 6 = 11520$ cu. ft., and $11520 \div 384 = 30$ days.

OPERATION.

$80 \times 6 \times 4 = 1920$ cu. ft. = solid measure of first wall.
$\frac{1}{6}$ of $1920 = 320$ cu. ft. = work done in 15 days by 1 man.
$\frac{1}{15}$ of $320 = \frac{320}{15}$ cu. ft. = work done in 1 day by 1 man.
$\frac{320}{15} \times 18 = 384$ cu. ft. = work done in 1 day by 18 men.
$240 \times 8 \times 6 = 11520$ cu. ft. = solid measure of second wall.
$\frac{11520}{384} = 30$ days = time for 18 men to do the work.

Or, $\frac{240 \times 8 \times 6}{1} \div \frac{1}{15}$ of $\frac{1}{6}$ of $\frac{80 \times 6 \times 4}{1} \times \frac{18}{1} =$

$$\frac{\overset{3}{2\cancel{4}\cancel{0}} \times \overset{2}{\cancel{8}} \times \cancel{6}}{1} \times \frac{15}{1} \times \frac{\cancel{6}}{1} \times \frac{1}{\cancel{8}\cancel{0} \times \cancel{6} \times \cancel{4}} \times \frac{1}{\underset{3}{\cancel{1}\cancel{8}}} = 30.$$

42. If 96 lbs. of bread be sufficient to serve 5 men 12 days, how many days will 57 lbs. serve 19 men?

43. If a man travel 220 miles in 10 days, traveling 12 hours a day, in how many days will he travel 880 miles, traveling 16 hours a day?

44. If 9 men pay $135 for 5 weeks' board, how much must 8 men pay for 4 weeks' board?

45. If 12 men reap 80 acres in 6 days, in how many days will 25 men reap 200 acres?

46. If 4 men are paid 24 dollars for 3 days' labor, how many men may be employed 16 days for $96?

47. A wall, to be built to the height of 27 feet, was raised to the height of 9 feet by 12 men, in 6 days: how many men must be employed, to finish the wall in 4 days, at the same rate of working?

48. Two men bought a horse for $150; one paid $90, and the other, $60; they sold the horse, and gained $75: what did each gain?

ANALYSIS.—Each must have the same part of the gain that the money which he paid, is of the whole money paid. One paid $90, which is $\frac{90}{150} = \frac{3}{5}$ of $150, and he ought to receive $\frac{3}{5}$ of the gain; and the other paid $60, which is $\frac{60}{150} = \frac{2}{5}$ of $150, and he ought to receive $\frac{2}{5}$ of the gain. $\frac{3}{5}$ of $75 = $45 = gain of one; and $\frac{2}{5}$ of $75 = $30 = gain of the other.

OPERATION.

$90 = $\frac{90}{150} = \frac{3}{5}$ of cost; $\frac{3}{5}$ of 75 = $45 = gain of one.
$60 = $\frac{60}{150} = \frac{2}{5}$ of cost; $\frac{2}{5}$ of 75 = $30 = " other.

49. Three persons bought 2 barrels of flour for 15 dollars. The first one ate from them 2 months, the second 3 months, and the third 7 months: how much should each pay?

50. If two persons engage in a business, where one ad-

vances $875 and the other $625, and they gain $300, what is each one's share?

51. A, B, and C sent a drove of hogs to market, of which A owned 105, B 75, and C 120; on the way, 60 died: how many must each lose?

52. A man who has only $50, owes $75 to A, $150 to B, and $100 to C: how much ought he to pay to each?

53. A can do a piece of work in 4 days, and B can do the same in 6 days: in what time can they both do the work, if they labor together?

Analysis.—Since A can do the work in 4 days, in 1 day he can do $\frac{1}{4}$ of the work, and B can do $\frac{1}{6}$ of the work in 1 day: both can, in 1 day, do the sum of $\frac{1}{4}$ and $\frac{1}{6}$: $\frac{1}{4}+\frac{1}{6}=\frac{10}{24}=\frac{5}{12}$: since in 1 day they can do $\frac{5}{12}$ of the work, it will require, for the whole work, as many days as $\frac{5}{12}$ is contained times in 1. $1\div\frac{5}{12}=1\times\frac{12}{5}=\frac{12}{5}$ $=2\frac{2}{5}$ days.

OPERATION.

$\frac{1}{4}$ = what A can do in 1 day;
$\frac{1}{6}$ = what B " "
$\frac{1}{4}+\frac{1}{6}=\frac{10}{24}=\frac{5}{12}$ = what both can do in 1 day;
$1\div\frac{5}{12}=1\times\frac{12}{5}=\frac{12}{5}=2\frac{2}{5}$ days $\left\{\begin{array}{l}\text{required for A and B to}\\ \text{do the whole work.}\end{array}\right.$

54. A can build a shed in 6 days, and B can build it in 5 days: in what time can they, by working together, build the shed?

55. A father earns, in 9 days, $18, and his son earns the same amount in 15 days: in what time could they, together, earn the amount?

56. A laborer can dig a trench in 25 days, but with the assistance of a second laborer, he digs it in 16 days: in what time would the second laborer, alone, have dug it?

57. A can build a wall in 16 days, and B can do it in 21 days; they both worked on the wall; after working 5 days, B left it: in what time could A finish the work?

58. A can build a wall in 18 days, and B can do it in 24 days; A worked alone for 6 days, and was then assisted by B: how many days were employed in doing the whole work?

59. If a barrel of flour would last a family 6 weeks, and if it would last a second family 8 weeks, how long would $\frac{1}{2}$ of the barrel last both families?

60. Divide \$500 between 3 persons, giving to one \$$\frac{1}{2}$, as often as to the second \$$\frac{1}{4}$, and to the third \$$\frac{1}{5}$.

61. Divide \$176.40 among 3 persons, so that the first shall have twice as much as the second, and the third three times as much as the first: what is each one's share?

62. A person bought 3 lots of ground for \$6000; he paid \$150 more for the second than for the first, and \$350 more for the third than for the second: what was the cost of each?

63. Three men hire a pasture, for which they pay 66 dollars. The first puts in 2 horses for 3 weeks; the second, 6 horses for $2\frac{1}{2}$ weeks; the third, 9 horses for $1\frac{1}{3}$ weeks: how much ought each to pay?

ANALYSIS.—The pasturage of 2 horses for 3 weeks, would be the same as the pasturage of 1 horse 2 times 3 weeks, or 6 weeks; that of six horses, $2\frac{1}{2}$ weeks, the same as for 1 horse six times $2\frac{1}{2}$ weeks, or 15 weeks; and that of 9 horses $1\frac{1}{3}$ weeks, the same as 1 horse for 9 times $1\frac{1}{3}$ weeks, or 12 weeks. The three persons had an equivalent for the pasturage of 1 horse for $6+15+12=33$ weeks; therefore, the first must pay $\frac{6}{33}$; the second, $\frac{15}{33}$; and the third, $\frac{12}{33}$ of 66 dollars.

OPERATION.

$3 \times 2 = 6$; then, \$66 $\times \frac{6}{33} =$ \$12. 1st.
$2\frac{1}{2} \times 6 = 15$; " \$66 $\times \frac{15}{33} =$ \$30. 2d.
$1\frac{1}{3} \times 9 = 12$; " \$66 $\times \frac{12}{33} =$ \$24. 3d.

64. Two persons, A and B, enter into partnership, and gain \$175. A puts in 75 dollars for 4 months, and B puts in 100 dollars for 6 months: what is each one's share of the gain?

65. Three men engage to build a house for 580 dollars. The first one employed 4 hands; the second, 5 hands; and the third, 7 hands. The first man's hands worked 3 times as many days as the third, and the second man's hands twice as many days as the third man's hands: how much must each receive?

ALLIGATION.

350. Alligation is the process of mixing substances in such a manner that the value of the compound may be equal to the sum of the values of the several ingredients.

ALLIGATION MEDIAL.

351. Alligation Medial is the process of finding the mean price of a mixture, when the quantity of each simple and its price, are known.

1. A merchant mixes 8 lb. of tea, worth 75 cents a pound, with 16 lb., worth $1.02 a pound: what is the price of the mixture per pound?

Analysis.—The quantity, 8 lb. of tea, at 75 cents a pound, costs $6; and 16 lb., at $1.02, costs $16.32: hence, the mixture, = 24 lb., costs $22.32; and the price of 1 lb. of the mixture is found by dividing this cost by 24: Hence, to find the price of the mixture,

OPERATION.

8 lb. at 75 cts. = $ 6.00
16 lb. at $1.02 = $16.32
24 24) 22.32
$0.93

Rule.—I. *Find the cost of the entire mixture:*

II. *Divide the entire cost of the mixture by the sum of the simples, and the quotient will be the price of the mixture.*

Examples.

1. A farmer mixes 30 bushels of wheat, at 5s. per bushel, and 72 bushels of rye, at 3s., with 60 bushels of barley, at 2s.: what is the price of 1 bushel of the mixture?

2. A wine-merchant mixes 15 gallons of wine, at $1 per gallon, with 25 gallons of brandy, worth 75 cents per gallon: what should be the price of a gallon of the compound?

3. A grocer mixes 40 gallons of whisky, worth 31 cents per gallon, with 3 gallons of water, which costs nothing: what should be the price of a gallon of the mixture?

4. A goldsmith melts together 2 lb. of gold, of 22 carats

fine, 6 oz., of 20 carats fine, and 6 oz., of 16 carats fine: what is the fineness of the mixture?

5. On a certain day, the mercury in the thermometer was observed to average the following heights: from 6 in the morning to 9, 64°; from 9 to 12, 74°; from 12 to 3, 84°; and from 3 to 6, 70°: what was the mean temperature of the day?

ALLIGATION ALTERNATE.

352. ALLIGATION ALTERNATE is the process of finding what proportions must be taken of each of several simples, whose prices are known, to form a compound of a given price. It is the opposite of Alligation Medial, and may be proved by it.

Alligation Alternate is founded on an equality of Gain and Loss. In selling a mixture at a mean price, there is a *gain* on each simple *below* that price, and a *loss* on each simple *above* the average price. The gain must be just equal to the loss, otherwise the value of the compound would not be a mean value.

CASE I.

353. To find the proportional parts.

1. A farmer would mix oats at 3s. a bushel, rye at 6s., and wheat at 9s. a bushel, so that the mixture shall be worth 5 shillings a bushel: what proportion must be taken of each sort?

ANALYSIS.—Having written the simples in the order of their values, place the mean price at the left, and then rule 5 columns. The column U expresses that 1 unit is first taken of each simple; the column G expresses the gain on 1 unit; the column L, the loss on 1 unit; and the difference between the sums of the columns, shows that on 1 unit of each simple, there is a loss of 3.

Since the column of gains must balance the column of losses, we must take so many additional units as will give a gain of 3s. Since 1 unit gives a gain of 2s., $\frac{1}{2}$ of a unit will give a gain of 1s., and $\frac{3}{2}$ units will give a gain of 3s.: hence, $1 + \frac{3}{2} = \frac{5}{2}$ will give a gain of 5s., which will balance the losses. Write $\frac{5}{2}$ in the column Bal., and then multiply each proportional number by 2 (the denominator of the fraction), which will give the proportional parts, 5, 2, and 2, in integral numbers.

OPERATION.

			G.	L.	U.	Bal.	P.P.
5	Oats,	3	2		1	$\frac{5}{2}$	5
	Rye,	6		1	1	1	2
	Wheat,	9		4	1	1	2
			2	5;	5 — 2 = 3.		

Rule.—I. *Write the prices of the simples in a column, beginning with the lowest, and the mean price at the left:*

II. *Opposite each simple, write its gain or loss on 1, and write the 1 in the column of units:*

III. *Find the difference between the gains and losses, and divide it by the first gain or loss in the least column, and add the quotient to the corresponding number in the column* U, *and then write the sum and the other numbers of that column, in the column* Bal.:

IV. *When the quotient is fractional, multiply each number by the denominator of the fraction, and the several products will be the proportional parts in integral numbers, which write in column* P. P. *of Proportional Parts.*

Note.—The answers to the last, and to all similar questions, will be infinite in number. For, if the proportional numbers in column P. P. be multiplied by any number, integral or fractional, the products will denote proportional parts of the simples.

Examples.

1. What proportions of tea, at 24 cents, 30 cents, 33 cents, and 36 cents a pound, must be mixed together so that the mixture shall be worth 32 cents a pound ?

2. What proportions of coffee, at 16 cts., 20 cts., and 28 cts. per pound, must be mixed together so that the compound shall be worth 24 cts. per pound ?

3. A goldsmith has gold of 16, of 18, of 23, and of 24 carats fine: what part must be taken of each, so that the mixture shall be 21 carats fine ?

4. What portion of brandy, at 14s. per gallon; of old Madeira, at 24s. per gallon; of new Madeira, at 21s. per gallon; and of brandy, at 10s. per gallon, must be mixed together so that the mixture shall be worth 18s. per gallon?

5. How many gallons of water must be mixed with wine at $3 per gallon, and wine at $5 per gallon, that the mixture may be sold at $2 per gallon?

CASE II.

354. When the quantity of one simple is given.

1. How much wheat, at 9s. a bushel, must be mixed with 20 bushels of oats, worth 3 shillings a bushel, that the mixture may be worth 5 shillings a bushel?

ANALYSIS.—First, find the proportional numbers, as in the last case: they are, 2 of oats and 1 of wheat. Hence, for every 2 bushels of oats, we take 1 of wheat; and since we take 20 bushels of oats, we take as many of wheat as 2 is contained times in 20, which is 10.

RULE.—I. *Find the proportional numbers, and divide the given quantity by its proportional number:*

II. *Multiply each remaining proportional number by this ratio, and the products will denote the quantities to be taken of each.*

EXAMPLES.

1. How much wine, at 5s., at 5s. 6d., and 6s. per gallon, must be mixed with 4 gallons, at 4s. per gallon, so that the mixture shall be worth 5s. 4d. per gallon?

2. A farmer would mix 14 bushels of wheat, at $1.20 per bushel, with rye at 72 cts., barley at 48 cts., and oats at 36 cts.: how much must be taken of each sort to make the mixture worth 64 cts. per bushel?

3. There is a mixture made of wheat at 4s. per bushel, rye at 3s., barley at 2s., with 12 bushels of oats at 18d. per bushel: how much is taken of each sort when the mixture is worth 3s. 6d.?

4. A distiller would mix 40 gallons of French brandy, at 12s. per gallon, with English at 7s., and spirits at 4s. per gal.: what quantity must be taken of each sort, that the mixture may be afforded at 8s. per gallon?

CASE III.

355. When the quantity of the mixture is given.

1. A merchant would make up a cask of wine, containing 50 gallons, with wine worth 16s., 18s., and 22s. a gallon, in such a way that the mixture may be worth 20s. a gallon: how much must he take of each sort?

Analysis.—Find the proportional parts: they are, 1, 1, and 3. Now, these numbers must be taken as many times as their sum, 5, is contained times in 50, which is 10: hence, there are 10 gallons of the first, 10 of the second, and 30 of the third: Hence,

Rule.—I. *Find the proportional parts:*

II. *Divide the quantity of the mixture by the sum of the proportional parts:*

III. *Multiply this ratio by the parts separately, and each product will denote the quantity of the corresponding simple.*

Examples.

1. A grocer has four sorts of sugar, worth 12d., 10d., 6d., and 4d. per pound; he would make a mixture of 144 pounds, worth 8d. per pound: what quantity must be taken of each sort?

2. A grocer, having four sorts of tea, worth 5s., 6s., 8s., and 9s. per pound, wishes a mixture of 87 pounds, worth 7s. per pound: how much must he take of each sort?

3. A silversmith has four sorts of gold, viz., of 24 carats fine, of 22 carats fine, of 20 carats fine, and of 15 carats fine; he would make a mixture of 42 oz., of 17 carats fine: how much must be taken of each sort?

Proof.—All the examples of Alligation Medial may be proved by Alligation Alternate.

INVOLUTION.

356. The POWER of a number, is the product which arises from multiplying the number successively by itself. The number, so multiplied, is called the *root* of the power.

The *first power* is the number itself, or the *root:*

The *second power* is the product of the root by itself:

The *third power* is the product when the root is taken 3 times as a factor:

The *fourth power*, when it is taken 4 times:

The *fifth power*, when it is taken 5 times, &c.

357. The EXPONENT of a power, is the number denoting how many times the root is taken as a factor. It is written a little at the right, and over the root: thus, if the equal factor or root is 4,

$$\begin{array}{rl}
4 = & 4, \text{ the 1st power of 4.} \\
4^2 = 4 \times 4 = & 16, \text{ the 2d power of 4.} \\
4^3 = 4 \times 4 \times 4 = & 64, \text{ the 3d power of 4.} \\
4^4 = 4 \times 4 \times 4 \times 4 = & 256, \text{ the 4th power of 4.} \\
4^5 = 4 \times 4 \times 4 \times 4 \times 4 = & 1024, \text{ the 5th power of 4.}
\end{array}$$

358. INVOLUTION is the process of finding the powers of numbers.

There are three things connected with every power: 1st, The root; 2d, The exponent; and 3d, The power or result of the multiplication.

In finding a power, the root is always the first power; hence, the number of multiplications is 1 less than the exponent.

RULE.—*Multiply the number by itself as many times, less 1, as there are units in the exponent, and the last product will be the power.*

EXAMPLES.

1. Square of $\frac{1}{2}$.
2. Cube of $\frac{1}{8}$.
3. Cube of 12.
4. 3d power of 125.
5. 4th power of 9.
6. 5th power of 16.
7. 2d power of 225.
8. Cube of 321.
9. 4th power of 215.
10. Square of 36049.
11. Cube of .25.
12. 4th power of 8.628.

EVOLUTION.

359. Evolution is the process of finding the equal factors when we know the power.

The *square root* of a number is one of its two equal factors: thus, 6 is the square root of 36, because $6 \times 6 = 36$.

The *cube root* of a number is one of its three equal factors: thus, 3 is the cube root of 27, because $3 \times 3 \times 3 = 27$.

The sign $\sqrt{}$, is called the radical sign. When placed before a number, it denotes that its square root is to be extracted: Thus, $\sqrt{36} = 6$.

We denote the cube root by the same sign, by writing 3 over it: thus, $\sqrt[3]{27}$, denotes the cube root of 27, which is equal to 3. The small figure, 3, placed over the radical, is the *index* of the root.

EXTRACTION OF THE SQUARE ROOT.

360. The Square Root of a number, as already defined, is one of its two equal factors. When the two equal factors can be found, *exactly*, the number is called a *perfect square:* and when they can only be found approximately, the number is called an *imperfect square.*

361. The first ten numbers, and their squares, are,

Roots,	1,	2,	3,	4,	5,	6,	7,	8,	9,	10.
Squares,	1,	4,	9,	16,	25,	36,	49,	64,	81,	100.

The numbers in the first line are the square roots of the numbers in the second: hence, the square root of any perfect square, expressed by two figures, will be expressed by a single figure.

Note.—The square root of a number less than 100 will be less than 10, while the square root of a number greater than 100 will be greater than 10.

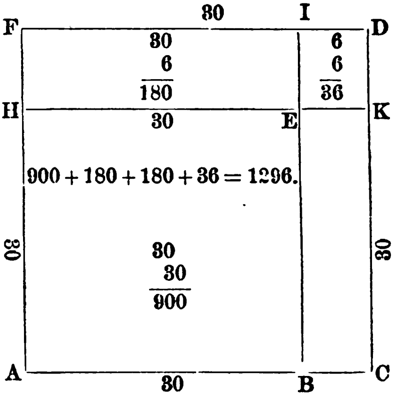
362. What is the square of 36 = 3 tens + 6 units?

ANALYSIS.—36 = 3 tens + 6 units, is first to be taken 6 units' times, giving $6^2 + 3 \times 6$: then, taking it 3 tens' times, we have $3 \times 6 + 3^2$, and the sum is, $3^2 + 2\,(3 \times 6) + 6^2$: that is,

$$\begin{array}{r} 3 + 6 \\ 3 + 6 \\ \hline 3 \times 6 + 6 \\ 3^2 + 3 \times 6 \quad\;\; \\ \hline 3^2 + 2\,(3 \times 6) + 6 \end{array}$$

The square of a number is equal to the square of the tens plus twice the product of the tens by the units, plus the square of the units.

The same may be shown by the figure:



Let the line A B represent the 3 tens or 30, and B C the six units.

Let A D be a square on A C, and A E a square on the ten's line A B.

Then E D will be a square on the unit line 6, and the rectangle, E F, will be the product of HE, which is equal to the ten's line, by I E, which is equal to the unit line. Also, the rectangle B K will be the product of E B, which is equal to the ten's line, by the unit line, B C. But the whole square on A C is made up of the square A E, the two rectangles, F E and E C, and the square E D.

1. Let it now be required to extract the square root of 1296.

ANALYSIS.—Since the number contains more than two places of figures, its root will contain tens and units. But as the square of one ten is one hundred, it follows that the square of the tens of the required root must be found in the two figures on the left of 96. Hence, we point off the number into periods of two figures each, giving 12 hundreds and 96 units.

The greatest perfect square in 12 hundreds, is 9 hundreds, the root of which is 3 tens, or 30. We square 3 tens, which gives 9 hundred, place 9 under the hundreds' place, and subtract; this takes away the square of the tens, and leaves 396, *which is twice the product of the tens by the units plus the square of the units.*

OPERATION.

$$\begin{array}{r|l} & \dot{1}\dot{2}\ \dot{9}\dot{6}\ (\ 39 \\ & 9 \\ \hline 66\ ) & 396 \\ & 396 \\ \hline \end{array}$$

If now, we double the divisor, and then divide this remainder, exclusive of the right-hand figure (since that figure can not enter into the product of the tens by the units), by it, the quotient will be the units' figure of the root. If we annex this figure to the augmented divisor, and then multiply the whole divisor, thus increased, by it, the product will be twice the tens by the units plus the square of the units; and hence, we have found both figures of the root.

Hence, for the extraction of the square root, we have the following

RULE.—I. *Separate the given number into periods of two figures each, by setting a dot over the place of units, a second over the place of hundreds, and on each alternate figure at the left:*

II. *Note the greatest square contained in the period on the left, and place its root on the right, after the manner of a quotient in division. Subtract the square of this root from the first period, and to the remainder bring down the second period for a dividend:*

III. *Double the root thus found for a trial divisor, and place it on the left of the dividend. Find how many times the trial divisor is contained in the dividend, exclusive of the right-hand figure, and place the quotient in the root, and also annex it to the divisor:*

IV. *Multiply the divisor thus increased, by the last figure of the root; subtract the product from the dividend and to the remainder bring down the next period for a new dividend:*

V. *Double the whole root thus found, for a new trial divisor, and continue the operation as before, until all the periods are brought down.*

Notes.—1. The left-hand period may contain but one figure; each of the others will contain two.

2. If any trial divisor is greater than its dividend, the corresponding quotient figure will be a cipher.

3. If the product of the divisor by any figure of the root exceeds the corresponding dividend, the quotient figure is too large, and must be diminished.

4. There will be as many figures in the root as there are periods in the given number.

5. If the given number is not a perfect square, there will be a remainder after all the periods are brought down. In this case, periods of ciphers may be annexed, forming new periods, each of which will give one decimal place in the root.

Examples.

1. What is the square root of 263169?

OPERATION.

$$
\begin{array}{rl}
 & 2\dot{6}\ 3\dot{1}\ 6\dot{9}\ (\,513 \\
 & 25 \\ \cline{2-2}
101\,) & 131 \\
 & 101 \\ \cline{2-2}
1023\,) & 3069 \\
 & 3069 \\ \cline{2-2}
\end{array}
$$

Analysis.—We first place a dot over the 9, making the right-hand period 69. We then put a dot over the 1, and also over the 6, making three periods.

The greatest perfect square in 26, is 25, the root of which is 5. Placing 5 in the root, subtracting its square from 26, and bringing down the next period, 31, we have 131 for a dividend, and by doubling the root, we have 10 for a trial divisor. Now, 10 is contained in 13, 1 time. Place 1 both in the root and in the divisor: then multiply 101 by 1; subtract the product, and bring down the next period.

We must now double the whole root, 51, for a new trial divisor; then dividing, we obtain 3, the third figure of the root.

2. What is the square root of 36729?
3. What is the square root of 2125?
4. What is the square root of 26883881?
5. What is the square root of 426409?
6. What is the square root of 2976412?
7. What is the square root of 91874261?

363. To extract the square root of a fraction.

1. What is the square root of .5?

Note.—We first annex one cipher, to make even decimal places. We then extract the root of the first period: to the remainder we annex two ciphers, forming a new period, and so on.

OPERATION.

.50 (.707 +
49
140) 100
000
1407) 10000
9849
151, Rem.

2. What is the square root of $\frac{4}{9}$?

Note.—The square root of a fraction is equal to the square root of the numerator divided by the square root of the denominator.

OPERATION.

$$\sqrt{\frac{4}{9}} = \frac{\sqrt{4}}{\sqrt{9}} = \frac{2}{3}.$$

3. What is the square root of $\frac{3}{4}$?

Note.—When the terms are not perfect squares, reduce the common fraction to a decimal fraction, and then extract the square root of the decimal.

OPERATION.

$\frac{3}{4} = .75$;

$\sqrt{\frac{3}{4}} = \sqrt{.75} = .8660 +$.

Rule.—I. *If the fraction is a decimal, point off the periods from the decimal point to the right, annexing ciphers if necessary, so that each period shall contain two places, and then extract the root as in integral numbers:*

II. *If the fraction is a common fraction, and its terms perfect squares, extract the square root of the numerator and denominator separately; if they are not perfect squares, reduce the fraction to a decimal, and then extract the square root of the decimal.*

Examples.

What are the square roots of the following numbers:

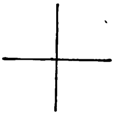
1. Of 3?
2. Of 11?
3. Of 1069?
4. Of 2268741?
5. Of 7596796?
6. Of 36372961?

7. Of 22071204?
8. Of 3271.4207?
9. Of 4795.25731?
10. Of 4.372594?
11. Of .0025?
12. Of .00032754?
13. Of .00103041?
14. Of 4.426816?
15. Of $8\frac{3}{6}$?
16. Of $9\frac{3}{4}$?
17. Of $\frac{64}{125}$?
18. Of $\frac{125}{729}$?
19. Of $\frac{2304}{5184}$?
20. Of $\frac{2704}{4225}$?

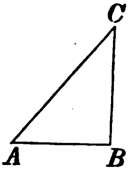
APPLICATIONS IN SQUARE ROOT.

364. A TRIANGLE is a plain figure, which has three sides and three angles.

If a straight line meets another straight line, making the adjacent angles equal, each is called, a *right angle;* and the lines are said to be perpendicular to each other.

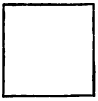


365. A RIGHT-ANGLED TRIANGLE is one which has one right angle. In the right-angled triangle ABC, the side AC, opposite the right angle B, is called, the *hypothenuse;* the side AB, the *base;* and the side BC, the *perpendicular.*

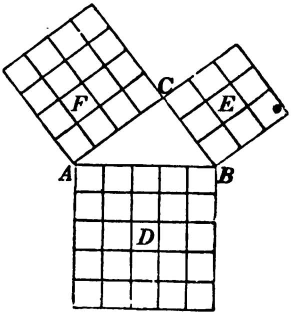


366. We have seen (Art. 196), that the area of a square is equal to the product of two of its equal sides, which is the square of one side. Hence, the square root of the area of a square, will be the side itself.

Thus, if the area of the square in the figure is 25, the square root of 25, which is 5, will denote the side.

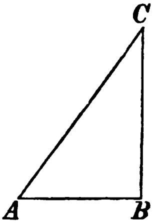


367. In a right-angled triangle, the square described on the hypothenuse is equal to the sum of the squares described on the other two sides.



Thus, if ACB be a right-angled triangle, right-angled at C, then will the large square, D, described on the hypothenuse AB, be equal to the sum of the squares F and E, described on the sides AC and CB. This is called, the carpenter's theorem. By counting the small squares in the large square, D, you will find their number equal to that contained in the small squares F and E. In this triangle, the hypothenuse AB = 5, AC = 4, and CB = 3. Any numbers having the same ratio as 5, 4, and 3, such as 10, 8, and 6; 20, 16, and 12, &c., will represent the sides of a right-angled triangle.

1. Wishing to know the distance from A to the top of a tower, I measured the height of the tower, and found it to be 40 feet; also the distance from A to B, and found it 30 feet: what was the distance from A to C?



$$\begin{aligned} AB &= 30; \quad AB^2 = 30^2 = 900 \\ BC &= 40; \quad BC^2 = 40^2 = \underline{1600} \\ AC^2 &= AB^2 + BC^2 = 2500 \\ AC &= \sqrt{2500} = 50 \text{ feet.} \end{aligned}$$

Hence, when the base and perpendicular are known, and the hypothenuse is required,

Square the base and square the perpendicular, add the results, and then extract the square root of their sum.

2. What is the length of a rafter that will reach from the eaves to the ridge-pole of a house, when the height of the roof is 15 feet, and the width of the building, 40 feet?

368. **To find one side, when we know the hypothenuse and other side.**

1. The length of a ladder which will reach from the mid-

dle of a street, 80 feet wide, to the eaves of a house, is 50 feet: what is the height of the house?

Analysis.—Since the square of the length of the ladder is equal to the sum of the squares of half the street and the height of the house, the square of the length of the ladder diminished by the square of half the street, will be equal to the square of the height of the house: Hence,

Square the hypothenuse and the known side, and take the difference; the square root of the difference will be the other side.

Examples.

1. If an acre of land be laid out in a square form, what will be the length of each side in rods?

2. What will be the length of the side of a square, in rods, that shall contain 100 acres?

3. A general has an army of 7225 men: how many must be put in each line, in order to place them in a square form?

4. Two persons start from the same point; one travels due east 50 miles, the other due south 84 miles: how far are they apart?

5. What is the length, in rods, of one side of a square that shall contain 12 acres?

6. A company of speculators bought a tract of land for $6724, each agreeing to pay as many dollars as there were partners: how many partners were there?

7. A farmer wishes to set out an orchard of 3844 trees, so that the number of rows shall be equal to the number of trees in each row: wnat will be the number of rows?

8. How many rods of fence will inclose a square field of 10 acres?

9. If a line 150 feet long, will reach from the top of a steeple 120 feet high, to the opposite side of the street, what is the width of the street?

10. What is the length of a brace whose ends are each 3½ feet from the angle made by the post and beam?

CUBE ROOT.

369. The CUBE ROOT of a number is one of three equal factors of the number.

To extract the cube root of a number, is to find a factor which, multiplied into itself *twice*, will produce the given number.

Thus, 2 is the cube root of 8; for, $2 \times 2 \times 2 = 8$: and 3 is the cube root of 27; for, $3 \times 3 \times 3 = 27$.

1,	2,	3,	4,	5,	6,	7,	8,	9.
1	8	27	64	125	216	343	512	729

The numbers in the first line are the cube roots of the corresponding numbers of the second.

370. A PERFECT CUBE is one in which the factors can be found, *exactly*. An IMPERFECT CUBE, is a number in which the equal factors can only be found, approximately.

By examining the numbers, in the lines above, we see,

1st, That the cube of units cannot give a higher order than hundreds.

2d, That since the cube of one ten (10) is 1000, and the cube of 9 tens (90), 81000, *the cube of tens will not give a lower denomination than thousands, nor a higher denomination than hundreds of thousands.*

Hence, if a number contains more than three figures, its cube root will contain more than one: if it contains more than six, its root will contain more than two, and so on; every additional three figures giving one additional figure in the root; and the figures which remain at the left hand, although less than three, will also give a figure in the root. This law explains the reason for pointing off into periods of three figures each.

371. Let us now see how the cube of any number, as 16, is formed. Sixteen is composed of 1 ten and 6 units, and may be written $10+6$. To find the cube of 16, or of $10+6$, we must multiply the number by itself twice.

$$\begin{array}{lrrrr}
\text{To do this, we place the number thus,} & & 16 = & 10+ & 6 \\
 & & & 10+ & 6 \\ \cline{4-5}
\text{Product by the units, ...} & & & 60+ & 36 \\
\text{Product by the tens, ...} & & 100+ & 60 & \\ \cline{3-5}
\text{Square of 16, ...} & & 100+ & 120+ & 36 \\
\text{Multiply again by 16, ...} & & & 10+ & 6 \\ \cline{3-5}
\text{Product by the units, ...} & & 600+ & 720+ & 216 \\
\text{Product by the tens, ...} & 1000+ & 1200+ & 360 & \\ \cline{2-5}
\text{Cube of 16, ...} & 1000+ & 1800+ & 1080+ & 216
\end{array}$$

1. By examining the parts of this number, it is seen that the first part, 1000, is the *cube of the tens;* that is,

$$10 \times 10 \times 10 = 1000.$$

2. The second part, 1800, is *three times the square of the tens multiplied by the units;* that is,

$$3 \times (10)^2 \times 6 = 3 \times 100 \times 6 = 1800.$$

3. The third part, 1080, *is three times the square of the units multiplied by the tens;* that is,

$$3 \times 6^2 \times 10 = 3 \times 36 \times 10 = 1080.$$

4. The fourth part is *the cube of the units;* that is,

$$6^3 = 6 \times 6 \times 6 = 216.$$

1. What is the cube root of the number 4096?

OPERATION.

$$\begin{array}{rlll}
 & \dot{4} & 09\dot{6} & (16 \\
 & 1 & & \\ \cline{2-3}
1^2 \times 3 = 3\,) & 3 & 0 & (9\text{–}8\text{–}7\text{–}6 \\
16^3 = & 4 & 096. &
\end{array}$$

ANALYSIS.—Since the number contains more than three figures, we know that the root will contain at least units and tens.

Separating the three right-hand figures from the 4, we know that the cube of the tens will be found in the 4; and 1 is the greatest cube in 4.

Hence, we place the root 1 on the right, and this is the tens of the required root. We then cube 1, and subtract the result from the first period 4, and to the remainder we bring down the first figure, 0, of the next period.

We have seen that the second part of the cube of 16, viz., 1800, is *three times the square of the tens multiplied by the units;* and hence, it can have no significant figure of a less denomination than hundreds. It must, therefore, make up a part of the 30 hundreds above. But this 30 hundreds also contains all the hundreds which come from the third and fourth parts of the cube of 16. If it were not so, the 30 hundreds, divided by three times the square of the tens, would give the unit figure exactly.

Forming a divisor of three times the square of the tens, we find the quotient to be ten; but this we know to be too large. Placing 9 in the root and cubing 19, we find the result to be 6859. Then trying 8, we find the cube of 8 still too large; but when we take 6, we find the exact number. Hence, the cube root of 4096 is 16.

372. Hence, to find the cube root of a number,

Rule.—I. *Separate the given number into periods of three figures each, by placing a dot over the place of units, a second over the place of thousands, and so on over each third figure to the left; the left-hand period will often contain less than three places of figures:*

II. *Note the greatest perfect cube in the first period, and set its root on the right, after the manner of a quotient in division. Subtract the cube of this number from the first period, and to the remainder bring down the first figure of the next period for a dividend:*

III. *Take three times the square of the root just found for a trial divisor, and see how often it is contained in the dividend, and place the quotient for a second figure of the root. Then cube the figures of the root thus found; and if their cube be greater than the first two periods of the given number, diminish the last figure; but if it be less, subtract it from the first two periods, and to the remainder bring down the first figure of the next period for a new dividend:*

IV. *Take three times the square of the whole root for a second trial divisor, and find a third figure of the root. Cube the whole root thus found, and subtract the result from the first three periods of the given number when it is less than that number; but if it is greater, diminish the figure of the root: proceed in a similar way for all the periods.*

EXAMPLES.

1. What is the cube root of 99252847 ?

OPERATION.

$$\begin{array}{rl} & 9\dot{9}\ 25\dot{2}\ 84\dot{7}\ (\ 463 \\ 4^3 = & \underline{64} \\ 4^2\times 3 = 48\ ) & 352,\ \text{dividend.} \\ \text{First two periods, . . .} & 9\dot{9}\ 25\dot{2} \\ (46)^3 = 46\times 46\times 46 = & \underline{97\ 336} \\ 3\times(46)^2 = 6348\ ) & 19168,\ \text{2d dividend.} \\ \text{The first three periods, .} & 9\dot{9}\ 25\dot{2}\ 84\dot{7} \\ (463)^3 \qquad = & \underline{99\ 252\ 847} \end{array}$$

Find the cube roots of the following numbers:

1. Of 389017 ?
2. Of 5735339 ?
3. Of 32461759 ?
4. Of 84604519 ?
5. Of 259694072 ?
6. Of 48228544 ?

373. **To extract the cube root of a decimal fraction.**

Rule.—*Annex ciphers to the decimal, if necessary, so that it shall consist of 3, 6, 9, &c., places. Then put the first point over the place of thousandths, the second over the place of millionths, aud so on over every third place to the right; after which, extract the root as in whole numbers.*

Notes.—1. There will be as many decimal places in the root, as there are periods in the given number.

2. The same rule applies, when the given number is composed of a whole number and a decimal.

3. If, in extracting the root of a number, there is a remainder after all the periods have been brought down, periods of ciphers may be annexed, by considering them as decimals.

EXAMPLES.

Find the cube roots of the following numbers:

1. Of .157464 ?
2. Of .870983875 ?
3. Of 12.977875 ?
4. Of .751089429 ?
5. Of .353393243 ?
6. Of 3.408862625 ?

374. To extract the cube root of a common fraction.

Rule.—I. *Reduce compound fractions to simple ones, mixed numbers to improper fractions, and then reduce the fraction to its lowest terms:*

II. *Extract the cube root of the numerator and denominator separately, if they have exact roots; but if either of them has not an exact root, reduce the fraction to a decimal and extract the root as in the last case.*

Examples.

Find the cube roots of the following fractions:

1. Of $\frac{250}{686}$?
2. Of $31\frac{15}{343}$?
3. Of $\frac{324}{1500}$?
4. Of $\frac{4}{7}$?
5. Of $\frac{5}{9}$?
6. Of $\frac{2}{3}$?

Applications.

1. What must be the length, depth, and breadth of a box, when these dimensions are all equal, and the box contains 4913 cubic feet?

2. The solidity of a cubical block is 21952 cubic yards: what is the length of each side? What is the area of the surface?

3. A cellar is 25 feet long, 20 feet wide, and $8\frac{1}{2}$ feet deep: what will be the dimensions of another cellar of equal capacity, in the form of a cube?

4. What will be the length of one side of a cubical granary that shall contain 2500 bushels of grain?

5. How many small cubes, of 2 inches on a side, can be sawed out of a cube 2 feet on a side, if nothing is lost in sawing?

6. What will be the side of a cube that shall be equal to the contents of a stick of timber containing 1728 cubic feet?

7. A stick of timber is 54 feet long, and 2 feet square: what would be its dimensions, if it had the form of a cube?

NOTES.—1. Bodies are said to be similar, when their like parts are proportional.

2. It is found that the contents of similar bodies are to each other as the cubes of their like dimensions.

3. All bodies named in the examples below, are supposed to be similar.

8. If a sphere of 4 feet in diameter contains 33.5104 cubic feet, what will be the contents of a sphere 8 feet in diameter?

$$4^3 : 8^3 :: 33.5104 : \textit{Ans.}$$

9. If the contents of a sphere 14 inches in diameter is 1436.7584 cubic inches, what will be the diameter of a sphere which contains 11494.0672 cubic inches?

10. If a ball weighing 32 pounds is 6 inches in diameter, what will be the diameter of a ball weighing 2048 pounds?

11. If a haystack 24 feet in height, contains 8 tons of hay, what will be the height of a similar stack that shall contain but 1 ton?

◆

ARITHMETICAL PROGRESSION.

375. An ARITHMETICAL PROGRESSION is a series of numbers in which each is derived from the preceding one, by the addition or subtraction of the same number.

The number added or subtracted is called, the *common difference.*

376. If the common difference is added, the series is called, an *increasing series.*

Thus, if we begin with 2, and add the common difference 3, we have,

2, 5, 8, 11, 14, 17, 20, 23, &c.,

which is an *increasing series.*

If we begin with 23, and subtract the common difference 3, we have,

23, 20, 17, 14, 11, 8, 5, &c.,

which is a *decreasing series.*

The several numbers are called, the *terms* of the progression or series: the first and last are called, the *extremes*, and the intermediate terms are called, *means*.

377. In every arithmetical progression there are five parts:

1st, The first term;
2d, The last term;
3d, The common difference;
4th, The number of terms;
5th, The sum of all the terms.

If any three of these parts are known or given, the remaining ones can be determined.

CASE I.

378. Knowing the first term, the common difference, and the number of terms, to find the last term.

1. The first term is 3, the common difference 2, and the number of terms 19: what is the last term?

ANALYSIS.—By considering the manner in which the increasing progression is formed, we see that the 2d term is obtained by adding the common difference to the 1st terms; the 3d, by adding the common difference to the 2d; the 4th, by adding the common difference to the 3d, and so on; *the number of additions being 1 less than the number of terms found.*

OPERATION.

18, No. less 1.
2, com. dif.
36
3, 1st term.
39, last term.

But instead of making the additions, we may multiply the common difference by the number of additions, that is, by 1 less than the number of terms, and add the first term to the product: Hence,

RULE.—*Multiply the common difference by 1 less than the number of terms; if the progression is increasing, add the product to the first term, and the sum will be the last term; if it is decreasing, subtract the product from the first term, and the difference will be the last term.*

EXAMPLES.

1. A man bought 50 yards of cloth, for which he was to pay 6 cents for the 1st yard, 9 cents for the 2d, 12 cents for the 3d, and so on, increasing by the common difference 3: how much did he pay for the last yard?

2. A man puts out $100 at simple interest, at 7 per cent; at the end of the 1st year it will have increased to $107, at the end of the 2d year to $114, and so on, increasing $7 each year: what will be the amount at the end of 16 years?

3. What is the 40th term of an arithmetical progression, of which the first term is 1, and the common difference 1?

4. What is the 30th term of a descending progression, of which the first term is 60, and the common difference 2?

5. A person had 35 children and grandchildren, and it so happened that the difference of their ages was 18 months, and the age of the eldest was 60 years: how old was the youngest?

CASE II.

379. Knowing the two extremes and the number of terms, to find the common difference.

1. The extremes of an arithmetical progression are 8 and 104, and the number of terms 25: what is the common difference?

ANALYSIS.—Since the common difference multiplied by 1 less than the number of terms, gives a product equal to the difference of the extremes, if we *divide* the difference of the extremes by 1 less than the number of terms, the quotient will be the *common difference:* Hence,

OPERATION.

$$\begin{array}{r} 104 \\ \underline{8} \\ 25 - 1 = 24\ )\ \overline{96}\ (\ 4 \end{array}$$

RULE.—*Subtract the less extreme from the greater, and divide the remainder by 1 less than the number of terms; the quotient will be the common difference.*

Examples.

1. A man has 8 sons; the youngest is 4 years old, and the eldest 32: their ages increase in arithmetical progression: what is the common difference of their ages?

2. A man is to travel from New York to a certain place in 12 days; to go 3 miles the first day, increasing every day by the same number of miles; the last day's journey is 58 miles: required the daily increase.

3. A man hired a workman for a month of 26 working days, and agreed to pay him 50 cents for the first day, with a uniform daily increase; on the last day he paid $1.50: what was the daily increase?

CASE III

380. To find the sum of the terms of an arithmetical progression.

1. What is the sum of the series whose first term is 3, common difference 2, and last term 19?

Given series.	3+	5+	7+	9+	11+	13+	15+	17+	19=	99
Same; order of terms inverted.	19+	17+	15+	13+	11+	9+	7+	5+	3=	99
Sum of both.	22	22	22	22	22	22	22	22	22=	198

Analysis.—The two series are the same; hence, their sum is equal to twice the given series. But their sum is equal to the sum of the two extremes, 3 and 19, taken as many times as there are terms; and the given series is equal to half this sum, or to the sum of the extremes multiplied by half the number of terms.

Rule.—*Add the extremes together, and multiply their sum by half the number of terms; the product will be the sum of the series.*

Examples.

1. The extremes are 2 and 100, and the number of terms 22: what is the sum of the series?

Analysis.—We first add together the two extremes, and then multiply by half the number of terms.

OPERATION.

2,	1st term.
100,	last term.
102,	sum of extremes.
11,	half the number of terms.
1122,	sum of series.

2. How many strokes does the hammer of a clock strike in 12 hours?

3. The first term of a series is 2, the common difference 4, and the number of terms 9: what is the last term and sum of the series?

4. James, a smart chap, having learned arithmetical progression, told his father that he would chop a load of wood of 15 logs, at 2 cents the first log, with a regular increase of 1 cent for each additional log: how much did James receive for chopping the wood?

5. An invalid wishes to gain strength by regular and increasing exercise; his physician assures him that he can walk 1 mile the first day, and increase the distance half a mile for each of the 24 following days: how far will he walk?

6. If 100 eggs are placed in a right line, exactly one yard from each other, and the first one yard from a basket, what distance will a man travel who gathers them up singly, and places them in the basket?

GEOMETRICAL PROGRESSION.

381. A Geometrical Progression is a series of terms, each of which is derived from the preceding one by multiplying it by a constant number. The constant multiplier is called the *ratio* of the progression.

382. If the *ratio* is *greater* than 1, each term is greater than the preceding one, and the series is said to be *increasing.*

If the *ratio* is *less* than 1, each term is less than the preceding one, and the series is said to be *decreasing;* thus,

1, 2, 4, 8, 16, 32, &c.—ratio 2—increasing series:
32, 16, 8, 4, 2, 1, &c.—ratio $\frac{1}{2}$—decreasing series.

The several numbers are called *terms* of the progression. The first and last are called the *extremes*, and the intermediate terms are called *means.*

383. In every Geometrical, as well as in every Arithmetical Progression, there are five parts:

1st, The first term;
2d, The last term;
3d, The common ratio;
4th, The number of terms;
5th, The sum of all the terms.

If any three of these parts are known, or given, the remaining ones can be determined.

CASE I.

384. Having given the first term, the ratio, and the number of terms, to find the last term.

1. The first term is 3, and the ratio 2: what is the 6th term?

OPERATION.

$$\begin{array}{rl} 2\times2\times2\times2\times2=2^5=32 & \\ \underline{3} & \text{1st term} \\ \textit{Ans. } 96 & \end{array}$$

ANALYSIS.—The second term is formed by multiplying the first term by the ratio; the third term by multiplying the second term by the ratio, and so on; the number of multiplications *being 1 less than the number of terms.*

$$\begin{array}{rl} 3 = 3, & \text{1st term,} \\ 3\times2 = 6, & \text{2d term,} \\ 3\times2\times2 = 3\times2^2 = 12, & \text{3d term,} \\ 3\times2\times2\times2 = 3\times2^3 = 24, & \text{4th term, \&c.} \end{array}$$

Therefore, *the last term is equal to the first term multiplied by the ratio raised to a power 1 less than the number of terms.*

Rule.—*Raise the ratio to a power whose exponent is 1 less than the number of terms, and then multiply this power by the first term.*

Examples.

1. The first term of a decreasing progression is 192; the ratio $\frac{1}{2}$, and the number of terms 7: what is the last term?

Note.—The 6th power of the ratio, ($\frac{1}{2}$), is $\frac{1}{64}$; and this, multiplied by the first term 192, gives the last term, 3.

OPERATION.

$$\left(\tfrac{1}{2}\right)^6 = \tfrac{1}{64}.$$
$$192 \times \tfrac{1}{64} = 3.$$

2. A man purchased 12 pears; he was to pay 1 farthing for the first, 2 farthings for the second, 4 for the third, and so on, doubling each time: what did he pay for the last?

3. The first term of a decreasing progression is 1024, the ratio $\frac{1}{2}$: what is the 9th term?

4. The first term of an increasing progression is 4, and the common ratio 3: what is the 10th term?

5. A gentleman dying, left nine sons, and bequeathed his estate in the following manner: To his executors, $50; to his youngest son twice as much as the executors, and each other son double the amount of the son next younger: what was the eldest son's portion?

6. A man bought 12 yards of cloth, giving 3 cents for the first yard, 6 for the second, 12 for the third, &c.: what did he pay for the last yard?

CASE II.

385. **Knowing the two extremes and the ratio, to find the sum of the terms.**

1. What is the sum of the terms, in the progression 1, 4, 16, 64?

Analysis.—If we multiply the terms of the progression by the ratio 4, we have a second progression, 4, 16, 64, 256, which is 4 *times as great* as the first. If from this we subtract the *first*, the remainder, 256−1,

OPERATION.

$$\begin{array}{rr} 4+16+64+256= & 4\text{ times.} \\ \underline{1+4+16+64\quad\;\; =} & \underline{\text{once.}} \\ 256-1=3 & \text{times.} \end{array}$$

$$\frac{256-1}{3}=\frac{255}{3}=85\text{, sum.}$$

will be 3 *times* as great as the first; and if the remainder be divided by 3, the quotient will be the sum of the terms of the first progression. But 256 is the product of the last term of the given progression multiplied by the ratio, 1 is the first term, and the divisor, 3, is 1 less than the ratio: Hence,

Rule.—*Multiply the last term by the ratio; take the difference between the product and the first term, and divide the remainder by the difference between 1 and the ratio.*

Note.—When the progression is *increasing*, the first term is subtracted from the product of the last term by the ratio, and the divisor is found by subtracting 1 from the ratio. When the progression is *decreasing*, the product of the last term by the ratio is subtracted from the first term, and the ratio is subtracted from 1.

Examples.

1. The first term of a progression is 2, the ratio 3, and the last term 4375: what is the sum of the terms?

2. The first term of a progression is 128, the ratio $\frac{1}{2}$, and the last term 2: what is the sum of the terms?

3. The first term is 3, the ratio 2, and the last term 192: what is the sum of the series?

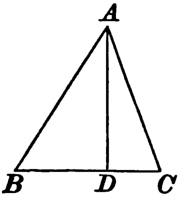
4. A gentleman gave his daughter in marriage on New Year's day, and gave her husband 1s. toward her portion, and was to double it on the first day of every month during the year: what was her portion?

5. A man bought 10 bushels of wheat, on the condition that he should pay 1 cent for the 1st bushel, 3 for the 2d, 9 for the 3d, and so on to the last: what did he pay for the last bushel, and for the 10 bushels?

6. A man has 6 children: to the youngest he gives $150; to the 2d, $300; to the 3d, $600, and so on, to each twice as much as to the one before: how much did the eldest receive, and what was the amount received by them all?

MENSURATION.

386. A TRIANGLE is a portion of a plane, bounded by three straight lines. It has three sides and three angles.



BC is called, the *base;* and AD, perpendicular to BC, the *altitude.*

387. To find the area of a triangle.

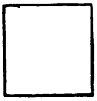
RULE.—*Multiply the base by half the altitude, and the product will be the area.* (*Bk.* IV. *Prop.* VI.)*

EXAMPLES.

1. The base BC, of a triangle, is 40 yards, and the perpendicular AD, 20 yards: what is the area?

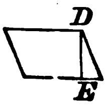
2. In a triangular field, the base is 40 chains, and the perpendicular 15 chains: how much does it contain? (Art. 194.)

3. There is a triangular field, of which the base is 35 rods, and the perpendicular 26 rods: what are its contents?



388. A SQUARE is a figure having four equal sides, and all its angles right angles.

389. A RECTANGLE is a four-sided figure, like a square, in which the sides are perpendicular to each other, but the adjacent sides are not equal.



390. A PARALLELOGRAM is a four-sided figure which has its opposite sides equal and parallel, but its angles not right angles.

The line DE, perpendicular to the base, is called, the altitude.

---

* Davies' Legendre.

391. To find the area of a square, rectangle, or parallelogram.

Rule.—*Multiply the base by the perpendicular height, and the product will be the area.* (*Bk.* IV., *Prop.* V.)

Examples.

1. What is the area of a square field, of which the sides are each 33.08 chains?

2. What is the area of a square piece of land, of which the sides are 27 chains?

3. What is the area of a square piece of land, of which the sides are 25 rods each?

4. What are the contents of a rectangular field, the length of which is 40 rods, and the breadth 20 rods?

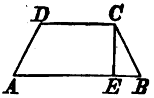
5. What are the contents of a field 40 rods square?

6. What are the contents of a rectangular field, 15 chains long, and 5 chains broad?

7. How many acres in a field 27 chains long and 69 rods broad?

8. The base of a parallelogram is 271 yards, and the perpendicular height 360 feet: what is the area?

392. A Trapezoid is a four-sided figure, ABCD, having two of its sides, AB, DC, parallel. The perpendicular, CE, is called, the altitude.



393. To find the area of a trapezoid.

Rule.—*Multiply half the sum of the two parallel lines by the altitude, and the product will be the area.* (*Bk.* IV. *Prop.* VII.)

Examples.

1. Required the area of the trapezoid ABCD, having given AB=321.51 ft., DC=214.24 ft., and CE=171.16 ft.

2. What is the area of a trapezoid, the parallel sides of which are 12.41 and 8.22 chains, and the perpendicular distance between them 5.15 chains?

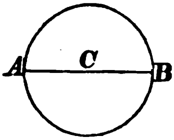
3. Required the area of a trapezoid whose parallel sides are 25 feet 6 inches, and 18 feet 9 inches, and the perpendicular distance between them 10 feet and 5 inches.

4. Required the area of a trapezoid whose parallel sides are 20.5 and 12.25, and the perpendicular distance between them 10.75 yards.

5. What is the area of a trapezoid whose parallel sides are 7.50 chains and 12.25 chains, and the perpendicular height 15.40 chains?

6. What are the contents, when the parallel sides are 20 and 32 chains, and the perpendicular distance between them 26 chains?

394. A CIRCLE is a portion of a plane, bounded by a curved line, called the *circumference.* Every point of the circumference is equally distant from a certain point within, called the *centre:* thus, C is the centre, and any line, as ACB, passing through the centre, is called, a *diameter.*



If the diameter of a circle is 1, the circumference will be 3.1416. Hence, *if we know the diameter, we may find the circumference by multiplying by* 3.1416; *or, if we know the circumference, we may find the diameter by dividing by* 3.1416.

EXAMPLES.

1. The diameter of a circle is 4: what is the circumference?
2. The diameter of a circle is 93: what is the circumference?
3. The diameter of a circle is 20: what is the circumference?
4. What is the diameter of a circle whose circumf. is 78.54?
5. What is the diameter of a circle whose circumference is 11652.1944?
6. What is the diameter of a circle whose circumf. is 6850?

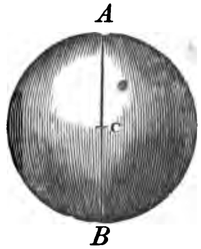
395. To find the area or contents of a circle.

RULE.—*Multiply the square of the radius by the decimal 3.1416. (Bk.* V., *Prop.* XIV., Cor. 2.)

EXAMPLES.

1. What is the area of a circle whose diameter is 6?
2. What is the area of a circle whose diameter is 10?
3. What is the area of a circle whose diameter is 7?
4. How many square yards in a circle whose diam. is $3\frac{1}{2}$ ft.?

396. A SPHERE is a figure bounded by a curved surface, all the points of which are equally distant from a certain point within, called the centre. The line AB, passing through its centre C, is called, the diameter of the sphere, and AC, its radius.



397. To find the surface of a sphere.

RULE.—*Multiply the square of the diameter by* 3.1416. (*Bk.* VIII., *Prop.* X., *Cor.*)

EXAMPLES.

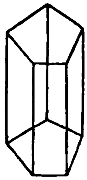
1. What is the surface of a sphere whose diameter is 12?
2. What is the surface of a sphere whose diameter is 7?
3. Required the number of square inches in the surface of a sphere whose diameter is 2 feet, or 24 inches.
4. How many square miles on the earth's surface, supposing it a sphere, whose diameter is 7912 miles.

398. To find the contents of a sphere.

RULE.—*Multiply the surface by the radius, and divide the product by 3: the quotient will be the contents.* (*Bk.* VIII., *Prop.* XIV.)

EXAMPLES.

1. What are the contents of a sphere whose diameter is 12?
2. What are the contents of a sphere whose diameter is 4?
3. What are the contents of a sphere whose diam. is 14 in.?
4. What are the contents of a sphere whose diam. is 6 ft.?



399. A prism is a figure whose ends are equal plane figures, and whose faces are parallelograms. The sum of the sides which bound the base is called the *perimeter* of the base; and the sum of the parallelograms which bound the solid, is called the *convex surface.*

400. To find the convex surface of a right prism.

Rule.—*Multiply the perimeter of the base by the perpendicular height, and the product will be the convex surface.* (*Bk.* VII., *Prop.* I.)

Examples.

1. What is the convex surface of a prism whose base is bounded by five equal sides, each of which is 35 feet, the altitude being 26 feet?

2. What is the convex surface when there are eight equal sides, each 15 feet in length, and the altitude is 12 feet?

401. To find the solid contents of a prism.

Rule.—*Multiply the area of the base by the altitude, and the product will be the contents.* (*Bk.* VII., *Prop.* XIV.)

Examples.

1. What are the contents of a square prism, each side of the square which forms the base being 15, and the altitude of the prism 20 feet?

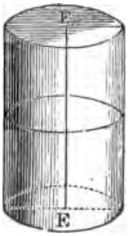
2. What are the contents of a cube, each side of which is 24 inches?

3. How many cubic feet in a block of marble, of which the length is 3 feet 2 inches, breadth 2 feet 8 inches, and height or thickness 2 feet 6 inches?

4. How many gallons of water will a cistern contain, whose dimensions are the same as in the last example?

5. Required the contents of a triangular prism whose height is 10 feet, and area of the base 350?

402. A cylinder is a figure with circular ends. The line E F is called the axis, or altitude; and the circular surface, the *convex surface* of the cylinder.



403. To find the convex surface.

Rule.—*Multiply the circumference of the base by the altitude, and the product will be the convex surface.* (*Bk.* VIII., *Prop.* I.)

Examples.

1. What is the convex surface of a cylinder, the diameter of whose base is 20, and the altitude 50?

2. What is the convex surface of a cylinder, whose altitude is 14 feet, and the circumference of its base 8 feet 4 inches?

3. Find the convex surface of a cylinder, in square inches, the diameter of whose base is 30 inches, and altitude 5 feet.

404. To find the contents of a cylinder.

Rule.—*Multiply the area of the base by the altitude: the product will be the contents.* (*Bk.* VIII., *Prop.* II.)

Examples.

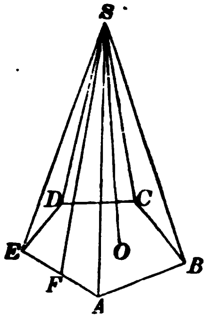
1. Required the contents of a cylinder, of which the altitude is 12 feet, and the diameter of the base 15 feet?

2. What are the contents of a cylinder, the diameter of whose base is 20, and the altitude 29?

3. How many barrels of wine will a cylindrical vat fill, the diameter of whose base is 12, and the altitude 30?

4. What are the contents, in hogsheads, of a cylindrical cistern, the diameter of whose base is 16, and altitude 9?

5. What are the contents of a cylinder, the diameter of whose base is 50, and altitude 15?



405. A pyramid is a figure formed by several triangular planes united at the same point S, and terminating in the different sides of a plane figure, as A B C D E. The altitude of the pyramid is the line S O, drawn perpendicular at the base.

406. To find the contents of a pyramid.

Rule.—*Multiply the area of the base by one-third of the altitude.* (*Bk.* VII., *Prop.* XVII.)

Examples.

1. Required the contents of a pyramid, of which the area of the base is 95, and the altitude 15?

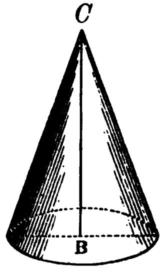
2. What are the contents of a pyramid, the area of whose base is 260, and the altitude 24?

3. What are the contents of a pyramid, the area of whose base is 207, and altitude 18?

4. What are the contents of a pyramid, the area of whose base is 403, and altitude 36?

5. What are the contents of a pyramid, the area of whose base is 270, and altitude 16?

6. A pyramid has a rectangular base, the sides of which are 25 and 12: the altitude of the pyramid is 36: what are its contents?



407. A cone is a figure with a circular base, and tapering to a point called the *vertex*. The point C is the vertex, and the line C B is called the axis, or altitude.

408. To find the contents of a cone.

Rule.—*Multiply the area of the base by the altitude, and divide the product by 3.* (*Bk.* VIII., *Prop.* V.)

Examples.

1. Required the contents of a cone, the diameter of whose base is 5, and the altitude 10.

2. What are the contents of a cone, the diameter of whose base is 18, and the altitude 27?

3. What are the contents of a cone, the diameter of whose base is 20, and the altitude 30?

4. What are the contents of a cone, whose altitude is 27 feet, and the diameter of the base 10 feet?

5. What are the contents of a cone, whose altitude is 12 feet, and the diameter of its base 15 feet?

◆ ◆ ◆

GAUGING.

409. Gauging is a process for determining the capacity or contents of casks.

The mean diameter of a cask is found by adding to the head diameter, two-thirds of the difference between the bung and head diameters, or, if the staves are not much curved, by adding six-tenths. This reduces the cask to a cylinder. Then, to find the contents, we multiply the square of the mean diameter by the decimal .7854, and the product by the length. This will give the contents in cubic inches. Then, if we divide by 231, we have the contents in gallons (Art. 199).

Multiply the length by the square of the mean diameter, then by the decimal .7854, and divide by 231.

OPERATION.

$$l \times d^2 \times \frac{.7854}{231} = l \times d^2 \times .0034$$

If, then, we divide the decimal .7854 by 231, the quotient, carried to four places of decimals, is .0034, and this decimal multiplied by the square of the mean diameter and by the length of the cask, will give the contents in gallons.

410. Hence, for gauging or measuring casks, we have the following

Rule.—*Multiply the length by the square of the mean diameter; then multiply by 34, and point off four decimal places, and the product will then express gallons and the decimals of a gallon.*

Examples.

1. How many gallons in a cask whose bung diameter is 36 inches, head diameter 30 inches, and length 50 inches?

We first find the difference of the diameters, of which we take two-thirds, and add to the head diameter. We then multiply the square of the mean diameter, the length, and 34 together, and point off four decimal places in the product.

OPERATION.

$36 - 30 = 6$

$\frac{2}{3}$ of $6 = 4$

$30 + 4 = 34$

$34^2 = 1156$

$1156 \times 50 \times 34 = 196.52$ gal.

2. What is the number of gallons in a cask whose bung diameter is 38 inches, head diameter 32 inches, and length 42 inches?

3. How many gallons in a cask whose length is 36 inches, bung diameter 35 inches, and head diameter 30 inches?

4. How many gallons in a cask whose length is 40 inches, head diameter 34 inches, and bung diameter 38 inches?

5. A water-tub holds 147 gallons; the pipe usually brings in 14 gallons in 9 minutes; the tap discharges at a medium, 40 gallons in 31 minutes. Now, supposing these to be left open, and the water to be turned on at 2 o'clock in the morning; a servant at 5 shuts the tap, and is solicitous to know at what time the tub will be filled, in case the water continues to flow.

Promiscuous Examples.

1. Sound travels about 1142 feet in a second; now, if the flash of a cannon is seen at the moment it is fired, and the report heard 45 seconds after, what distance would the observer be from the gun?

2. Two persons depart from the same place; one travels 32, and the other 36 miles a day; if they travel in the same direction, how far will they be apart at the end of 19 days, and how far, if they travel in contrary directions?

3. A traveler leaves New Haven at 8 o'clock on Monday morning, and walks toward Albany, at the rate of 3 miles an hour; another traveler sets out from Albany at 4 o'clock on the same evening, and walks toward New Haven, at the rate of 4 miles an hour: now, supposing the distance to be 130 miles, where on the road will they meet?

4. Two persons, A and B, are indebted to C; A owes $2173, which is the least debt, and the difference of the debts is $371: what is the amount of their indebtedness?

5. What number, added to the 43d part of 4429, will make the sum 240?

6. What number is that which, being multiplied by $\frac{2}{3}$, will produce $\frac{1}{4}$?

7. A tailor had a piece of cloth containing $24\frac{1}{2}$ yards, from which he cut $6\frac{5}{8}$ yards: how much was there left?

8. From $\frac{3}{4}$ of $\frac{6\frac{4}{5}}{12}$, take $\frac{1}{3}$ of $\frac{1\frac{7}{8}}{5}$.

9. What is the difference between $3\frac{3}{8}+7\frac{5}{6}$, and $4+2\frac{1}{12}$?

10. The product of two numbers is 2.26, and one of the numbers is .25: what is the other?

11. If the divisor of a certain number be $6.66\frac{2}{3}$, and the quotient $\frac{5}{8}$, what will be the dividend?

12. A merchant bought 13 packages of goods, for which he paid $326: what will 39 packages cost, at the same rate?

13. How many bushels of oats, at $62\frac{1}{2}$ cents a bushel, will pay for 4250 feet of lumber, at $7.50 per thousand?

14. Bought 2 hhd. of sugar, which weighed as follows: the 1st, 5 cwt. 1 qr. 18 lb.; the 2d, 6 cwt. 10 lb.: what did it cost, at 7 cents per pound?

15. How many hours between the 4th of Sept., 1854, at 3 P. M., and the 20th day of April, 1855, at 10 A. M.?

16. If $\frac{5}{6}$ of a gallon of wine costs $\frac{5}{8}$ of a dollar, what will $\frac{3}{8}$ of a hogshead cost?

17. The sum of two numbers is 425, and their difference 1.625: what are the numbers?

18. The sum of two numbers is $\frac{4}{7}$, and their difference $\frac{1}{3}$: what are the numbers?

19. What is the difference between twice five and fifty, and twice fifty-five?

20. What number is that which being multiplied by three-thousandths, the product will be 2637?

21. What is the difference between half a dozen dozens and six dozen dozens?

22. The slow, or parade step, is 70 paces per minute, at 28 inches each pace: how fast is that per hour?

23. A person dying, divided his property between his widow and his four sons; to his widow he gave $1780, and to each of his sons, $1250; he had been 25$\frac{1}{2}$ years in business, and had cleared, on an average, $126 a year: how much had he when he began business?

24. How many planks, 15 feet long and 15 inches wide, will floor a barn, 60$\frac{1}{2}$ feet long and 33$\frac{1}{2}$ feet wide?

25. A room 30 feet long and 18 feet wide, is to be covered with painted cloth $\frac{3}{4}$ of a yard wide: how many yards will cover it?

26. There was a company of soldiers, of whom $\frac{1}{8}$ were on guard, $\frac{1}{6}$ preparing dinner, and the remainder, 85 men, were drilling: how many were there in the company?

27. A person owned $\frac{3}{5}$ of a mine, and sold $\frac{3}{4}$ of his interest for $1710: what was the value of the entire mine?

28. In a certain orchard, $\frac{1}{2}$ of the trees bear apples, $\frac{1}{4}$ of them bear peaches, $\frac{1}{6}$ of them plums, 120 of them cherries, and 80 of them pears: how many trees are there in the orchard?

29. A, B, and C trade together, and gain $120, which is to be shared according to each one's stock; A put in $140, B $300, and C $160: what is each man's share?

30. Four persons traded together, on a capital of $6000, of which A put in $\frac{1}{2}$, B put in $\frac{1}{4}$, C put in $\frac{1}{6}$, and D the

rest; at the end of 4 years, they had gained $4728: what was each one's share of the gain?

31. A can do a piece of work in 12 days, and B can do the same work in 18 days: how long will it take both, if they work together?

32. If a barrel of flour will last one family 7½ months, a second family 9 months, and a third 11¼ months, how long will it last the three families together?

33. Suppose I have $\frac{3}{16}$ of a ship worth $1200: what part have I left after selling ⅖ of ⅘ of my share, and what is it worth?

34. What number is that which, being multiplied by ⅔ of ⅘ of 1½, the product will be 1?

35. Divide $420 among three persons, so that the second shall have ¾ as much as the first, and the third ½ as much as the other two.

36. Divide $10429.50 among three persons, so that as often as one gets $4, the second will get $6, and the third $7.

37. A gentleman whose annual income is £1500, spends 20 guineas a week: does he save, or run in debt, and how much?

38. A lady being asked her age, and not wishing to give a direct answer, said: "I have 9 children, and three years elapsed between the birth of each of them; the eldest was born when I was 19 years old, and the youngest is now exactly 19:" what was her age?

39. A wall of 700 yards in length, was to be built in 29 days; 12 men were employed on it for 11 days, and only completed 220 yards: how many men must be added, to complete the wall in the required time?

40. A besieged garrison, consisting of 360 men, was provisioned for 6 months, but hearing of no relief at the end of 5 months, dismissed so many of the garrison, that the remaining provisions lasted 5 months: how many men were sent away?

41. A farmer exchanged 70 bushels of rye, at $0.92 per bushel, for 40 bushels of wheat, at $1.37½ a bushel, and received the balance in oats, at $0.40 per bushel: how many bushels of oats did he receive?

42. If a quantity of provisions serves 1500 men 12 weeks, at the rate of 20 ounces a day for each man, how many men will the same provisions maintain for 20 weeks, at the rate of 8 ounces a day for each man?

43. How many bricks, 8 inches long and 4 inches wide, will pave a yard that is 100 feet by 50 feet?

Can 44. How many stones, 2 feet long, 1 foot wide, and 6 inches thick, will build a wall 12 yards long, 2 yards high, and 4 feet thick?

45. If 20 men perform a work in 12 days, how many men will accomplish thrice as much in one-fifth of the time?

46. Twelve workmen, working 12 hours a day, have made, in 12 days, 12 pieces of cloth, each piece 75 yards long: how many pieces of the same stuff would have been made, each piece 25 yards long, if there had been 7 more workmen?

Can 47. A person was born on the 1st day of Oct., 1801, at 6 o'clock in the morning: what was his age on the 21st of Sept., 1854, at half-past 4 in the afternoon?

Can 48. A man went to sea at 17 years of age; 8 years after, he had a son born, who lived 46 years, and died before his father: after which the father lived twice twenty years, and died: what was the age of the father?

49. A can do a piece of work, alone, in 10 days, and B in 13 days: in what time can they do it if they work together?

50. A cistern, containing 60 gallons of water, has three unequal pipes for discharging it; the largest will empty it in one hour, the second in two hours, and the third in three hours; in what time will the cistern be emptied if they run together?

51. A man bought ⅝ of the capital of a cotton factory, at par; he retained ⅘ of his purchase, and sold the balance for $5000, which was 15 per cent. advance on the cost: what was the whole capital of the factory?

52. Bought a cow for $30 cash, and sold her for $35 at a credit of 8 months: reckoning the interest at 6 per cent., how much did I gain?

53. If, when I sell cloth for 8s. 9d. per yard, I gain 12 per cent., what per cent. will be gained when it is sold for 10s. 6d. per yard?

54. How much stock, at par value, can be purchased for $8500, at 8½ per cent. premium, ¼ per cent. being paid to the broker?

55. Divide $500 among 4 persons, so that when A has ½ of a dollar, B shall have ⅓, C, ¼, and D, ⅕.

56. Three persons purchase a piece of property for $9202; the first gave a certain sum, the second three times as much,

and the third one and a half times as much as the other two: what did each pay?

57. A ship has a leak, by which it would fill and sink in 15 hours, but, by means of a pump, it could be emptied, if full, in 16 hours. Now, if the pump is worked from the time the leak begins, how long before the ship will sink?

58. A reservoir of water has two cocks to supply it; the first would fill it in 40 minutes, and the second in 50. It has likewise a discharging cock, by which it may be emptied, when full, in 25 minutes. Now, if all the cocks are opened at once, and the water runs uniformly as we have supposed, how long before the cistern will be filled?

59. If a house is 50 feet wide, and the post which supports the ridge-pole is 12 feet high, what will be the length of the rafters?

60. A man had 12 sons; the youngest was 3 years old and the eldest 58, and their ages increased in arithmetical progression: what was the common difference of their ages?

61. A man bought 10 bushels of wheat, on the condition that he should pay 1 cent for the 1st bushel, 3 for the 2d, 9 for the 3d, and so on to the last: what did he pay for the last bushel, and for the 10 bushels?

62. There is a mixture made of wheat at 4s. per bushel, rye at 3s., barley at 2s., with 12 bushels of oats at 18d. per bushel: what proportion must be taken of each sort, to make the mixture worth 3s. 6d. per bushel?

63. What length must be cut off a board 8½ inches broad, to contain a square foot?

64. What is the difference between the interest of $2500, for 4 years 9 months, at 6 per cent., and half that sum for twice the time, at half the same rate per cent.?

65. A person lent a certain sum at 4 per cent. per annum; had this remained at interest 3 years, he would have received for principal and interest, $9676.80: what was the principal?

66. In what time will $2377.50 amount to $2852.42, at 4 per cent. per annum?

67. A man purchased a building lot, containing 3600 square feet, at the cost of $1.50 per foot, on which he built a store at an expense of $3000. He paid yearly $180.66 for repairs and taxes: what annual rent must he receive, to obtain 10 per cent. on the cost?

68. A's note, of \$7851.04, was dated Sept. 5th, 1837, on which were indorsed the following payments, viz.: Nov. 13th, 1839, \$416.98; May 10th, 1840, \$152: what was due March 1st, 1841, the interest being 6 per cent.?

69. If 1 pound of tea be equal in value to 50 oranges, and 70 oranges be worth 84 lemons: what is the value of a pound of tea, when a lemon is worth 2 cents?

70. A person bought 160 oranges at 2 for a penny, and 180 more at 3 for a penny; after which he sold them out at the rate of 5 for 2 pence: did he make or lose, and how much?

71. A snail, in getting up a pole 20 feet high, was observed to climb up 8 feet every day, but to descend 4 feet every night: in what time did he reach the top of the pole?

72. What is the height of a wall, which is $14\frac{1}{2}$ yards in length, and $\frac{7}{10}$ of a yard in thickness, and which has cost \$406, it having been paid for at the rate of \$10 per cubic yard?

73. What will be the duty on 225 bags of coffee, each weighing gross 160 lbs., invoiced at 6 cts. per lb.; 2 per cent. being the actual tare, and 20 per cent. the duty?

74. A house is 40 feet from the ground to the eaves, and it is required to find the length of a ladder which will reach the eaves; supposing the foot of the ladder cannot be placed nearer to the house than 30 feet.

75. A person dying, worth \$5460, left a wife and 2 children, a son and daughter, absent in a foreign country. He directed that, if his son returned, the mother should have one-third of the estate, and the son the remainder; but if the daughter returned, she should have one-third, and the mother the remainder. Now it so happened that they both returned: how must the estate be divided to fulfil the father's intentions?

76. If a cylindrical cistern is 8 feet in diameter and 6 feet in height, how many hogsheads of water will it contain?

77. If A is 40 years old and B is 16, how many years since A was 9 times as old as B?

78. A can earn a certain sum of money in 20 days: A and B together can earn the same sum in 6 days: how long will it take B, alone, to earn the same sum?

79. A and B can perform a certain piece of work in 6 days, B and C in 7 days, and A and C in 14 days: in what time *would each* do it alone?

METRIC SYSTEM OF WEIGHTS AND MEASURES.

The primary base, in this system, for all denominations of weights and measures, is the one-ten-millionth part of the distance from the equator to the pole, measured on the earth's surface. It is called a METER, and is equal to 39.37 inches, very nearly.

The change from the base, in all the denominations, is according to the decimal scale of tens: that is, the units increase ten times, at each step, in the ascending scale, and decrease ten times, at each step, in the descending scale.

MEASURES OF LENGTH.

Base, 1 meter = 39.37 inches, nearly.

TABLE.

Ascending Scale.					Descending Scale.		
1 Myriameter.	1 Kilometer.	1 Hectometer.	1 Decameter.	1 METER. UNIT.	1 Decimeter.	1 Centimeter.	1 Millimeter.

The names, in the ascending scale, are formed by prefixing to the base, Meter, the words, Deca (ten), Hecto (one hundred), Kilo (one thousand), Myria (ten thousand), from the Greek numerals; and in the descending scale, by prefixing Deci (tenth), Centi (hundredth), Milli (thousandth), from the Latin numerals. Hence, the name of a unit indicates whether it is greater or less than the standard ; and, also, how many times. The table is thus read:

10 millimeters make 1 centimeter.
10 centimeters make 1 decimeter.
10 decimeters make 1 METER.
10 METERS make 1 decameter.
10 decameters make 1 hectometer.
10 hectometers make 1 kilometer.
10 kilometers make 1 myriameter.

Table of Equivalents.

Myriameter.	Kilometer.	Hectometer.	Decameter.	METER.	Decimeter.	Centimeter.	Millimeter.
.	.	.	.	.	.	1=	10
.	.	.	.	.	1=	10=	100
.	.	.	.	1=	10=	100=	1,000
.	.	.	1=	10=	100=	1,000=	10,000
.	.	1=	10=	100=	1,000=	10,000=	100,000
.	1=	10=	100=	1,000=	10,000=	100,000=	1,000,000
1=	10=	100=	1,000=	10,000=	100,000=	1,000,000=	10,000,000

Table of Equivalents in English Measure.

1 Millimeter = 0.0394 inches, nearly.
1 Centimeter = 0.3937 "
1 Decimeter = 3.9370 "
1 METER = 39.37 in. = 3.280833 ft.
1 Decameter = 32 ft. 9.7 in.
1 Hectometer = 19 rd. 14 ft. 7 in.
1 Kilometer = 4 fur. 38 rd. 13 ft. 10 in.
1 Myriameter = 6 mi. 1 fur. 28 rd. 6 ft. 4 in.

Besides a clear apprehension of the length of the base, 1 *meter, it* is well to consider the length of the largest unit, the myriameter, equal to nearly 6 and one-fourth miles; and also the length of the smallest unit, the millimeter, about four-hundredths of an inch. Compare, also, each of the smaller measures, the decimeter and centimeter, with the inch.

When, in the metric system, the value of any single unit is fixed in the mind, the values of all the others may be readily apprehended, since they always arise from multiplying or dividing by 10.

NOTE.—In all the tables, the UNIT is in small capitals, and should be constantly referred to.

Methods of Reading.

The number 25365.897 meters, is read, in English,

Twenty-five thousand three hundred and sixty-five meters, and 897 thousandths of a meter. But in the language of the metric system, it may be read,

Two myriameters, 5 kilometers, 3 hectometers, 6 decameters, 5 meters, 8 decimeters, 9 centimeters, and 7 millimeters. It may also be read, beginning with the lowest denomination, 7 millimeters, 9 centimeters, &c., &c.

In reading, remember that the unit of any place is *ten* times as great as the unit of the place next at the right, and *one-tenth* of the unit of the place next at the left. Hence, the change from one unit to another, and the methods of reduction and reading, are identical with those in the system of decimal currency.

1. Write, numerate, and read, five hundred and ninety-six hectometers.

2. Write, numerate, and read, eighty-nine thousand and forty-one centimeters.

Questions.—What is the primary base of the metric system? To what portion of the earth's surface is it equal? What is its length? What is the ascending scale from the meter? What is the descending scale? What is the length of a myriameter? According to what law do the different units increase and decrease?

MEASURES OF SURFACES OR SQUARE MEASURE.

Base, 1 Are = the square whose side is 10 meters.
= 119.6 square yards, nearly.
= 4 perches or square rods, nearly.

The unit of surface is a square whose side is 10 meters. It is called an ARE, and is equal to 100 square meters.

TABLE.

1 Hectare.	1 ARE. UNIT.	1 Centare.

The table is thus read:

100 centares	make	1 ARE.
100 ares	make	1 hectare.

Table of Equivalents.

Hectare.	Are.	Centare.
	1 =	100
1 =	100 =	10,000

Equivalents in acres, roods, and perches.

1 Centare = 1.195985 sq. yards, nearly.
1 ARE = 3.95367 perches.
1 Hectare = 2A. 1R. 35.367P.

MEASURES OF VOLUMES.

Base, 1 liter = the cube or the decimeter.
= 61.023378 cubic inches.
= a little more than a wine quart.

Questions.—What is the primary base of the measure for surfaces? To what is it equal, in square yards? What are the denominations, beginning with the least? To what is the centare equal? To what is *the hectare* equal?

The unit for the measure of volume is the cube whose edge is one-tenth of the meter—that is, a cube whose edge is 3.937 inches. This cube is called a LITER, and is one-thousandth part of the cube constructed on the meter, as an edge.

TABLE.

Ascending Scale.				Descending Scale.		
1 Kiloliter, or Stere.	1 Hectoliter.	1 Decaliter.	1 LITER. UNIT.	1 Deciliter.	1 Centiliter.	1 Milliliter.

The table is thus read:

10 milliliters make 1 centiliter.
10 centiliters make 1 deciliter.
10 deciliters make 1 liter.
10 liters make 1 decaliter.
10 decaliters make 1 hectoliter.
10 hectoliters make 1 kiloliter, or stere.

Table of Equivalents.

Kiloliter, or Stere.	Hectoliter.	Decaliter.	LITER.	Deciliter.	Centiliter.	Milliliter.
.	.	.	.	.	1 =	10
.	.	.	.	1 =	10 =	100
.	.	.	1 =	10 =	100 =	1,000
.	.	1 =	10 =	100 =	1,000 =	10,000
.	1 =	10 =	100 =	1,000 =	10,000 =	100,000
1 =	10 =	100 =	1,000 =	10,000 =	100,000 =	1,000,000

NOTE.—The kiloliter, or stere, is the cube constructed on the meter, as an edge. Hence, the liter is one-thousandth part of the kiloliter.

Equivalents in Cubic Measure.

1 milliliter = .061023 cubic inches.
1 centiliter = .610234 cubic inches.
1 deciliter = 6.102338 cubic inches.
1 LITER = 61.023378 cubic inches.
1 decaliter = 610.233779 cubic inches.
1 hectoliter = 6102.337795 cu. in. = 3.5314454 cu. ft.
1 kiloliter, or stere = 61023.377953 cu. in. = 35.314454 cu. ft.

NOTE.—Law of change in the units, and methods of reading, are the same as in linear measure.

DRY MEASURE.

EQUIVALENTS IN THE WINCHESTER BUSHEL.

Since 1 bushel = 2150.4 cu. in.; 1 pk. = 537.6 cu. in.; 1 qt. = 67.2 cu. in.; 1 pt. = 33.6 cu. in.; therefore,

1 milliliter = .001816 pints.
1 centiliter = .018161 pints.
1 deciliter = .181611 pints.
1 LITER = 1.816112 pints.
1 decaliter = 1 pk. 1.08056 qt.
1 hectoliter = 2 bu. 3 pk. 2 qt. 1.6112 pt.
1 kiloliter, or stere = 28 bu. 1 pk. 4 qt. 0.112 pt.

NOTE.—The liter, or standard, is a little less than 1 quart, and the stere, nearly 30 Winchester bushels.

LIQUID MEASURE.

EQUIVALENTS IN THE WINE GALLON.

Since 1 wine gallon contains 231 cubic inches, 1 quart will contain 57.75 cubic inches; 1 pint, 28.875 cubic inches; and 1 gill, 7.21875 cubic inches; we have,

---

Questions.—What is the unit for the measure of volumes? To what is it equal in cubic inches? What part is it of the cube on the meter? Name all the denominations of volume. What is the unit of Dry Measure? To what is it equal? To what is the stere or kiloliter equal?

1 milliliter = 0.008453 gills.
1 centiliter = .084534 gills.
1 deciliter = .845345 gills.
1 LITER = 1 qt. .11336 pt.
1 decaliter = 2 gal. 2 qt. 1 pt. .1336 pt.
1 hectoliter = 26 gal. 1 qt. 1 pt. 1.344 gills.
1 kiloliter, or stere = 1 tun, 12 gal. 0 qt. 1 pt. 1.44 gill.

WEIGHTS.

Base, 1 gram = weight of a cubic centimeter of rain-water.
= 15.432 grains, Troy, nearly.
= .0352746 ounces, Avoirdupois, nearly.

The unit of weight is also equal to the one-millionth part of the weight of a cubic meter of pure rain-water, weighed in vacuum. It is called a GRAM, and is equal to 15.432 grains, Troy, which is equal to .0352746 ounces, Avoirdupois, very nearly.

TABLE.

Ascending Scale.							Descending Scale.		
1 Millier, tonneau	1 Quintal.	1 Myriagram.	1 Kilogram.	1 Hectogram.	1 Decagram.	1 GRAM. UNIT.	1 Decigram.	1 Centigram.	1 Milligram.

The table is thus read:

10 milligrams make 1 centigram.
10 centigrams make 1 decigram.
10 decigrams make 1 gram.
10 grams make 1 decagram.
10 decagrams make 1 hectogram.
10 hectograms make 1 kilogram.
10 kilograms make 1 myriagram.
10 myriagrams make 1 quintal.
10 quintals make 1 millier, or tonneau.

Table of Equivalents.

Millier, tonneau.	Quintal.	Myriagram.	Kilogram.	Hectogram.	Decagram.	GRAM.	Decigram.	Centigram.	Milligram.
.	.	.	.	.	.	.	.	1=	10
.	.	.	.	.	.	.	1=	10=	100
.	.	.	.	.	.	1=	10=	100=	1,000
.	.	.	.	.	1=	10=	100=	1,000=	10,000
.	.	.	.	1=	10=	100=	1,000=	10,000=	100,000
.	.	.	1=	10=	100=	1,000=	10,000=	100,000=	1,000,000
.	.	1=	10=	100=	1,000=	10,000=	100,000=	1,000,000=	10,000,000
.	1=	10=	100=	1,000=	10,000=	100,000=	1,000,000=	10,000,000=	100,000,000
1=	10=	100=	1,000=	10,000=	100,000=	1,000,000=	10,000,000=	100,000,000=	1,000,000,000

Equivalents in Avoirdupois and Troy Weights.

1 Milligram	=	0.0154	grains, Troy.
1 Centigram	=	0.1543	grains, "
1 Decigram	=	1.5432	grains, "
1 GRAM	=	15.4327	grains, "
1 Decagram	=	0.3527	ounces, Avoirdupois.
1 Hectogram	=	3.5274	ounces, "
1 Kilogram	=	2.2046	pounds, "
1 Myriagram	=	22.046	pounds, "
1 Quintal	=	220.46	pounds, "
1 Millier, or ton.	=	2204.6	pounds, "

NOTE.—Law of change in the units, and methods of reading, the same as in Linear Measure.

NATURE OF THE METRIC SYSTEM.

The Metric system is based on the METER. From the meter, three other units are derived; and the four constitute the primary units of the system. They are:

Questions.—What is the unit of weight? To what is it equal in Troy weight? To what is it equal in Avoirdupois? Name all the units of the weight, from the lowest to the highest. To what is the *millier, or* tonneau, equal?

METER = 39.37 inches, nearly: unit of length.
ARE = a square on 10 meters: unit of surface.
LITER = a cube whose edge is a decimeter: unit of volume.
GRAM = the weight of a cube of rain-water, each edge of which is a centimeter: unit of weight.

From these four units all others are derived, according to the decimal scale.

Every system of Weights and Measures *must* have an *invariable unit* for its base; and every other unit of the entire system should be derived from it, according to a fixed law.

The French Government, in order to obtain an invariable unit, measured a degree of the arc of a meridian on the earth's surface; and from this computed the length of the meridional arc from the equator to the pole. This length they divided into ten million equal parts, and then took one of these parts for the unit of length, and called it a METER. The length of this meter is equal to 1 yard, 3 inches, and 37 hundredths of an inch, very nearly. Thus they obtained the length of the unit which is the base of the Metric System of Weights and Measures.

The next step was to fix the *law*, by which the other units should be obtained from the base. The scale of tens was adopted.

PRONUNCIATION.

ME'TER.	ARE.	LI'TER.	GRAM.
Mil'li-me-ter.		Mil'li-lï-ter.	Mil'li-gram.
Cen'ti-me-ter.	Cen'târe.	Cen'ti-lï-ter.	Cen'ti-gram.
Dec'i-me-ter.		Dec'i-lï-ter.	Dec'i-gram.
Dee'a-me-ter.		Dee'a-lï-ter.	Dee'a-gram.
Hee'to-me-ter.	Hee'târe.	Hee'to-lï-ter.	Hee'to-gram.
Kil'o-me-ter.		Kil'o-lï-ter.	Kil'o-gram.
Myr'i-a-me-ter.		Myr'i-a-lïter.	Myr'i-a-gram.

TO CHANGE FROM ONE SYSTEM TO THE OTHER.

To change, in Linear Measure, from the Metric to the Common system.

RULE.

Multiply the meters and decimals of a meter by 3.280833 (the value of a meter), *and the product will be the result in feet.*

To change from the Common to the Metric system.

RULE.

Reduce the linear measure to feet and decimals of a foot, and then divide by 3.280833: the quotient will be the result in meters and decimals of a meter.

EXAMPLES.

1. In 5961.874 meters, how many feet and inches?
2. In 874163 meters, and 37 hectometers, how many feet and inches?
3. Express 320 rods, 5 yards and 6 inches in the Metric Measures.
4. Express 1 mile, 3 furlongs, 39 rods and 5 yards in the Metric Measures.

To change, in Square Measure, from the Metric to the Common system.

RULE.

Reduce the number to ARES *and decimals of the* ARE: *then multiply by 3.95367, and the product will be the result in perches.*

To change, from the Common system, to the Metric system.

RULE.

Find the value of the number in perches and decimals of a perch: then divide by 3.95367, and the quotient will be the result in ARES *and decimals of the* ARE.

EXAMPLES.

1. In 6127 ares, 4 hectares and 3 centares, how many acres, roods and perches?

2. In 327 ares, 15 hectares and 89 centares, how many square feet?

3. In 4 acres, 3 perches and 200 square feet, how many hectares, ares and centares?

4. In 1375 square yards and 250 square feet, how many hectares, ares and centares?

To change, in measures of volume, from the Metric to the Common system.

RULE.

Reduce the number to LITERS *and decimals of the* LITER: *then multiply by 61.023378, and the product will be the result in cubic inches.*

To change, in measures of volume, from the Common to the Metric system.

RULE.

Reduce the number to cubic inches: then divide by 61.023378, and the quotient will be the result in LITERS *and decimals of the* LITER.

EXAMPLES.

1. In 6 kiloliters, 9 hectoliters, 6 decaliters, 8 liters and 4 centiliters, how many cubic feet and inches?

2. In 8 hectoliters, 9 decaliters, 27 liters and 15 milliers, how many cubic yards, feet and inches?

3. Change 27 cubic yards, 16 cubic feet, and 16 cubic inches, to the Metric measures.

4. Change 40 cubic yards, 25 cubic feet and 1167 inches, to the Metric measures.

To change, in weights, from the Metric to the Common system.

RULE.

Reduce the number to grams and decimals of a gram: then multiply by 15.423, and the product will be the result in grains Troy; or, multiply by .0352746, and the product will be ounces in Avoirdupois.

To change, in weights, from the Common to the Metric system.

RULE.

Reduce the number to Troy grains, or to Avoirdupois ounces: then divide by 15.423, or by .0352746, and the quotient will be GRAMS *and decimals of the* GRAM.

EXAMPLES.

1. Change 4 quintals, 6 kilograms, 4 decagrams, 7 grams and 6 centigrams, to Avoirdupois and Troy weights.

2. Change 2 milliers, 6 myriagrams, 9 grams, 4 decagrams and 9 milligrams, to Troy and Avoirdupois.

3. Change 1 T. 3 cwt. 3 qr. 20 lb. 6 oz., to the Metric weights.

4. Change 16 lb. 11 oz. 4 pwt. 19 gr., Troy, to the Metric weights.

Ques.—In linear measure, how do you change from the Metric to the Common system? How do you change from the Common to the Metric system?

In square measure, how do you change from the Metric to the Common system? How do you change from the Common to the Metric system?

In measures of volume, how do you change from the Metric to the Common system? How do you change from the Common to the Metric system?

In weights, how do you change from the Metric to the Common tem? How do you change from the Common to the Metric syst?

ANSWERS.

PAGE. EX. ANS. EX. ANS. EX. ANS. EX. ANS. EX. ANS. EX. ANS.
17. ‖ 1 | 30 ‖ 2 | 80 ‖ 3 | 9 ‖ 4 | 8654 ‖ 5 | 90876 ‖ 6 | 154970
17. ‖ 7 | 90813 ‖ 8 | 780571 ‖ 9 | 9023029 ‖ 10 | 86981028
17. ‖ 11 | 1409060760503 ‖ 12 | 13800769703009 ‖ 13
17. ‖ 9074008321060000201 ‖ 14 | 86981028 ‖ 15 | 169765432
19. ‖ 1 | 105 ‖ 2 | 302 ‖ 3 | 519 ‖ 4 | 1004 ‖ 5 | 8701 ‖ 6 | 40406
19. ‖ 7 | 58061 ‖ 8 | 99999 ‖ 9 | 406049 ‖ 10 | 641721
19. ‖ 11 | 1421602 ‖ 12 | 9621016 ‖ 13 | 94807409 ‖ 14 | 4000-
19. ‖ 306909 ‖ 15 | 49000000949065 ‖ 16 | 990000000099990999
19. ‖ 17 | 409000000000000000209106 ‖ 21. ‖ 1 | 209 ‖ 2 | 5005
21. ‖ 3 | 12012 ‖ 6 | 100101 ‖ 9 | 47204851 ‖ 10 | 6049072000-
21. ‖ 407861 ‖ 11 | 8994608502005064999 ‖ 12 | 59059059059-
21. ‖ 959 ‖ 13 | 12111 ‖ 14 | 9000000065 ‖ 15 | 304000001321941
21. ‖ 16 | 9640706080213 ‖ 17 | 305104001045074 ‖ 18 |
21. ‖ 80301006004620 ‖ 26. ‖ 1 | 196 ‖ 2 | 749 ‖ 3 | 1347
26. ‖ 4 | 566 ‖ 5 | 1183 ‖ 6 | 1397 ‖ 7 | 999 ‖ 8 | 689 ‖ 9 | 9789
26. ‖ 10 | 9799 ‖ 11 | 12089 ‖ 12 | 26901 ‖ 13 | 28637
26. ‖ 14 | 203933 ‖ 15 | 23272 ‖ 16 | 233642 ‖ 17 | 247481
26. ‖ 18 | 1994439 ‖ 19 | 175874 ‖ 20 | 172775 ‖ 21 | 98967
27. ‖ 22 | 10742750 ‖ 25 | 787676921 ‖ 26 | 100570011
27. ‖ 27 | 15371791930 ‖ 28 | 577 ‖ 29 | 7689 ‖ 30 | 502616
27. ‖ 31 | 799999 ‖ 32 | 43 ‖ 33 | 73 ‖ 34 | 888 ‖ 35 | 6846
28. ‖ 36 | 9798 ‖ 37 | 8601 ‖ 38 | 7032 ‖ 39 | 979 ‖ 40 | 559
28. ‖ 41 | 26754 ‖ 42 | 730528 ‖ 43 | 7047897 ‖ 44 | 25687540
28. ‖ 45 | 297303078 ‖ 46 | 13115375 ‖ 47 | 39428059 ‖ 48 |
29. ‖ 140700034 ‖ 49 | 1819857171537 ‖ 50 | 1105364 ‖ 51 |
29. ‖ 1079167 ‖ 52 | 1118969 ‖ 53 | 1665400 ‖ 30. ‖ 1 | 365
30. ‖ 2 | 137 ‖ 3 | 4025 ‖ 4 | 2800 ‖ 5 | 5567 ‖ 6 | 16375
30. ‖ 7 | 8 *h.*, 20 *c.*, 8 *o.*, 12 *c.*, 36 *c.*, 17 *y. c.*, 320 *s.*—all, 421
31. ‖ 8 | 392 ‖ 9 | 1880 ‖ 10 | 84237 ‖ 11 | 1100 ‖ 12 | 4107
31. ‖ 13 | 34671660 ‖ 14 | 112869 ‖ 15 | 2576406 ‖ 16 | 370

32. ‖ 17 | 1199–4596 ‖ 18 | 42390529–4530902 ‖ 19 | 1287462

32. ‖ 20 | 50994 ‖ 21 | 94341 ‖ 22 | 143985 ‖ 23 | 2728110

32. ‖ 24 | 1862 ‖ **35.** ‖ 2 | 632 ‖ 3 | 1010 ‖ 4 | 11123 ‖ 5 | 341

36. ‖ 6 | 111342 ‖ 7 | 724221 ‖ 8 | 10403360 ‖ 9 | 17102

36. ‖ 10 | 7513725 ‖ 11 | 58185356 ‖ 12 | 114865607

36. ‖ 13 | 2774 ‖ 14 | 1737 ‖ 15 | 85991 ‖ 16 | 27087 ‖ 17 | 9926

36. ‖ 18 | 20001 ‖ 19 | 99999 ‖ 20 | 1396765 ‖ 21 | 9949994

36. ‖ 22 | 260822 ‖ 23 | 2935621 ‖ 24 | 50391719 ‖ 25 | 28443

36. ‖ 26 | 99246591 ‖ 27 | 999999 ‖ 28 | 776462 ‖ 29 | 185-

36. ‖ 61747 ‖ 30 | 4244083 ‖ 31 | 8013105 ‖ 32 | 52528 ‖ 33 |

36. ‖ 56001996606 ‖ 34 | 14098198916 ‖ 35 | 37097042013300

37. ‖ 1 | 1115 ‖ 2 | 1894 ‖ 3 | 10 ‖ 4 | 45 ‖ 5 | 67 ‖ 6 | 62

37. ‖ 7 | 785608 ‖ 8 | 37 ‖ 9 | 5057 ‖ 10 | 3632 ‖ 11 | 696

38. ‖ 12 | 1825 ‖ 13 | 1732 ‖ 14 | ‖ 15 | 1759 ‖ 16 | 79

38. ‖ 17 | 175502 ‖ 18 | 1860805 ‖ 19 | 239 ‖ 20 | 250–1500

38. ‖ 21 | 1340–4020 ‖ 22 | 2769818 ‖ 23 | 145 ‖ 24 | 7906

38. ‖ 25 | 2d–5 ‖ 26 | 15914260 ‖ 27 | 20463760 ‖ 28 | 25579

39. ‖ 29 | 2276525 ‖ 1 | 29045 ‖ 2 | 418 ‖ 3 | 714 ‖ 4 | 5795

40. ‖ 5 | 390 ‖ 6 | 224980 ‖ 7 | 230–527 ‖ 8 | 19553068

40. ‖ 9 | 919 ‖ 10 | 55 ‖ 11 | 28223 ‖ 12 | 1706 ‖ 13 | 3818

40. ‖ 14 | 11854617 ‖ **44.** ‖ 1 | 867901 ‖ 2 | 557808 ‖ 3 | 2036223

44. ‖ 4 | 191616 ‖ 5 | 3903175 ‖ 6 | 5462172 ‖ 7 | 12533346

44. ‖ 8 | 320 ‖ 9 | 27144 ‖ 10 | 536392 ‖ 11 | 1352060 ‖ 12

44. ‖ 212912 ‖ 13 | 000000 ‖ 14 | 182982 ‖ 15 | 347535 ‖ 16 | 936

44. ‖ 17 | 1236 ‖ 18 | 298 ‖ 19 | 26766 ‖ 20 | 28511 ‖ 21 | 5382

44. ‖ 22 | 1485 ‖ **46.** ‖ 1 | 30660 ‖ 2 | 7913576 ‖ 3 | 1723682

46. ‖ 4 | 4280822 ‖ 5 | 19014604 ‖ 6 | 85564584 ‖ 7 | 2183178497

46. ‖ 8 | 93939864472 ‖ 9 | 395061696 ‖ 10 | 393916488

46. ‖ 11 | 65948806 ‖ 12 | 36914176 ‖ 13 | 85950000 ‖ 14 | 3320-

46. ‖ 863272 ‖ 15 | 816515040 ‖ 16 | 68959488 ‖ 17 | 62415-

46. ‖ 197790 ‖ 18 | 105062176 ‖ 19 | 294360066 ‖ 20 | 4984-

46. ‖ 155396 ‖ 21 | 406070736 ‖ 22 | 800105244 ‖ 23 | 1227-

47. ‖ 697160 ‖ 24 | 330445150 ‖ 25 | 36742802152 ‖ 26 |

47. ‖ 350152703494 ‖ 27 | 47190263648 ‖ 28 | 6119311582584

47. ‖ 29 | 2981473412994 ‖ 30 | 79817443789174 ‖ 31 | 35

47. ‖ 843685 ‖ 32 | 267293339604 ‖ 33 | 214007086881

47. ‖ 34 | 764819895290424 ‖ 35 | 2324334767296 ‖ 36 |

49. ‖ 443362670734173 ‖ 1 | 3924 ‖ 2 | 274032 ‖ 3 | 19180896

49. ‖ 4 | 15076944 ‖ 5 | 50618898 ‖ 6 | 7430778 ‖ 7 | 553248

50. ‖ 1 | 2540 ‖ 2 | 64800 ‖ 3 | 7987000 ‖ 4 | 98400000

50. ‖ 5 | 375000 ‖ 6 | 67040000 ‖ 7 | 214100 ‖ 8 | 87200000

50. ‖ 2 | 4368560000 ‖ 4 | 1315170000000 ‖ 5 | 259175000000

50. ‖ 6 | 1960310474010 ‖ 7 | 1484000 ‖ 8 | 109215040000

50. ‖ 9 | 5210018850000 ‖ **51.** ‖ 2 | 480 ‖ 3 | 168 ‖ 4 | 3087

51. ‖ 5 | 119568 ‖ 6 | 291 ‖ 7 | 11680 ‖ 8 | 4415 ‖ 9 | 123–2214

52. ‖ 10 | 18755 ‖ 11 | 984072 ‖ 12 | 24427326 ‖ 13 | 349440

52. ‖ 14 | 1057500 ‖ 15 | 150000 ‖ 16 | 131250000 ‖ 17 | 5306

52. ‖ 18 | 1620–2220 ‖ 19 | 968710 ‖ 20 | 2720 ‖ 21 | 408–2040

53. ‖ 22 | 17979365 ‖ 23 | Gained $636 ‖ 24 | 19152500

53. ‖ 25 | 1211 ‖ 26 | 4044 ‖ **57.** ‖ 3 | 18421 ‖ 4 | 134684

57. ‖ 5 | $63770\frac{6}{7}$ ‖ 6 | $349361\frac{4}{5}$ ‖ 7 | 43217 ‖ 8 | $104177\frac{2}{4}$

57. ‖ 9 | 12828 ‖ 10 | $46490\frac{3}{6}$ ‖ 11 | 15840087 ‖ 12 | 9486312

57. ‖ 13 | $2264702\frac{2}{3}$ ‖ 14 | $2343381\frac{2}{9}$ ‖ 15 | 390946494 ‖ 16 |

57. ‖ 81456626 ‖ 17 | $7544181\frac{5}{9}$ ‖ 18 | $9429537\frac{2}{8}$ ‖ 19 | $74909989\frac{3}{9}$

57. ‖ 20 | $2809934424\frac{4}{5}$ ‖ 21 | $3925434\frac{3}{7}$ ‖ 22 | $55828109\frac{2}{3}$

58. ‖ 23 | 13178 ‖ 24 | $87066\frac{1}{7}$ ‖ 25 | $469\frac{4}{8}$ ‖ 26 | $2262\frac{6}{7}$ ‖ 27 |

58. ‖ 10560 ‖ 28 | $4177\frac{1}{9}$ ‖ 29 | 23040 ‖ 30 | 327 ‖ 31 | 60534

60. ‖ 1 | 5 ‖ 2 | 140–14 rem. ‖ 3 | 3283 ‖ 4 | 194877 ‖ 5 | 65

61. ‖ 6 | 327 ‖ 7 | 302 ‖ 8 | 1049 ‖ 9 | 16 ‖ 10 | 399

61. ‖ 11 | 3283 ‖ 12 | $17544\frac{11}{64}$ ‖ 13 | 1345 ‖ 14 | $332627\frac{12}{238}$

61. ‖ 15 | $795073\frac{41}{78}$ ‖ 16 | $194877\frac{48}{74}$ ‖ 17 | 3283 ‖ 18 | $11572\frac{10}{162}$

61. ‖ 19 | $2017\frac{108}{216}$ ‖ 20 | 39407 ‖ 21 | 40367 ‖ 22 | 6704984

61. ‖ 23 | 78795 ‖ 24 | $10110\frac{9}{4674}$ ‖ 25 | $3097\frac{33788}{38473}$

61. ‖ 26 | $307140\frac{121}{912314}$ ‖ 27 | $34960078\frac{346}{8405}$ ‖ 28 | $80496\frac{11707}{57606}$

61. ‖ 29 | $1672940\frac{165534}{2104722}$ ‖ 30 | $206008604\frac{24}{2708830423}$

61. ‖ 31 | $30001000\frac{6347}{57143}$ ‖ 32 | $9948157977\frac{81605}{678957}$

61. ‖ 33 | $1935468\frac{14976}{37149}$ ‖ 34 | $15395919\frac{12214}{37140}$

61. ‖ 35 | $14243757748\frac{35411}{4143}$ ‖ 36 | $15395919\frac{12714}{37149}$ ‖ 37 |

61. ‖ $3008292243\frac{50442}{57007}$ ‖ 38 | 123456789 ‖ 39 | $41996\frac{2}{63}$ ‖ 40 | 208
63. ‖ 2 | 718328 ‖ 3 | 4871000 ‖ 1 | 105 ‖ 2 | 387 ‖ 3 | 133
63. ‖ 4 | 201 ‖ 5 | 387 ‖ 6 | 1935 ‖ 7 | 1809 ‖ 8 | 12864.
65. ‖ 2 | $17085\frac{29}{56}$ ‖ 3 | $67639\frac{21}{72}$ ‖ 4 | $6129\frac{11}{110}$ ‖ 5 | $3095\frac{87}{144}$
65. ‖ 6 | $45561\frac{16}{42}$ ‖ 7 | $1392\frac{27}{105}$ ‖ 8 | $245\frac{14}{110}$ ‖ 9 | $405\frac{141}{231}$
65. ‖ 1 | $4976\frac{3}{10}$ ‖ 2 | 76412 ‖ 3 | $496\frac{321}{1000}$ ‖ 4 | $6\frac{4978}{10000}$.
66. ‖ 1 | $4\frac{146327}{2100000}$ ‖ 2 | 146 ‖ 3 | $91\frac{135803}{400700}$ ‖ 4 | $156557\frac{34400}{36500}$
66. ‖ 5 | $253\frac{21700}{73000}$ ‖ 6 | $2247\frac{26649}{31500}$ ‖ 7 | $2364\frac{2833}{3700}$
66. ‖ 8 | $130\frac{455090}{504000}$ ‖ 9 | $10\frac{2370469}{85100000}$ ‖ 10 | $13\frac{6790397}{69870000}$.
67. ‖ 1 | 900 ‖ 2 | 552 ‖ 3 | 2313 ‖ 4 | 4285 ‖ 5 | 514 ‖ 6 | 6494–6178.
68. ‖ 7 | 329 ‖ 8 | 45 ‖ 9 | 85 ‖ 10 | 84032 ‖ 11 | 312, or
68. ‖ 2 per bbl. ‖ 12 | Loses 26 ‖ 13 | 23 ‖ 14 | 75
68. ‖ 15 | 4562 ‖ 16 | 281 ‖ 17 | $85\frac{148}{365}$ ‖ 18 | $714\frac{10}{35}$ ‖ 19 | 3
69. ‖ 20 | 110 ‖ 21 | 2 ‖ 22 | 42 ‖ 23 | 20 ‖ 24 | 36
69. ‖ 25 | 146 ‖ 26 | 886144 ‖ 27 | 22886826 ‖ 28 | 351
69 ‖ 29 | Gained 140 ‖ 30 | Lost 1625.
70. ‖ 31 | 9 ‖ 32 | 185—157 ‖ 33 | 5 ‖ 34 | 276 ‖ 35 | 17376
70. ‖ 36 | 140 ‖ 37 | 7545 ‖ 38 | $1\frac{32}{64}$ ‖ 39 | 456 ‖ 40 | 6483.
71. ‖ 41 | Lost 952 ‖ 42 | Each son, 13763; daughter, 12923
71. ‖ 43 | 60 ‖ 44 | 8250 ‖ 45 | Gained 12 ‖ 46 | 56350
71. ‖ 47 | 1st and 2d, 5000; 3d, 5850 ‖ 48 | 69000.
73. ‖ 1 | 3 and 3 ‖ 2 | 3 and 5 ‖ 3 | 2, 2, 2, and 3
73. ‖ 4 | 2, 2, 2, and 2 ‖ 5 | 2, 3, and 3 ‖ 6 | 2, 2, 2, 2, and 2
73. ‖ 7 | 2, 2, 2, 2, and 3 ‖ 8 | 2, 2, 2, and 7 ‖ 9 | 3, 3, and 7
73. ‖ 10 | 2, 2, and 19 ‖ 74. ‖ 1 | 3 ‖ 2 | 3 and 7 ‖ 3 | 3 and 7
74. ‖ 4 | 2 and 7 ‖ 5 | 2, 3, and 7 ‖ 6 | 3, 5, and 7
76. ‖ 3 | $1\frac{4}{7}$ ‖ 4 | 1 ‖ 5 | 11 ‖ 6 | 55 ‖ 7 | 8 ‖ 8 | 348 ‖ 9 | 46
76. ‖ 10 | 36 ‖ 11 | 7056 ‖ 12 | 80 ‖ 13 | 26 ‖ 14 | $40\frac{1}{3}$
77. ‖ 15 | $9\frac{1}{3}$ ‖ 16 | 40 ‖ 17 | 57 ‖ 18 | 15 ‖ 19 | $5\frac{5}{14}$ ‖ 20 | 58
77. ‖ 21 | 4 ‖ 22 | 3 ‖ 78. ‖ 1 | 24 ‖ 2 | 72 ‖ 79. ‖ 3 | 840
79. ‖ 4 | 147 ‖ 5 | 840 ‖ 6 | 196 ‖ 7 | 78 ‖ 8 | 84 ‖ 9 | 100
79. ‖ 10 | 156 ‖ 80. ‖ 1 | 6 ‖ 2 | 9 ‖ 3 | 6 ‖ 4 | 5 ‖ 5 | 6 ‖ 6 | 5
80. ‖ 7 | 14 ‖ 8 | 42 ‖ 81. ‖ 1 | 24 ‖ 2 | 4 ‖ 3 | 45 ‖ 4 | 630
81. ‖ 5 | 267 ‖ 6 | 396 ‖ 7 | 12 ‖ 8 | 8 ‖ 9 | 4 ‖ 10 | 3

87. ‖ 1 | $\frac{24}{8}$ ‖ 2 | $\frac{35}{5}$ ‖ 3 | $\frac{9}{7}$ ‖ 4 | $\frac{112}{19}$ ‖ 5 | $\frac{140}{6}$ ‖ 6 | $\frac{4175}{81}$

88. ‖ 1 | $\frac{3}{2}$ and $\frac{3}{1}=3$ ‖ 2 | $\frac{16}{16}$, $\frac{16}{8}$, and $\frac{16}{4}$ ‖ 3 | $\frac{4}{24}$, $\frac{4}{12}$, and $\frac{4}{8}$

88. ‖ 4 | $\frac{19}{42}$, $\frac{19}{21}$, and $\frac{19}{14}$ ‖ 5 | $\frac{37}{21}$, $\frac{37}{7}$, and $\frac{37}{6}$ ‖ 6 | $\frac{151}{40}$ and $\frac{151}{20}$

88. ‖ 1 | $\frac{14}{15}$ and $\frac{4}{15}$ ‖ 2 | $\frac{2}{56}$ ‖ 3 | $\frac{16}{117}$ and $\frac{50}{117}$ ‖ 4 | $\frac{80}{530}$, $\frac{40}{530}$

88. ‖ and $\frac{64}{530}$ ‖ 5 | $\frac{32}{127}$, $\frac{16}{127}$, and $\frac{8}{127}$ ‖ 6 | $\frac{14}{114}$, $\frac{2}{114}$, and $\frac{6}{11}$

89. ‖ 1 | $\frac{1}{4}$ ‖ 2 | $\frac{1}{35}$ ‖ 3 | $\frac{17}{120}$ ‖ 4 | $\frac{41}{1105}$ ‖ 2 | $\frac{21}{36}$, $\frac{28}{48}$, $\frac{35}{60}$, $\frac{42}{72}$

89. ‖ and $\frac{63}{108}$ ‖ 3 | $\frac{150}{168}$, $\frac{225}{252}$, $\frac{300}{336}$, $\frac{375}{420}$, and $\frac{450}{504}$ ‖ 90. ‖ 3 | $\frac{16}{64}$,

90. ‖ $\frac{8}{32}$, $\frac{4}{16}$, and $\frac{2}{8}$ ‖ 4 | $\frac{30}{90}$, $\frac{20}{60}$, $\frac{15}{45}$, $\frac{12}{36}$, $\frac{10}{30}$, $\frac{6}{18}$, and $\frac{5}{15}$

91. ‖ 1 | $\frac{108}{9}$ ‖ 2 | $\frac{690}{15}$ ‖ 3 | $\frac{182}{7}$ ‖ 4 | $\frac{7120}{40}$ ‖ 5 | $\frac{27362}{114}$

91. ‖ 6 | $\frac{216}{4}$ ‖ 7 | $\frac{384}{4}$ ‖ 8 | $\frac{6816}{16}$ ‖ 1 | $\frac{287}{6}$ ‖ 2 | $\frac{139}{8}$ ‖ 3 | $\frac{849}{20}$

91. ‖ 4 | $\frac{8129}{13}$ ‖ 5 | $\frac{23006}{112}$ ‖ 6 | $\frac{2033}{24}$ ‖ 7 | $\frac{5617}{365}$ ‖ 8 | $\frac{550050}{600}$

91. ‖ 9 | $\frac{1909}{76}$, $\frac{9382}{60}$ ‖ 92. ‖ 2 | $12\frac{3}{8}$ ‖ 3 | $2\frac{5}{7}$ ‖ 4 | $5\frac{6}{9}$ ‖ 5 | 5

92. ‖ 6 | $2\frac{77}{125}$, 24, $7\frac{1700}{6941}$, $13\frac{47712}{72301}$ ‖ 7 | $219\frac{2}{4}$ ‖ 8 | $20\frac{307}{3114}$,

92. ‖ $45\frac{22}{109}$, $9\frac{135750}{278436}$ ‖ 9 | $31504\frac{4453}{4674}$, 1345, $7947\frac{5294}{7803}$

93. ‖ 1 | $\frac{4}{5}$ ‖ 2 | $\frac{3}{4}$ ‖ 3 | $\frac{3}{4}$ ‖ 4 | $\frac{1}{4}$ ‖ 5 | $\frac{7}{8}$ ‖ 6 | $\frac{1}{6}$ ‖ 7 | $\frac{1}{9}$ ‖ 8 | $\frac{17}{33}$

93. ‖ 9 | $\frac{7}{15}$ ‖ 10 | $\frac{1}{3}$ ‖ 11 | $\frac{1}{8}$ ‖ 12 | $\frac{5}{8}$ ‖ 13 | $\frac{117}{265}$ ‖ 14 | $\frac{2}{13}$

93. ‖ 15 | $\frac{7}{9}$ ‖ 16 | $\frac{7}{9}$ ‖ 94. ‖ 1 | $\frac{5}{42}$ ‖ 2 | $\frac{90}{126}=\frac{5}{7}$ ‖ 3 | $\frac{3}{5}$

94. ‖ 4 | $\frac{819}{8}=102\frac{3}{8}$ ‖ 5 | $\frac{30}{14}=2\frac{1}{7}$ ‖ 6 | $\frac{106343}{324}=328\frac{71}{324}$

95. ‖ 2 | $\frac{5}{7}$ ‖ 3 | $\frac{45}{1664}$ ‖ 4 | $\frac{2}{209}$ ‖ 5 | $\frac{63}{8}=7\frac{7}{8}$ ‖ 6 | $\frac{169}{36}=4\frac{25}{36}$

95. ‖ 7 | $\frac{16}{21}$ ‖ 8 | $\frac{9}{875}$ ‖ 96. ‖ 1 | $\frac{63}{105}$, $\frac{70}{105}$, $\frac{15}{105}$ ‖ 2 | $\frac{110}{495}$, $\frac{180}{495}$,

96. ‖ $\frac{297}{495}$ ‖ 3 | $\frac{360}{504}$, $\frac{63}{504}$, $\frac{280}{504}$ ‖ 4 | $\frac{98}{42}$, $\frac{3}{42}$ ‖ 5 | $\frac{231}{42}$, $\frac{12}{42}$, $\frac{168}{42}$

96. ‖ 6 | $\frac{95}{60}$, $\frac{36}{60}$ ‖ 7 | $\frac{525}{600}$, $\frac{1080}{600}$, $\frac{22200}{600}$ ‖ 8 | $\frac{300}{75}$, $\frac{93}{75}$, $\frac{1550}{75}$

96. ‖ 9 | $\frac{1080}{144}$, $\frac{248}{144}$, $\frac{900}{144}$ ‖ 10 | $\frac{518}{126}$, $\frac{1026}{126}$, $\frac{315}{126}$ ‖ 12 | $\frac{3}{15}$, $\frac{5}{15}$

96. ‖ 13 | $\frac{2}{12}$, $\frac{1}{12}$, $\frac{9}{12}$ ‖ 14 | $\frac{12}{28}$, $\frac{8}{28}$, $\frac{8}{28}$ ‖ 15 | $\frac{15}{24}$, $\frac{92}{24}$, $\frac{18}{24}$

96. ‖ 16 | $\frac{77}{12}$, $\frac{114}{12}$, $\frac{60}{12}$ ‖ 17 | $\frac{282}{36}$, $\frac{16}{36}$, $\frac{9}{36}$, $\frac{4}{36}$ ‖ 97. ‖ 1 | $\frac{38}{4}$

97. ‖ $\frac{40}{45}$. $\frac{9}{45}$ ‖ 2 | $\frac{122}{8}$, $\frac{51}{8}$, $\frac{44}{8}$ ‖ 3 | $\frac{18}{90}$, $\frac{15}{90}$. $\frac{80}{90}$ ‖ 4 | $\frac{67}{120}$

97. ‖ $\frac{18}{120}$, $\frac{300}{120}$ ‖ 5 | $\frac{41}{60}$, $\frac{165}{60}$, $\frac{200}{60}$ ‖ 6 | $\frac{225}{72}$, $\frac{312}{72}$ $\frac{600}{72}$

97. ‖ 7 | $\frac{6}{12}$, $\frac{8}{12}$, $\frac{9}{12}$, $\frac{10}{12}$ ‖ 8 | $\frac{35}{84}$, $\frac{528}{84}$ ‖ 9 | $\frac{36}{90}$, $\frac{60}{90}$, $\frac{50}{90}$, $\frac{6}{9}$

97. ‖ 10 | $\frac{16}{48}$, $\frac{36}{48}$, $\frac{40}{48}$, $\frac{42}{48}$, $\frac{33}{48}$ ‖ 99. ‖ 1 | $\frac{13}{2}=6\frac{1}{2}$ ‖ 2 | $2\frac{1}{7}$ ‖ 3 | $4\frac{2}{3}$

99. ‖ 4 | $1\frac{11}{14}$ ‖ 5 | $1\frac{7}{30}$ ‖ 6 | $1\frac{11}{16}$ ‖ 7 | $2\frac{5}{6}$ ‖ 8 | $2\frac{29}{40}$ ‖ 9 | $10\frac{71}{90}$

99. ‖ 10 | $2\frac{179}{504}$ ‖ 11 | $2\frac{29}{120}$ ‖ 12 | $1\frac{25}{126}$ ‖ 13 | $1\frac{187}{1008}$ ‖ 14 | $1\frac{4}{5}$

99. ‖ 16 | $25\frac{33}{40}$ ‖ 17 | $52\frac{4}{9}$ ‖ 18 | $20\frac{17}{70}$ ‖ 19 | $21\frac{87}{116}$

99. ‖ 20 | $1426\frac{33}{50}$ ‖ 21 | $\frac{7}{36}$ ‖ 22 | $23\frac{14}{15}$ ‖ 23 | $17\frac{1}{20}$

99. ‖ 24 | $96\frac{29}{50}$ ‖ 25 | $64\frac{232}{2625}$ ‖ 2 | $\frac{17}{72}$, $\frac{16}{60} = \frac{4}{15}$ ‖ 3 | $\frac{20}{91}$,

99. ‖ $\frac{22}{120} = \frac{11}{60}$, $\frac{17}{60}$ ‖ 4 | $\frac{16}{48}$, $\frac{15}{54}$, $\frac{18}{45}$ ‖ 100. ‖ 2 | $\frac{5}{7}$ ‖ 3 | $\frac{3}{9} = \frac{1}{3}$

100. ‖ 4 | $\frac{58}{365}$ ‖ 5 | $\frac{166}{105} = 1\frac{61}{105}$ ‖ 101. ‖ 1 | $\frac{17}{63}$ ‖ 2 | $\frac{12}{56}$

101. ‖ 3 | $\frac{37}{221}$ ‖ 4 | $\frac{33}{100}$ ‖ 5 | $1\frac{1}{34}$ ‖ 6 | $4\frac{1}{32}$ ‖ 7 | $\frac{5}{33}$

101. ‖ 8 | $2\frac{17}{21}$ ‖ 9 | $\frac{31}{756}$ ‖ 11 | $68\frac{2}{15}$ ‖ 12 | $82\frac{7}{40}$ ‖ 14 | $10\frac{53}{72}$

101. ‖ 15 | $6\frac{41}{63}$ ‖ 16 | $8\frac{48}{55}$ ‖ 17 | $72\frac{19}{24}$ ‖ 19 | $\frac{2}{48}$, $\frac{7}{60}$, $\frac{8}{153}$,

102. ‖ $\frac{17}{740}$, $\frac{4}{165}$, $\frac{72}{4480}$ ‖ 1 | $3\frac{1}{12}$ ‖ 2 | $6\frac{5}{7}$ ‖ 3 | 35 ‖ 4 | $42\frac{1}{3}$

103. ‖ 5 | 123 ‖ 6 | $16\frac{2}{3}$ ‖ 7 | $8\frac{1}{3}$ ‖ 8 | 10 ‖ 9 | $13\frac{1}{2}$ ‖ 10 | $8\frac{8}{11}$

103. ‖ 11 | $20\frac{4}{5}$ ‖ 12 | $58\frac{1}{2}$ ‖ 13 | $50\frac{1}{7}$ ‖ 104. ‖ 1 | 21 ‖ 2 | $38\frac{1}{2}$

104. ‖ 3 | 75 ‖ 4 | $26\frac{2}{3}$ ‖ 5 | $5\frac{1}{3}$ ‖ 6 | $43\frac{3}{4}$ ‖ 7 | 180 ‖ 8 | $36\frac{4}{5}$

104. ‖ 9 | $16\frac{1}{5}$ ‖ 10 | 440 ‖ 105. ‖ 1 | $\frac{7}{10}$ ‖ 2 | $\frac{5}{13}$ ‖ 3 | $\frac{7}{32}$

105. ‖ 4 | $\frac{3}{7}$ ‖ 5 | $\frac{3}{4}$ ‖ 6 | $\frac{4}{7}$ ‖ 7 | $\frac{7}{20}$ ‖ 8 | $\frac{15}{28}$ ‖ 1 | $\frac{9}{40}$ ‖ 2 | $\frac{9}{20}$

106. ‖ 3 | $\frac{17}{18}$ ‖ 4 | $8\frac{1}{3}$ ‖ 5 | 675 ‖ 6 | $20\frac{4}{7}$ ‖ 8 | $636\frac{1}{2}$ ‖ 9 | 114

106. ‖ 10 | 1344 ‖ 11 | $20\frac{1}{2}$ ‖ 12 | $51\frac{2}{3}$ ‖ 13 | 90 ‖ 14 | 25

106. ‖ 15 | 2 ‖ 16 | $\frac{7}{44}$ ‖ 17 | $546\frac{5}{6}$ ‖ 18 | $1\frac{31}{64}$ ‖ 19 | 1295

106. ‖ 20 | $6\frac{2}{3}$ ‖ 21 | $60\frac{3}{7}$ ‖ 107. ‖ 2 | 40 ‖ 3 | 42 ‖ 4 | 81

108. ‖ 1 | $\frac{5}{22}$ ‖ 2 | $\frac{2}{37}$ ‖ 3 | $1\frac{8}{19}$ ‖ 4 | $\frac{2}{177}$ ‖ 5 | $\frac{3}{19}$ ‖ 6 | $\frac{1}{26}$

108. ‖ 7 | $\frac{1}{44}$ ‖ 8 | $\frac{2}{279}$ ‖ 9 | $\frac{3}{20}$ ‖ 10 | $\frac{2}{21}$ ‖ 11 | $\frac{2}{9}$ ‖ 12 | $\frac{3}{16}$

108. ‖ 13 | $\frac{1}{10}$ ‖ 14 | $\frac{1}{3}$ ‖ 15 | $\frac{2}{9}$ ‖ 16 | $\frac{8}{15}$ ‖ 109. ‖ 1 | 16

109. ‖ 2 | $226\frac{2}{15}$ ‖ 3 | $94\frac{1}{2}$ ‖ 4 | $513\frac{1}{3}$ ‖ 5 | $9\frac{9}{11}$ ‖ 6 | $11\frac{3}{7}$

109. ‖ 7 | $14\frac{2}{5}$ ‖ 110. ‖ 1 | $\frac{24}{25}$ ‖ 2 | $\frac{13}{33}$ ‖ 3 | $4\frac{2}{3}$ ‖ 4 | $\frac{2}{3}$

110. ‖ 5 | $6\frac{4}{11}$ ‖ 6 | $2\frac{5}{8}$ ‖ 7 | $5\frac{1}{2}$ ‖ 8 | 7 ‖ 9 | $\frac{3}{4}$ ‖ 10 | 14

110. ‖ 11 | $10\frac{2}{3}$ ‖ 12 | $7\frac{1}{9}$ ‖ 13 | 4 ‖ 14 | $5\frac{35}{48}$ ‖ 15 | $9\frac{1}{3}$

110. ‖ 16 | $\frac{1}{2}$ ‖ 17 | 27 ‖ 18 | $9\frac{1}{3}$ ‖ 19 | $3\frac{3}{20}$ ‖ 111. ‖ 1 | $\frac{14}{15}$

111. ‖ 2 | 10 ‖ 3 | $2\frac{3}{46}$ ‖ 4 | $\frac{7}{8}$ ‖ 5 | $\frac{12}{55}$ ‖ 6 | $2\frac{57}{59}$ ‖ 7 | $1\frac{97}{350}$

111. ‖ 8 | $8\frac{89}{311}$ ‖ 9 | $\frac{6}{35}$ ‖ 10 | $\frac{2}{45}$ ‖ 112. ‖ 1 | $1\frac{121}{280}$ ‖ 2 | $1\frac{3}{40}$

112. ‖ 3 | $2\frac{11}{24}$ ‖ 4 | 6 ‖ 5 | $4\frac{97}{112}$ ‖ 6 | $136\frac{11}{12}$ = James’;

112. ‖ $161\frac{19}{24}$ = Joseph’s; $177\frac{23}{120}$ = Daniel’s. ‖ 7 | $2\frac{1}{5}$ ‖ 8 | $9\frac{31}{70}$

113. ‖ 9 | $6\frac{1}{4}$ ‖ 10 | $19\frac{1}{3}$ ‖ 11 | $3\frac{120554}{143325}$ ‖ 12 | $8\frac{88}{171}$

113.	13	$\frac{108}{245}$	14	$25\frac{65}{173}$ each.	15	\$125	16	49200		
113.	17	$86\frac{1}{16}$	18	$\frac{5}{24}$	19	$6\frac{6}{19}$	20	$3\frac{109}{162}$	21	
113.	$671\frac{11}{32}$	22	$1\frac{1}{5}$	23	$26\frac{2}{3}$	24	$\frac{650}{2277}$	25	$2\frac{2141}{7250}$	
117.	1	.03	2	.016	3	.0017	4	.32	5	.0165
117.	6	18.03	7	12.009	8	16.012	9	14.65	10	22.1
117.	11	10000	12	10000	13	10000	14	1000000		
117.	15	10000	16	100000	17	1000000				
117.	18	1000000	23	41.3	24	16.000003	25	5.09		
117.	26	65.015	27	80.000003	28	2.000300	29			
117.	400.092	30	3000.0021	31	47.00021	32	1500-			
117.	.000003	33	39.640	34	3000.000840	35	.650			
120.	1	1303.9805	2	428.677893	3	169.371	4			
120.	1.5413	5	444.0924	6	1215.7304	7	246.067			
121.	8	389.989	9	71.2100	10	494.521	11	$.1\dot{1}$		
121.	12	1.215009	13	23001044.500059	14	.560596				
121.	15	74435.0309	16	33541.7500	17	513.13997				
122.	18	2096.32335	19	215318.146799	1	3294.9121				
122.	2	249.72501	3	9.888890	4	395.9992	5	.999		
122.	6	6377.9	7	365.007497	8	20.9943	9	260-		
123.	.4708953	10	10.030181	11	2.0094	12	34999-			
123.	.965	13	4238.60807	14	126.831874057	15				
123.	63.879674	16	106.9993	17	1.1215	18	.001			
124.	1	.036588	2	.365491	3	742.0361960				
124.	4	.001000001	5	.000000000147	6	9308.37				
124.	7	311.2751050254	8	.25	9	.0025	10	.0238416		
124.	11	3.04392632	12	17.2975	13	14.274	14			
124.	4.543944	15	240.1	16	5676.4	17	46.95			
124.	18	1.051279	19	.00025015788028	20	2.39015				
124.	21	.000016	22	.000274855	23	.00182002625				
125.	24	67.9—679	25	2.694—269.4	26	7.5—7500				
125.	27	10049000	126.	1	1.11	2	4.261			
126.	3	33.331	4	1.0001	5	4123.5	6	1175.07		
126	7	12.52534, 125.2534, 1252.534, 12525.34, 125253.4								
126	8	16.21987, 1621.987, 16219.87, 162198.7, 1621987								

127. ‖ 9 | 2.769 ‖ 10 | .06479 ‖ 11 | .000056 ‖ 12 | 1260
127. ‖ 13 | 10000 ‖ 14 | 66.66666+ ‖ 15 | .33333$\frac{1}{3}$ ‖ 16 |
127. ‖ 8.31111+ ‖ 17 | .01563+ ‖ 18 | .002102+ ‖ 19 |
127. ‖ .78431+ ‖ 20 | 1.45764 ‖ 21 | 156.4 ‖ 22 | .016495
127. ‖ 23 | 21940 ‖ 24 | 30100 ‖ 25 | 10—100—1000 ‖ 26 |
127. ‖ 12—120—1200 ‖ 27 | .75632+ ‖ 28 | .0001764+
28. ‖ 1 | .42857$\frac{1}{7}$ ‖ 2 | .88235$\frac{5}{17}$ ‖ 3 | .08571$\frac{15}{35}$ ‖ 4 | .25
128. ‖ -00797+ ‖ 5 | 025-.74358+-.003 ‖ 6 | .5-.002801+
128. ‖ 7 | 1.49653+ ‖ 8 | 1.33333$\frac{1}{3}$—.16299+—.79228+
128. ‖ 9 | .6375 ‖ 10 | 4.075 ‖ 11 | 8.136 ‖ 12 | .00875
128. ‖ 13 | .2976 ‖ 14 | .006875 ‖ 15 | .01171875 ‖ 16 |
128. ‖ .135546875 ‖ 17 | .0001 ‖ 18 | .222464 ‖ 1 | $\frac{1}{25}$
128. ‖ 2 | $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$ ‖ 3 | 4$\frac{1}{5}$, $\frac{7}{8}$, $\frac{3}{8}$ ‖ 4 | 3$\frac{67}{1000}$, 8$\frac{11}{40}$ ‖ 5 |
128. ‖ $\frac{49}{100000}$ ‖ 6 | $\frac{5}{16}$, $\frac{3}{4}$ ‖ 7 | $\frac{5}{16}$ ‖ 8 | $\frac{363}{800}$ ‖ 1 | 45.8958$\frac{1}{3}$
128. ‖ 2 | 4681.2991071$\frac{3}{7}$ ‖ 3 | 227.313001 ‖ 4 | 125.1011
129. ‖ 5 | 112.29 ‖ 6 | 45.401 ‖ 7 | .75 ‖ 8 | 16.63 ‖ 9 | 559.875
129. ‖ 10 | .6597$\frac{2}{9}$ ‖ 11 | 1922.6666$\frac{2}{3}$ ‖ 12 | 46.875 ‖ 13 |
129. ‖ 445334.90625 ‖ 14 | .10486+ ‖ 15 | .08333$\frac{1}{3}$ ‖ 16 |
129. ‖ .0470204+ ‖ 17 | 188 ‖ 18 | 1000 ‖ 19 | 2.4305$\frac{5}{9}$
129. ‖ 20 | 2.06730$\frac{30}{39}$ ‖ 131. ‖ 1 | \$39.397 ‖ 2 | 12.003
131. ‖ 3 | 147.04 ‖ 4 | 148.004 ‖ 5 | 4.006 ‖ 6 | 9.069
131. ‖ 7 | 10.132 ‖ 8 | \$25 ‖ 9 | \$.1875 ‖ 10 | \$.047
132. ‖ 1 | 560—964.7—10.6 ‖ 2 | 1600—4562.5—108.75
132. ‖ 3 | 4156—69000—750 ‖ 1 | \$45.689 ‖ 2 | \$67.94—
132. ‖ 679.4 ‖ 3 | 3765.94 ‖ 4 | 4754.6 ‖ 5 | E. 6.7506
132. ‖ 6 | \$37.496 ‖ 7 | \$47.049 ‖ 133. ‖ 1 | \$6.73 ‖ 2 | \$5.125
133. ‖ 3 | \$16.696 ‖ 4 | \$83.575 ‖ 5 | \$252.321 ‖ 6 | 330.9[illegible]5
133. ‖ 1 | \$73.436 ‖ 2 | 219.614 ‖ 134. ‖ 3 | 132.475 ‖ 4 | 99.1
134. ‖ 5 | 52.371 ‖ 6 | 1843.94 ‖ 7 | 656.369 ‖ 8 | 22.334
134. ‖ 9 | 7.952 ‖ 10 | 14.405 ‖ 11 | 3632.6875 ‖ 135. ‖ 1 | 105
135. ‖ .965 ‖ 2 | 8366.302 ‖ 3 | 600.979 ‖ 4 | 324.911 ‖ 5
135. ‖ 2319.67 ‖ 6 | 5.999, 9.742, .744, 87.345 ‖ 7 | 106.524
135. ‖ 8 | 170.056 ‖ 9 | 44.377 ‖ 10 | 2512.50 ‖ 11 | 51.997
136. ‖ 12 | 2.50 ‖ 13 | 945.361 ‖ 14 | 2906.961 ‖ 15 | 24.625

136. ‖ 16 | 12.43 ‖ 17 | 59.827 ‖ 18 | 2223.1465 ‖ 19 | 65-
136. ‖ .1295 ‖ 20 | Diff. 8279.155; gain, 329.087 ‖ 21 | 932.802
137. ‖ 1 | 51.00 ‖ 2 | 20.625 ‖ 3 | 375 ‖ 4 | 117.21875
137. ‖ 5 | 81.8125 ‖ 6 | 82.25 ‖ 7 | 11.25 ‖ 8 | 210.295
137. ‖ 9 | 28.00 ‖ 10 | 104.00 ‖ 11 | $128.8392\frac{6}{7}$ ‖ 12 | $101.56\frac{1}{4}$
137. ‖ 13 | 17591.25 ‖ 14 | $33.97031\frac{1}{4}$ ‖ 138. ‖ 1 | 3.51
138. ‖ 2 | 3.84266+ ‖ 3 | .06 ‖ 4 | $.666\frac{2}{3}$ ‖ 5 | 16.80392+
138. ‖ 6 | 796.178+ ‖ 7 | 2.12 ‖ 8 | .375 ‖ 9 | 1.125 ‖ 10 | .14
139. ‖ 11 | 3.50 ‖ 12 | 23.07692+ ‖ 13 | .90 ‖ 14 | 8.22826+
139. ‖ 15 | 15 ‖ 16 | 25.5 ‖ 17 | 9 ‖ 18 | 20 ‖ 19 | 32.5
139. ‖ 20 | 16 ‖ 21 | .60 ‖ 22 | 3 ‖ 23 | 112 ‖ 24 | 12
140. ‖ 1 | 12.50 ‖ 2 | 62.50 ‖ 3 | 3.4375 ‖ 4 | 81.25 ‖ 5 | 22.5
140. ‖ 6 | 415 ‖ 7 | $45.33333\frac{1}{3}$ ‖ 141. ‖ 1 | 412.875
141. ‖ 2 | 3718.50 ‖ 3 | 9984.50 ‖ 4 | 27631.50 ‖ 5 | 7657.56
141. ‖ 6 | 2977.26 ‖ 1 | 21.40 ‖ 2 | $60.14\frac{1}{4}$ ‖ 3 | 6.11712
142. ‖ 4 | 8.40 ‖ 5 | 164.8115 ‖ 6 | $7654.90\frac{1}{4}$ ‖ 1 | 5.96124
142. ‖ 2 | 23.597 ‖ 3 | 98.720875 ‖ 4 | 7.34125 ‖ 5 | $139.907\frac{7}{16}$
143. ‖ 1 | 75.385 ‖ 2 | 25.25 ‖ 3 | $66.66666\frac{2}{3}$ ‖ 4 | 2.50
143. ‖ 1 | 91.87096+ ‖ 2 | 96 ‖ 3 | 16 ‖ 144. ‖ 1 | 625
144. ‖ 2 | 5.25, 7.00, 13.78125 ‖ 3 | 16.000816+; 64.65194+
144. ‖ 4 | 80.50 ‖ 5 | .06 ‖ 6 | 10 wood; $18.857142\frac{}{}$ coal
144. ‖ 7 | .1875 ‖ 8 | 414.75 ‖ 9 | 3480 yd.; 4.50 per yard.
144. ‖ 10 | $2.33333\frac{1}{3}$ ‖ 11 | 11000, w.'s; 5500, ch.'s ‖ 12 | 23.16
145. ‖ 13 | 155 ‖ 14 | 547.92 ‖ 15 | 916 ‖ 16 | 1, 100, 100
145. ‖ 17 | $122.76616\frac{2}{3}$ ‖ 18 | 177 ‖ 19 | $15.68\frac{28}{29}$ ‖ 20 | .92
146. ‖ 21 | 19.8 ‖ 22 | 104.126 ‖ 23 | 27.685 ‖ 24 | 290.82
146. ‖ 25 | 90277.70 ‖ 151. ‖ 2 | 30183 ‖ 3 | 84226 ‖ 4 | 391679
151. ‖ 5 | 84 ‖ 6 | £1 12s. 3d. 1far. ‖ 7 | £25 14s. 1d.
153. ‖ 3 | 316767 ‖ 4 | 359 mi. 7 fur. 28 rd. ‖ 5 | 3796602
153. ‖ 6 | 8201 ‖ 7 | 240700858 ‖ 8 | 109° $21\frac{1}{2}$ mi. 7 fur. 1 rd
153. ‖ $3\frac{1}{2}$ yd. 2 ft. 8 in., or 109° 22 mi. 3 fur. 1 rd. 4 yd. 1 ft. 2 in
153. ‖ 9 | 4744 ‖ 10 | 5 ft. 2 in. ‖ 1 | 5 ch. 60 l. ‖ 2 | 2682 ft
153. ‖ 10.8 in. ‖ 3 | 172 ch. 58 l. ‖ 154. ‖ 1 | 575 ‖ 2 | 35 yd
154. ‖ 3 qr. 3 na. ‖ 3 | 980 ‖ 4 | 623 ‖ 5 | 204 yd 3 qr. 2 na

154. ‖ 6 | 28 E. Fl. 1 qr. ‖ 7 | 95 E. E. 4 qr. ‖ **157.** ‖ 3 | 3157
157. ‖ 4 | 762300 ‖ 5 | $260\frac{1}{9}$ ‖ 6 | 93 A. 2 R. 12 P.
157. ‖ 7 | 35 mi. 563 A. 1 R. 19 P. ‖ 8 | 12584.25 ‖ 9 | 15.25
159. ‖ 3 | 592704 ‖ 4 | 200 C. ft., 3200 cu. ft. ‖ 5 | 5 C. 2 C ft.
159. ‖ 6 | 21870 C. 4 C ft. ‖ 7 | 204 T. 11 cu. ft. 1292 cu. in.
160. ‖ 3 | 12602 ‖ 4 | 10 T. 1 pi. ‖ 5 | 25 T. 1 gal. ‖ 6 | 56.64
161. ‖ 3 | 23808 ‖ 4 | 844 ‖ 5 | 272 ‖ 6 | 1 ch. 29 bu. 3 pk. 6 qt
163. ‖ 3 | 2790366 ‖ 4 | 903136 ‖ 5 | 5 T. 8 cwt. 3 qr. 24 lb.
163. ‖ 13 oz. 14 dr. ‖ 6 | 28 T. 4 cwt. 1 qr. 21 lb. ‖ 7 | 6 T.
163. ‖ 2 cwt. 4 lb. 13 oz. 14 dr. ‖ 8 | 2998128 ‖ 9 | 212 T
163. ‖ 14 cwt. 1 qr. 7 lb. ‖ 10 | 118.995–$10 ‖ 11 | 431.68–160
164. ‖ 3 | 148340 ‖ 4 | 1 lb. 1 oz. 10 pwt. 10 gr. ‖ 5 | 25 lb.
164. ‖ 9 oz. 0 pwt. 20 gr. ‖ 6 | 678618 ‖ 7 | 36 lb. 7 oz. 14 pwt.
164. ‖ 8 | 38901 ‖ 9 | 6496 ‖ 10 | 657 ‖ **165.** ‖ 3 | 8011
165. ‖ 4 | 9113 ‖ 5 | 27 ℔ 9 ℥ 6 ʒ 1 ℈ ‖ 6 | 94 ℔ 11 ℥ 1 ʒ
165. ‖ 7 | 73918 ‖ 8 | 12 ℔ 9 ℥ 7 ʒ 2 ℈ 18 gr. ‖ **167.** ‖ 3 | 379-
168. ‖ 467108 ‖ 4 | 4 yr. 1 da. ‖ 5 | 24 yr. 1 da. 26 m. 58 sec.
168. ‖ 6 | 9 yr. 14 da. 17 hr. 16 m. 45 sec. ‖ 7 | 6600 ‖ 1 | 10765
168. ‖ 2 | 2592000 ‖ 3 | 7° 44′ 54″ ‖ 4 | 1 c. 5 s. 28° 15′
168. ‖ 5 | 3946800 ‖ 6 | 921625 ‖ 7 | 2° 23′ 9″ ‖ **170.** ‖ 1 | 62208
170. ‖ 2 | $24 ‖ 3 | 672 cu. ft.—$5\frac{1}{4}$ cords. ‖ 4 | 28.125 ‖ 5 |
170. ‖ $13165\frac{5}{7}$, $13165\frac{165}{231}$ ‖ 6 | 172.96875 ‖ 7 | 36288 ‖ 8 | 336
170. ‖ 9 | 223 ‖ 10 | 23856 ‖ 11 | .88 loss. ‖ 12 | .6448 A-
170. ‖ 48.4809 ‖ 13 | 14.34802+ ‖ 14 | 58097 ‖ 15 | 7 s. 15°
170. ‖ 24′ 40″ ‖ 16 | 12 ‖ 17 | 1244160 ‖ 18 | 3456 ‖ 19 |
170. ‖ $377\frac{1}{2}$ ‖ 20 | 48976 ‖ 21 | 478602432 ‖ 22 | 84 mi. 3 fur.
170. ‖ 4 rd. 3 yd. 2 ft. ‖ 23 | 5 A. 3 R. 35 P. $3\frac{1}{4}$ yd. 2 ft. 5 in.
171. ‖ 24 | 26880 ‖ 25 | 116280 ‖ 26 | 27 ‖ 27 | 4 ‖ 28 | 40
171. ‖ 29 | 23 wk. 5 da. 16 hr. ‖ 30 | 576 ‖ 31 | 110592 ‖ 32 |
171. ‖ 38 ‖ 33 | 36 ‖ 34 | $\frac{3}{4}$ ft. or 9 in. ‖ 35 | $755\frac{15}{27}$ ‖ 36 | 1125
172. ‖ 2 | $\frac{140}{1} = 140$ ‖ 3 | $\frac{3}{5}$ ‖ 4 | $\frac{15}{19}$ ‖ 5 | $\frac{7}{6} = 1\frac{1}{6}$ ‖ 6 | $\frac{4}{5}$
172. ‖ 7 | $\frac{2}{3}$ ‖ 8 | $\frac{7}{8}$ ‖ 9 | 200.00 ‖ 10 | 19800.0000 ‖ 11 |
172. ‖ 270.0000 ‖ 12 | 1071.468 ‖ 13 | 992.4480 ‖ 14 | 559.92
173. ‖ 2 | $\frac{4}{315}$ ‖ 3 | $\frac{5}{114}$ ‖ 4 | $\frac{1}{19800}$ ‖ 5 | $\frac{7}{576}$ ‖ 6 | .0390625

173. ‖ 7 | .000560781$\frac{1}{4}$ ‖ 8 | .00409375 ‖ 9 | .00007957+

173. ‖ 10 | 1.74484375 ‖ 11 | .0496814+ ‖ 12 | .000146484$\frac{3}{8}$

173. ‖ 13 | .02734375 ‖ 14 | .00035 ‖ 15 | .0097222+

173. ‖ 16 | 1.3125 ‖ 17 | 71.151136+ ‖ 18 | .003968+

174. ‖ 1 | 1 pi. 1 hhd. 31 gal. 2 qt. ‖ 2 | 3 qr. 2 na. 0.9 in

174. ‖ 3 | 3 wk. 1 da. 9 hr 36 m. ‖ 4 | 13 bu. 2 pk. ‖ 5 | 6 fur

174. ‖ 8 rd. 4 yd. 2 ft. 8 in. ‖ 6 | 3 cwt. 0 qr. 12 lb. 8 oz

174. ‖ 7 | 2 da. 13 hr. 42 m. 51$\frac{3}{7}$ sec. ‖ 8 | 2 bu. [illegible] pk. ‖ 9 | 50

174. ‖ gal. 1 qt. 1 pt. 0$\frac{4}{5}$ gi. ‖ 10 | 1 qr. 21 lb. 10 oz. 10$\frac{2}{3}$ dr.

174. ‖ 11 | 2 gal. 3$\frac{1}{5}$ gi. ‖ 12 | 2 R. 6 P. 4 yd. 5 ft. 127$\frac{5}{13}$ in.

174. ‖ 13 | 15 cwt. 3 qr. 3 lb. 15 oz. 2$\frac{10}{19}$ dr. ‖ 14 | 342 da. 4 hr. 30 m.

175. ‖ 1 | 12.00384 gr. ‖ 2 | 2 qr. 12 lb. 8 oz. ‖ 3 | 2 qt. 1 pt.

175. ‖ 4 | 6s. 9d. ‖ 5 | 6 cwt. 3 qr. ‖ 6 | 8 P. ‖ 7 | 1 hhd

175. ‖ 47 gal. 1 qt. ‖ 8 | 7 gal. 3 qt. 1 pt. ‖ 9 | 136 da. 21 hr.

176. ‖ 10 | 1s. 8d. 1$\frac{3}{5}$ far. ‖ 11 | 3 qr. 11 lb. ‖ 12 | 1 qr. 7 lb. 4 oz.

176. ‖ 13 | 19 hr. 40 m. 48 sec. ‖ 14 | 1 mi. 28 rd. 2 yd. 1 ft. 11.04 in.

176. ‖ 15 | 1 oz. 8 dr. ‖ 16 | 103 da. 23 hr. 59 m. 12.48 sec.

176. ‖ 17 | £1 0s. 11d. 0.16far. ‖ 18 | £1 17s. 7d. 0.8far.

176. ‖ 1 | $\frac{7}{8}$ ‖ 2 | $\frac{1}{18}$ ‖ 3 | $\frac{4}{11}$ ‖ 4 | $\frac{7}{25}$ ‖ 5 | $\frac{8}{9}$ ‖ 6 | $\frac{2503}{4500}$

177. ‖ 7 | $\frac{849}{1210}$ ‖ 8 | $\frac{115}{189}$ ‖ 9 | $\frac{9}{1000}$ ‖ 10 | $\frac{7}{9}$ ‖ 11 | $\frac{125}{1152}$

177. ‖ 12 | $\frac{17}{18}$ ‖ 13 | $\frac{71}{21120}$ ‖ 1 | 4.8899553+ ‖ 2 | 2.46944+

178. ‖ 3 | 1.25 ‖ 4 | 1.046875 ‖ 5 | 5.08333$\frac{1}{3}$ ‖ 6 | 4.765625

178. ‖ 7 | .472291$\frac{2}{3}$ ‖ 8 | .78875 ‖ 9 | 5.88125 ‖ 10 | .0055

178. ‖ 11 | .42859226+ ‖ 12 | .39201388$\frac{8}{9}$ ‖ 13 | 7.878125

178. ‖ 14 | .778515625 ‖ 15 | .15378472+ ‖ 179. ‖ 2 | 931

179. ‖ lb. 6 oz. 9 pwt. 5$\frac{13}{21}$ gr. ‖ 3 | 104 ℔ 3 ℥ 3 ʒ 2 ℈ 4 gr

179. ‖ 4 | 254 T. 19 cwt. 2 qr. 5 lb. 9$\frac{1}{10}$ oz. ‖ 5 | 50 T. 0 pi

179. ‖ 1 hhd. 38 gal. 3 qt. ‖ 6 | 138 ch. 30 bu. 3 pk. 5 qt

180. ‖ 7 | 172 yr. 2 mo. 1 wk. 4 da. 5$\frac{11}{20}$ hr. ‖ 8 | 29 s. 28° 32

180. ‖ 49″ ‖ 1 | 291 lb. 6 oz. 15 pwt. 22 gr. ‖ 2 | 432 L. 2 mi.

180. ‖ 4 fur. 39 rd. 4 yd. 2$\frac{1}{36}$ ft. ‖ 3 | 424 E. Fl. 0 qr. 3 na

180. ‖ 4 | 176 cu. yd. 18 cu. ft. 614 cu. in. ‖ 5 | 27 sq. mi. 277 A

180. ‖ 1 R. 0 P. 24$\frac{1}{2}$ yd. ‖ 6 | 244 lb. 5 oz. 4 pwt. 3 gr. ‖ 7 | 82 T.

180. ‖ 16 cwt. 16 lb. 1 oz. 7 dr. ‖ 8 | 41 T. 3 qr. 17 lb. 5 dr

181. ‖ 9 | 336 A. 1 R. 31 P. 210 sq. ft. 136 sq. in. ‖ 10 | 170 T
181. ‖ 11 cu. ft. 744 cu. in. ‖ 11 | 168 bu. 0 pk. 2 qt. ‖ 12 | 45 A
181. ‖ 3 R. 31 P. 38 sq. ft. 130 sq. in. ‖ 13 | 158 bu. 0 pk. 4 qt.
181. ‖ 14 | 2 T. 5 cwt. 2 qr. 21 lb. ‖ 15 | 85 yd. ‖ 16 |
181. ‖ 3 lb. 1 oz. 11 pwt. 17 gr. ‖ 17 | 322 mi. 6 fur. 11 rd. ‖ 18 |
182. ‖ 100 A. 1 R. 13 P. ‖ 1 | 12 cwt. 1 qr. 7 lb. 13 oz. $11\frac{3}{7}$ dr
182. ‖ 2 | 7 fur. 2 ft. 9 in. ‖ 3 | 1 mi. 3 fur. 18 rd. ‖ 4 | 1 cwt.
182. ‖ 2 qr. 2 lb. 13 oz. ‖ 5 | 5 da. 20 hr. 52 m. $15\frac{5}{19}$ sec.
182. ‖ 6 | 16s. 3d. 3.9far. ‖ 7 | 6 cwt. 3 qr. 21 lb. 5 oz. 8 dr.
183. ‖ 8 | 56 yd. ‖ 9 | 75 bu. 0 pk. $7\frac{1}{9}$ qt. ‖ 10 | 90 mi. 4 fur.
183. ‖ 15 rd. 1 yd. 0 ft. $11\frac{1}{7}$ in. ‖ 11 | 2 yd. 2 qr. $1\frac{1}{3}$ na. ‖ 12
184. ‖ 1 cwt. 1 qr. 7 lb. 7 oz. $\frac{64}{105}$ dr. ‖ 1 | 3 A. 0 R. 38 P.
184. ‖ 2 | 1 T. 14 cwt. 0 qr. 19 lb. ‖ 3 | 175 lb. 1 oz. 1 pwt. 3 gr.
184. ‖ 4 | 8 lb. 10 oz. 14 pwt. 4 gr. ‖ 5 | 5 T. 7 cwt. 1 qr. 23 lb.
184. ‖ 11 oz. ‖ 6 | 7 cwt. 2 qr. 20 lb. 11 oz. 5 dr. ‖ 7 | 124 T
184. ‖ 0 hhd. 59 gal. ‖ 8 | 14 yr. 46 wk. 4 da. 20 hr. 58 m. 54 sec.
184. ‖ 9 | 14 mi. 7 fur. 27 rd. 2 yd. 2 ft. 9 in. ‖ 10 | 46 A. 3 R.
185. ‖ 35 P. 23 yd. $8\frac{29}{36}$ ft. ‖ 11 | £5 17s. $6\frac{3}{4}$d. ‖ 12 | 1 s. 24°
185. ‖ 19′ $32\frac{7}{12}$″ ‖ 13 | 28 mo. 1 wk. 0 da. 20 hr. 20 min.
185. ‖ 14 | 84 yr. 11 mo. 2 wk. 5 da. ‖ 15 | £2 17s.
185. ‖ 16 | 1 lb. 11 oz. 19 pwt. 4 gr. ‖ 17 | 6 ℔ 10 ℥ 5 ʒ 1 ℈
185. ‖ 18 | 7 T. 18 cwt. 1 qr. 4 lb. 0 oz. 2 dr. ‖ 19 | 2 mi. 4 fur. 21 rd.
185. ‖ 20 | 68 lb. 10 oz. 3 pwt. 15 gr. ‖ 21 | 1 T. 17 cwt. 3 qr.
185. ‖ 7 lb. 14 oz. 2 dr. ‖ 22 | 84 ℔ 9 ℥ 4 ʒ 1 ℈ 14 gr.
185. ‖ 23 | 3 yd. 2 qr. 1 na. $\frac{1}{4}$ in. ‖ 24 | 4 C. 3 c. ft. 2 cu. ft.
186. ‖ 2 | 9 yr. 4 mo. 2 da. ‖ 3 | 21 yr. 9 mo. 5 da. ‖ 4 | 17 yr.
186. ‖ 1 mo. 3 da. ‖ 6 | 12 yr. 3 mo. 26 da. 22 hr. ‖ 7 | 30 yr.
186. ‖ 1 mo. 29 da. 12 hr. ‖ 8 | 7 yr. 9 mo. 1 da. ‖ 9 | 369 yr.
187. ‖ 9 mo. 14 da. ‖ 1 | 6 pwt. 15 gr. ‖ 2 | £1 9s. 3d.
187. ‖ 3 | 1 oz. 3 pwt. 3 gr. ‖ 4 | 11 hr. 59 m. $59\frac{1}{3}$ sec.
187. ‖ 5 | 3 yd. 1 ft. $11\frac{4}{7}$ in. ‖ 6 | 16 gal. 2 qt. 0 pt. $2\frac{26}{35}$ gi. ‖ 7 |
187. ‖ 11 pwt. 3 gr. ‖ 8 | 4 cwt. 1 qr. 12 lb. 15 oz. $5\frac{1}{35}$ dr.
187. ‖ 9 | 8 cwt. 3 qr. 5 lb. 13 oz. $0\frac{32}{70}$ dr. ‖ 10 | 3 lb. 5 oz. 16 pwt.
187. ‖ 16 gr. ‖ 11 | 1 rd. 1 yd. 2 ft. $5\frac{4}{7}$ in. ‖ 12 | 7 ℥ 5 ʒ 2 ℈ 10 gr.

187. ‖ 13 | 2 hhd. 25 gal. 3 qt. 0 pt. 0.292 gi. ‖ 14 | 2 wk.
187. ‖ 19 hr. 15 min. 58.464 sec. ‖ 15 | 4.8241065+
189. ‖ 3 | 56 mi. 5 fur. 4 rd. ‖ 4 | 27 s. 28° 22′ 45″
189. ‖ 5 | 32 yr. 3 mo. 18 da. 18 hr. ‖ 6 | 53 T. 3 cwt. 2 qr.
189. ‖ 16 lb. 4 oz. 8 dr. ‖ 8 | 25 bu. 3 pk. 1 qt. ‖ 9 | 8 C. 6 c. ft.
189. ‖ 10 | 17 yr. 5 mo. 3 da. ‖ 11 | 3 lb. 3 oz. 12 pwt. ‖ 12
189. ‖ 1 T. 19 cwt. 2 qr. 12 lb. ‖ 13 | 13 ℔ 7 ℥ 2 ʒ 1 ℈ 4 gr
190. ‖ 14 | 122 mi. 4 fur. $23\frac{3}{4}$ rd. ‖ 15 | 111 A. 2 R. 25 P
190. ‖ 16 | 267 yd. 0 qr. 3 na. ‖ 17 | 47 L. 1 mi. 7 fur. 8 rd
190. ‖ 18 | 95 hhd. 6 gal. ‖ 19 | 32 lb. 9 oz. 15 pwt. ‖ 20 |
190. ‖ 746 mi. 5 fur. ‖ 21 | 15° ‖ 22 | 56 T. 14 cwt. 3 qr. 15 lb.
190. ‖ 23 | £5 4s. 3d. ‖ 24 | 24 hhd. 22 gal. 1 qt. 1 pt. ‖ 25 |
191. ‖ 927 yd. ‖ 2 | 12 A. 2 R. 25 P. ‖ 3 | 5 L. 2 mi. 6 fur. 36 rd.
192. ‖ 4 | 2 bu. 3 pk. 4 qt. ‖ 5 | 2 cwt. 1 qr. 18 lb. $3\frac{1}{7}$ oz.
192. ‖ 6 | 5 yd. 2 qr. $0\frac{7}{9}$ na. ‖ 7 | 25 lb. 3 oz. 8 dr. ‖ 8 | 2° 34′
192. ‖ 16″ ‖ 9 | 49 gal. 2 qt. 1 pt. ‖ 10 | 5 bu. 1 pk. $6\frac{1}{4}$ qt.
192. ‖ 11 | 3 ℔ 4 ℥ 6 ʒ 1 ℈ 16 gr. ‖ 12 | 12 A. 2 R. 25 P.
192. ‖ 13 | 2 T. 7 cwt. ‖ 14 | 61 gal. 1 qt. 1 pt. ‖ 15 | £1 2s. $4\frac{1}{2}$d.
192. ‖ 16 | £21 9s. 8d. ‖ 17 | 1 pk. 2 qt. 0 pt. $2.952\frac{8}{21}$ gi.
192. ‖ 18 | 17 cwt. 3 qr. 18 lb. 12 oz. $2\frac{34}{79}$ dr. ‖ 19 | 2 hhd.
192. ‖ 54 gal. 3 qt. 1.65 pt. ‖ 20 | 24 mi. 7 fur. 4 rd
192. ‖ 21 | 14 lb. 0 oz. 8 pwt. 11 gr. ‖ 22 | 3 gal. 1.5 qt
193. ‖ 1 | 106 A. 3 R. $22\frac{4}{7}$ P. ‖ 2 | 4 bu. 3 pk. 2 qt.
193. ‖ 3 | 95 yr. 7 mo. 3 wk. 2 da. 8 hr. ‖ 4 | 2 cwt. 1 qr. 24 lb.
193. ‖ 5 | 1 lb. 1.73214+ oz. ‖ 6 | 15′ ‖ 7 | 2 bu. 7 qt.
193. ‖ 8 | 9 gal. 2 qt. 0.62711+ pt. ‖ 9 | 7110.00 ‖ 10 |
193. ‖ 12 cwt. 2 qr. 11 lb. ‖ 11 | 124 mi. 3 fur. 20 rd.
193. ‖ 12 | $4\frac{13}{119}$ ‖ 13 | 326 ‖ 14 | 24 reams 5 quires 12 sheets
194. ‖ 15 | $10\frac{17}{24}$ ‖ 16 | £63 14s. 8d. ‖ 17 | $8\frac{169}{3905}$
194. ‖ 18 | 1 hhd. 19 gal. 0 qt. 1 pt. ‖ 19 | $222\frac{192}{297} = 222\frac{64}{99}$
194. ‖ 20 | 61 mi. 5 fur. 29 rd. ‖ 21 | 20 T. 3 cwt. 1 qr. 13.125 lb
194. ‖ 22 | $3795, 24 T. 0 cwt. 2 qr. 20 lb. ‖ 23 | $8\frac{2}{35}$ ‖ 24 | $88\frac{54}{103}$
196. ‖ 2 | 11 hr. 55 m. 24 sec. A. M. ‖ 3 | 11 hr. 18 m. 28 sec. A. M
196. ‖ 4 | 10 hr. 59 m. 56 sec. A. M. ‖ 5 | 2 hr. 20 m. $3\frac{1}{3}$ sec. P. M

197. ‖ 6 | 6 hr. 0 m. 8 sec. A. M. ‖ 7 | 12 hr. 53 m. 56 sec. P. M
197. ‖ 2 | 78° 52′ W. ‖ 3 | 76° 37′ W. ‖ 4 | 74° 1′ 2″ W.
197. ‖ 5 | 33° 52′ 30″ W. ‖ **199.** ‖ 1 | 15 ft. 5′ ‖ 2 | 1 ft. 8′ 10′
199. ‖ 3 | 2 ft. 6′ 3″ 11‴ ‖ 4 | 5 ft. 8′ 2″ 1‴ ‖ 5 | 15 ft. 4′
199. ‖ 10″ 4‴ ‖ 6 | 1 ft. 11′ 10″ 11‴ ‖ 7 | 11 ft. 6′ 5″ 5″
199. ‖ 8 | 7 ft. 10′ 1″ 9‴ ‖ 9 | $\frac{5}{9}$ ‖ 10 | $\frac{89}{108}$ ‖ 11 | .36979+
199. ‖ 12 | .052083$\frac{1}{3}$ ‖ **201.** ‖ 1 | 77 sq. ft. ‖ 2 | 2 C. 5 c. ft
201. ‖ 3 | 87 ft. 1′ ‖ 4 | 27$\frac{51}{72}$ sq. ft. ‖ 5 | 165 sq ft. 6′ 8″
201. ‖ 6 | 105$\frac{15}{32}$ cu. ft. ‖ 7 | 866$\frac{11}{16}$ sq. ft. ‖ 8 | 39 C. 33 cu. ft.
201. ‖ 9 | 46$\frac{11}{324}$ sq. yd. ‖ 10 | 1 C. 4 c. ft. 3 cu. ft. ‖ 11 |
201. ‖ 158 cu. yd. 17$\frac{1}{3}$ cu. ft. ‖ 12 | 20.00$\frac{3}{4}$ ‖ 13 | 84 ft. 4′ 6″
201. ‖ 14 | 373.34483+ ‖ 15 | 905$\frac{85}{108}$ ‖ **203.** ‖ 1 | 2 ‖ 2 | 4
203. ‖ 3 | 2 ‖ 4 | $\frac{9\times9}{3\times9}=3$ ‖ 5 | $\frac{12\times12}{3\times4}=12$ ‖ 6 | $\frac{6\times10\times3}{5\times3\times2}=6$
203. ‖ 7 | $\frac{2}{9}$ ‖ 8 | $\frac{1}{4}$ ‖ 9 | $\frac{1}{5}$ ‖ 10 | $\frac{2}{3}$ ‖ 11 | $\frac{1}{4}$ ‖ 12 | $\frac{1}{4}$
203. ‖ 13 | $\frac{1}{4}$ ‖ 14 | $\frac{1}{7}$ ‖ 15 | $\frac{7}{8}$ ‖ 16 | $\frac{1}{11}$ ‖ **204.** ‖ 17 | $\frac{3}{10}$
204. ‖ 18 | $\frac{11}{42}$ ‖ 19 | $\frac{65}{138}$ ‖ 20 | $\frac{5}{49}$ ‖ 21 | $\frac{110}{279}$ ‖ 22 | $\frac{82}{115}$
204. ‖ 23 | $\frac{3191}{4350}$ ‖ 24 | 5$\frac{6869}{9372}$ ‖ 26 | 72 ‖ 27 | 12 ‖ 28 | 25
204. ‖ 29 | $\frac{1}{5}$ ‖ 30 | $\frac{28}{45}$ ‖ 31 | 2 ‖ 32 | $\frac{2}{3}$ ‖ 33 | $\frac{20}{81}$
206. ‖ 2 | 27 ‖ 3 | 40 ‖ 4 | 5 ‖ 5 | 20 ‖ 6 | $\frac{15}{112}$ ‖ 7 | 10
206. ‖ 8 | 3$\frac{1}{2}$ ‖ 9 | 15 ‖ **207.** ‖ 1 | 20$\frac{4}{25}$ ‖ 2 | 108 ‖ 3 | 56
207. ‖ 4 | 2 ‖ 5 | 56 ‖ **209.** ‖ 1 | 308 ‖ 2 | 165 ‖ 3 | 72
209. ‖ 4 | 1381.25 ‖ 5 | 3300 ‖ 6 | 160 ‖ 7 | 61.425
209. ‖ 8 | 10955 ‖ **210.** ‖ 9 | 20 ‖ 10 | 243.75 ‖ 11 | 201.75
210. ‖ 12 | 106$\frac{2}{3}$ ‖ 13 | 36000 ‖ 14 | 8 ‖ 15 | 27 ‖ 17 | 1.875
210. ‖ 18 | 8 ‖ 19 | 115.50 ‖ **211.** ‖ 20 | 29.25 ‖ 21 | 4.861$\frac{1}{9}$
211. ‖ 22 | £227 12s. 1d. ‖ 23 | 81$\frac{1}{3}$ ‖ 24 | 18 ‖ 25 | 40.32
211. ‖ 26 | A, 1787.50; B, 1283.75 ‖ 27 | 16 ‖ 28 | 22.50
211. ‖ 29 | .15421+ ‖ 30 | 6206.93$\frac{1}{3}$ ‖ 31 | 8 hr. 20 min
211. ‖ 32 | 6 ‖ 33 | 134 ‖ 34 | 20$\frac{14}{65}$ ‖ **212.** ‖ 35 | 51.1875
212. ‖ 36 | 5s. 9d. ‖ 37 | 378 ‖ 38 | 252 ‖ 39 | 24 ‖ 40 | 16
212. ‖ 41 | 72 ‖ 42 | 37 ‖ 43 | 64$\frac{4}{5}$ ‖ 44 | 12.13 ‖ 45 | 25
212. ‖ 46 | 28 ‖ 47 | 2 A. 2 R. 26$\frac{2}{3}$ P. ‖ 48 | 18.27

212. ‖ 49 | 168.7421+ ‖ **214.** ‖ 1 | 45 ‖ 2 | 150 ‖ 3 | 99
214. ‖ 4 | 232 ‖ 5 | $511\frac{1}{2}$ ‖ 6 | 18 ‖ 7 | 27 ‖ **215.** ‖ 8 | 72
215. ‖ 9 | $4\frac{1}{31}$ ‖ 10 | 1112 ‖ 11 | $2\frac{6}{7}$ ‖ 12 | $343\frac{1}{8}$ ‖ 13 | 15 ‖ 14 |
215. ‖ $38\frac{2}{5}$ ‖ 15 | 147.2144+ ‖ 16 | 192 ‖ 17 | 200 ‖ 18 | 36
217. ‖ 2 | $857.142\frac{6}{7}$; $142.857\frac{1}{7}$ ‖ 3 | 480 ; 750 ; 675
217. ‖ 4 | 800 ; 1200 ; 400 ‖ 5 | 66 ; 99 ; 110 ‖ 6 | 3160-
217. ‖ $.714\frac{2}{7}$; $1564.285\frac{5}{7}$ ‖ 7 | $1066.66\frac{2}{3}$; $1066.66\frac{2}{3}$; $1866.66\frac{2}{3}$
217. ‖ 8 | 675 ; 450 ; 225 ‖ **218.** ‖ 2 | 77 ; 260 ‖ 3 | 54 ; 38.50
218. ‖ 4 | 60.777+ ; 127.633+ ; 233.387+ ; 328.201+
218. ‖ 5 | $1666.666\frac{2}{3}$; $3888.888\frac{8}{9}$; $9444.444\frac{4}{9}$ ‖ 6 | 273.365+ ;
218. ‖ 476.635+ ‖ 7 | 1808.8669+ ; 1596.0591+ ; 1995.0738+
220. ‖ 2 | 16.25 ‖ 3 | 19.5 ‖ 4 | 39.375 ‖ 5 | 2.375
220. ‖ 6 | 155.48 ‖ 7 | 5 ‖ 8 | 8.93 ‖ 9 | 18.5487 ‖ 10 | 280
220. ‖ 11 | 893.13 ‖ 12 | 1015.6525 ‖ 13 | 2109.05125
220. ‖ 14 | 7.5 ‖ 15 | 2.2908 ‖ 16 | 3.50 ‖ 17 | .197225
220. ‖ 18 | 42.60 ‖ **221.** ‖ 19 | 432 ‖ 20 | 42 ‖ 21 | 24.25
221. ‖ 22 | 10.80 ‖ 23 | $26\frac{2}{3}$ per ct.; $3333.33\frac{1}{3}$ ‖ 24 | 1304.75
221. ‖ 1 | 25 ‖ 2 | 50 ‖ 3 | 40 ‖ **222.** ‖ 4 | 5 ‖ 5 | $1\frac{1}{29}$ ‖ 6 | 3
222. ‖ 7 | $11\frac{139}{301}$ ‖ 8 | $111\frac{1}{9}$ ‖ 9 | $2.32\frac{16}{37}$ ‖ 10 | 700
222. ‖ 11 | $23\frac{1}{13}$ ‖ 12 | $77\frac{1}{7}$ ‖ 13 | $15\frac{1}{22}$ ‖ 14 | 55 ‖ 15 | 20
222. ‖ 16 | $28\frac{384}{947}$ ‖ 17 | $12\frac{1}{2}$ ‖ **223.** ‖ 1 | 3840 ‖ 2 | 118.4
223. ‖ 3 | $401.785\frac{5}{7}$ ‖ 4 | 9900 ‖ 5 | $4.666\frac{2}{3}$ ‖ 6 | 95.23809+
223. ‖ 7 | 88 ‖ 8 | 27900 ‖ 9 | 20 ‖ 10 | 2000 ; 1630
224. ‖ 1 | 1378 ‖ 2 | 541.025 ‖ 3 | 52.7025 ; 1704.0475
224. ‖ 4 | 24862.50 ‖ 5 | 8443.75 ‖ 6 | 233.75
224. ‖ 7 | 207.4125 ‖ 8 | 196.1875 ‖ **225.** ‖ 9 | 17030 ‖ 10 |
225. ‖ $47\frac{1}{2}$ ‖ 11 | $2\frac{5}{6}$ ‖ **226.** ‖ 1 | $697.6744\frac{8}{43}$ ‖ 2 | 183.05
226. ‖ 3 | 19070.142+ ‖ 4 | 365 ‖ 5 | 550.122+
226. ‖ 6 | 12000 ; 600 ‖ 7 | 145.6311+ ‖ 8 | $4854.23\frac{1}{13}$
226. ‖ 9 | 5425.46153+ ‖ 10 | 20600 ; $137333\frac{1}{3}$
227. ‖ 1 | 40.25 ‖ 2 | 2600 ‖ 3 | .75 ‖ **228.** ‖ 1 | .377976+
228. ‖ 2 | 5415 ‖ 3 | .382 ‖ 4 | 9810 ‖ 1 | 225.00
228. ‖ 2 | 2390.625 ‖ 3 | 1485.60 ‖ **229.** ‖ 4 | 1.0625
229. ‖ 5 | .96653+ ‖ 6 | 1.03125 ‖ 7 | 2.054 ‖ 1 | 18

229. ‖ 2 | 13.00 ; 20 ‖ 3 | $14\frac{6}{11}$ ‖ 4 | $43\frac{1}{8}$ ‖ 230. ‖ 1 | 280
230. ‖ 2 | 200 ‖ 3 | 2275 ‖ 4 | 4778.0153+ ‖ 231. ‖ 1 | 43.77
231. ‖ 2 | 1312.50 ‖ 3 | 237.60 ; 158.40 ‖ 4 | 504
232. ‖ 5 | 450 ‖ 6 | 1320.00 ‖ 7 | 255.45 ‖ 8 | 9558.4375
232. ‖ 9 | 6500.00 ‖ 10 | 240 ‖ 11 | 2250 ‖ 12 | 33.75 ‖ 13 | 850
233. ‖ 4 | 14700.00 ‖ 5 | 187.50 ‖ 6 | 15968.75 ‖ 7 | 27156.2
233. ‖ 2 | 15000 ‖ 3 | 104 ‖ 4 | 20000 ‖ 5 | 12000
234. ‖ 1 | 39.00 ‖ 2 | 266.00 ‖ 3 | 4446.75 ‖ 4 | 642.60
234. ‖ 5 | 9.5067+ ‖ 6 | 331.1511+ ‖ 7 | 1158.0668
234. ‖ 8 | $22.9963\frac{1}{8}$ ‖ 9 | 4454.857 ‖ 10 | 30455.0224
234. ‖ 11 | $95.2291\frac{2}{3}$ ‖ 12 | $121.3257\frac{3}{4}$ ‖ 13 | 1315.389
234. ‖ 14 | 221.075 ‖ 15 | 1060.6258 ‖ 16 | 7285.285
234. ‖ 17 | 825.1914 ‖ 236. ‖ 1 | $10.80123\frac{1}{3}$ ‖ 2 | 2.7285
236. ‖ 3 | 309.56345 ‖ 4 | 35.14857 ‖ 5 | 30.5598 ‖ 6 |
236. ‖ 14.09793 ‖ 7 | 62.8992 ‖ 8 | 76.24393 ‖ 9 | 194.6178
237. ‖ 10 | 328.32 ‖ 11 | 1004.6976 ‖ 12 | 1183.69845
237. ‖ 13 | 1445.23831 ‖ 14 | 190.15024 ‖ 15 | 218.88
237. ‖ 16 | 600.449 ‖ 17 | $44.2893\frac{1}{3}$ ‖ 18 | 167.001625
237. ‖ 19 | 3126.20468 ‖ 20 | 9051.67312 ‖ 21 | 4968.9975
237. ‖ 22 | $778.1458\frac{1}{3}$ ‖ 23 | 1622.80002 ‖ 24 | $6972.68\frac{1}{3}$
237. ‖ 25 | 1316.00669 ‖ 238. ‖ 26 | 272.80 ‖ 27 | 39.92749
238. ‖ 28 | 928.07013 ‖ 29 | 529.925 ‖ 30 | 31.26875 ‖ 31 |
238. ‖ 9426.9783125 ‖ 32 | $245.49154\frac{1}{4}$ ‖ 33 | $33.32343\frac{3}{4}$
238. ‖ 34 | $28761.776\frac{2}{3}$ ‖ 35 | 5678.07186 ‖ 36 | 217.464
238. ‖ 37 | 58.14 ‖ 38 | 439.67266 ‖ 39 | 188.034925
239. ‖ 1 | £15 2s. 8d. 2far. ‖ 2 | £24 18s. $3\frac{1}{4}$d. ‖ 3 | £26
239. ‖ 10s. $11\frac{1}{4}$d. ‖ 4 | £331 1s. 6d. ‖ 241. ‖ 2 | 860.41952
241. ‖ 3 | 167.98369 ‖ 242 ‖ 2 | .07 ‖ 3 | 5 yr. ‖ 4 | 225
243. ‖ 5 | 7 yr. 4 mo. ‖ 6 | 1508.57142 ‖ 7 | 2 yr. 6.309 da.
243. ‖ 8 | 13.976 per ct. ‖ 9 | 58.5 per ct. ‖ 10 | 646.33237
243. ‖ 11 | 7 yr. 11 mo. 24 da. ‖ 244 ‖ 2 | 19.101
244. ‖ 3 | 36.5077 ‖ 4 | $404.07421\frac{7}{8}$ ‖ 5 | 291.60 ‖ 6
244. ‖ 271.8921 ‖ 7 | 185.77061 ‖ 8 | 281.3772, 31.3772
245. ‖ 9 | $38.44531\frac{1}{4}$ ‖ 10 | 171 60752 ‖ 11 | 118.5288

245. ‖ 12 | 315.2375 ‖ **248.** ‖ 1 | 1750 ‖ 2 | 1565.40284
249. ‖ 3 | 9677.50928 ‖ 4 | 223.28323 ‖ 5 | 5620.17636
249. ‖ 6 | 702.48545 ‖ 7 | 1.96407 ‖ 8 | 3869.40749
249. ‖ 9 | 2109.23681 ‖ 10 | 2763.694+ ‖ 11 | 4000
249. ‖ 12 | 6.47269 ‖ **250.** ‖ 1 | 6.32916 ‖ 2 | 10.50
250. ‖ 3 | 15240.54 ‖ **251.** ‖ 4 | 5.84083 ‖ 5 | 3393.504
251. ‖ 6 | 29.0097 ‖ 7 | 17.14875 ‖ 8 | 1591.09255
251. ‖ 9 | 122.81079 ‖ **252.** ‖ 1 | 344.666+ ‖ 2 | 5734.3270+
252. ‖ 3 | 695.643+ ‖ 4 | 1745.87401+ ‖ 5 | 376.46785
254. ‖ 1 | 3720.9375 ‖ 2 | 8668.935 ‖ 3 | 6748.60
254. ‖ 4 | 4583.94375 ‖ 5 | 3643.875 ‖ 6 | 971.411
254. ‖ 7 | 3891.62561 ‖ 8 | 745.14987 ‖ **255.** ‖ 1 | 5944.79166
256. ‖ 2 | 9226.06133 ‖ 1 | 1270.42801 ‖ 2 | 2016.11001
256. ‖ 3 | 16975.77519 ‖ **257.** ‖ 1 | 8 months $22\frac{4}{23}$ days
258. ‖ 2 | 9 mo. ‖ 3 | $137\frac{13}{31}$ ‖ **259.** ‖ 1 | $67\frac{6}{19}$ days, or
259. ‖ March 9th ‖ 2 | May 2d ‖ 3 | 51 days—Jan. 26th
260. ‖ 4 | May 31st, 1861 ‖ 5 | $168\frac{751}{1135}$ days, due
260. ‖ Dec. 20th, 1860 ‖ **261.** ‖ 2 | $391.91 ‖ **264.** ‖ 3 | 12.25
264. ‖ 4 | 6.25 ‖ **266.** ‖ 1 | 2812.50 ‖ 2 | $423.36
266. ‖ 3 | 251.45+ ‖ 4 | 1442.875 ‖ **268.** ‖ 2 | 112 ‖ 3 | 222
268. ‖ 4 | 19 ‖ 5 | 18 ‖ 8 | 9 ‖ 9 | $8.333\frac{1}{3}$ ‖ 10 | 36.42857
269. ‖ 11 | 21.25 ‖ 12 | $28.333\frac{1}{3}$ ‖ 13 | 32.8125 ‖ 14 | $30.833\frac{1}{3}$
269. ‖ 15 | 62 ‖ 16 | $3.111\frac{1}{9}$ ‖ 17 | 24 ‖ 18 | 112.50
269. ‖ 19 | 472.50 ‖ 20 | 6 ‖ 22 | 6.5625 ‖ 23 | 10
269. ‖ 24 | $9.166\frac{2}{3}$ ‖ 25 | 10500 ‖ **270.** ‖ 27 | 72 ‖ 28 | 108
270. ‖ 29 | 308—112 ‖ 30 | 5040 ‖ 32 | 12 ‖ 33 | 48
270. ‖ 34 | $2\frac{5}{8}$ ‖ 35 | $10\frac{10}{11}$ ‖ **271.** ‖ 37 | $10\frac{5}{13}$ ‖ 38 | $62\frac{6}{7}$
271. ‖ 39 | $10\frac{1}{2}$ yd. ‖ 40 | $10\frac{1}{5}$ ‖ **272.** ‖ 42 | $1\frac{7}{8}$ ‖ 43 | 30
272. ‖ 44 | 96 ‖ 45 | $7\frac{1}{5}$ ‖ 46 | 3 ‖ 47 | 36
272. ‖ 49 | 2.50 ; 3.75 ; 8.75 ‖ 50 | 175 ; 125
273. ‖ 51 | 21; 15; 24 ‖ 52 | 11.538+; 23.076+; 15.384+
273. ‖ 54 | $2\frac{8}{11}$ ‖ 55 | $5\frac{5}{8}$ ‖ 56 | $44\frac{4}{9}$ ‖ 57 | $7\frac{4}{21}$ ‖ 58 | $12\frac{6}{7}$
274. ‖ 59 | $1\frac{5}{7}$ ‖ 60 | 263.1578+; 131.5789+; 105.2631+
274. ‖ 61 | 19.60; 39.20; 117.60 ‖ 62 | $1783.333\frac{1}{3}$; 1933·

274. ‖ .333$\frac{1}{3}$; 2283.333$\frac{1}{3}$ ‖ 64 | 58.333$\frac{1}{3}$; 116.666$\frac{2}{3}$
274. ‖ 65 | 240; 200; 140 ‖ 275. ‖ 1 | 3 ‖ 2 | 84$\frac{3}{8}$
275. ‖ 3 | 28$\frac{36}{43}$ ‖ 4 | 20$\frac{2}{3}$ ‖ 276. ‖ 5 | 73
277. ‖ 1 | 1, 1, 6, 1 ‖ 2 | 1, 1, 3 ‖ 3 | 2, 2, 5, 2
278. ‖ 4 | 1, 1, 2, 1 ‖ 5 | 2, 1, 1, or 2 gallons of water
278. ‖ 1 | 4 at 5s.; 24 at 5s. 6d.; 4 at 6s. ‖ 2 | 24; 14; 14; 14
278. ‖ 3 | 12; 12; 12; 96 ‖ 279. ‖ 4 | 32; 32; 40
279. ‖ 1 | 36 of each ‖ 2 | 21$\frac{3}{4}$ of each ‖ 3 | 4, 4, 4, 30
280. ‖ 1 | $\frac{1}{4}$ ‖ 2 | $\frac{1}{512}$ ‖ 3 | 1728 ‖ 4 | 1953125 ‖ 5 | 6561
280. ‖ 6 | 1048576 ‖ 7 | 50625 ‖ 8 | 33076161
280. ‖ 9 | 2136750625 ‖ 10 | 1299530401 ‖ 11 | .015625
280. ‖ 12 | 5541.668535603456 ‖ 284. ‖ 2 | 191.64+
284. ‖ 3 | 46.0977+ ‖ 4 | 5184.966+ ‖ 5 | 653 ‖ 6 | 1725-
284. ‖ .228+ ‖ 7 | 9585.1062+ ‖ 285. ‖ 1 | 1.73205+
285. ‖ 2 | 3.31662+ ‖ 3 | 32.695+ ‖ 4 | 1506.23+
285. ‖ 5 | 2756.22+ ‖ 6 | 6031 ‖ 7 | 4698 ‖ 8 | 57.19+
285. ‖ 9 | 69.247+ ‖ 10 | 2.091+ ‖ 11 | .05 ‖ 12 | .01809+
285. ‖ 13 | .0321 ‖ 14 | 2.104 ‖ 15 | 2.91547
286. ‖ 16 | 3.12249+ ‖ 17 | .71554+ ‖ 18 | .41408+
286. ‖ 19 | $\frac{2}{3}$ ‖ 20 | $\frac{4}{5}$ ‖ 287. ‖ 2 | 25 ‖ 1 | 30
288. ‖ 1 | 12.649+ ‖ 2 | 126.491+ ‖ 3 | 85 ‖ 4 | 97.754+
288. ‖ 5 | 43.817+ ‖ 6 | 82 ‖ 7 | 62 ‖ 8 | 160 ‖ 9 | 90
288. ‖ 10 | 4.949+ ‖ 292. ‖ 1 | 73 ‖ 2 | 179 ‖ 3 | 319
292. ‖ 4 | 439 ‖ 5 | 638 ‖ 6 | 364 ‖ 1 | .54 ‖ 2 | .955
292. ‖ 3 | 2.35 ‖ 4 | .909 ‖ 5 | .707 ‖ 6 | 1.505
293. ‖ 1 | $\frac{5}{7}$ ‖ 2 | 3$\frac{1}{7}$ ‖ 3 | $\frac{3}{5}$ ‖ 4 | .829+ ‖ 5 | .822+
293. ‖ 6 | .873+ ‖ 1 | 17 ‖ 2 | 28—4704 ‖ 3 | 16.197+
293. ‖ 4 | 14.59+ ‖ 5 | 1728 ‖ 6 | 12 ‖ 7 | 6
294. ‖ 8 | 268.0832 ‖ 9 | 2 ft. 4 in. ‖ 10 | 2 ‖ 11 | 12
296. ‖ 1 | 1.53 ‖ 2 | 212 ‖ 3 | 40 ‖ 4 | 2 ‖ 5 | 9
297. ‖ 1 | 4 ‖ 2 | 5 ‖ 3 | 4 ‖ 298. ‖ 2 | 78 ‖ 3 | 34—162
298. ‖ 4 | 1.35 ‖ 5 | 175 ‖ 6 | 5 miles 1300 yards
300. ‖ 2 | £2 2s. 8d. ‖ 3 | 4 ‖ 4 | 78732 ‖ 5 | 25600
300. ‖ 6 | 6144 ‖ 301. ‖ 1 | 6561$\frac{1}{2}$ ‖ 2 | 254 ‖ 3 | 381

301. ‖ 4 | £204 15s. ‖ 5 | 196.83; 295.24 ‖ 6 | 4800; 9450

302. ‖ 1 | 400 ‖ 2 | 30 ‖ 3 | 2 A. 3 R. 15 P.

303. ‖ 1 | 109 A. 1 R. 28.5824 P. ‖ 2 | 72 A. 3 R. 24 P.

303. ‖ 3 | 3 A. 3 R. 25 P. ‖ 4 | 5 A. ‖ 5 | 10 A.

303. ‖ 6 | 7 A. 2 R. ‖ 7 | 46 A. 2 R. 12 P. ‖ 8 | 32520 sq. yd

303. ‖ 1 | 45849.485 ‖ 2 | 5 A. 1 R. 9.956 P. ‖ 304. ‖ 3

304. ‖ 230 ft. 5′ 7″ 6‴ ‖ 4 | 176.03125 ‖ 5 | 15 A. 33.2 P.

304. ‖ 6 | 67 A. 2 R. 16 P. ‖ 1 | 12.5664 ‖ 2 | 292 1688

304. ‖ 3 | 62.8320 ‖ 4 | 25 ‖ 5 | 3709 ‖ 6 | 2180.41+

305. ‖ 1 | 28.2744 ‖ 2 | 78.54 ‖ 3 | 38.4846 ‖ 4 | 1.069+

305. ‖ 1 | 452.3904 ‖ 2 | 153.9384 ‖ 3 | 1809.5616

305. ‖ 4 | 196663355.7504 ‖ 1 | 904.7808 ‖ 2 | 33.5104

305. ‖ 3 | 1436.7584 ‖ 4 | 113.0976 ‖ 306. ‖ 1 | 4550

306. ‖ 2 | 1440 ‖ 1 | 4500 ‖ 2 | 13824 ‖ 3 | $21\frac{1}{3}$

306. ‖ 4 | $157\frac{213}{231}$ ‖ 5 | 3500 ‖ 307. ‖ 1 | 3141.6

307. ‖ 2 | 116.666+ ‖ 3 | 5654.88 ‖ 1 | 2120.58 ‖ 2 | 9110.64

307. ‖ 3 | $805.7417\frac{1}{7}$ ‖ 4 | $214.8644\frac{4}{7}$ ‖ 5 | 29452.5

308. ‖ 1 | 475 ‖ 2 | 2080 ‖ 3 | 1242 ‖ 4 | 4836 ‖ 5 | 1440

308. ‖ 6 | 3600 ‖ 309. ‖ 1 | 65.45 ‖ 2 | 2290.2264

309. ‖ 3 | 3141.6 ‖ 4 | 706.86 ‖ 5 | 706.86 ‖ 310. ‖ 2 | 185-

310. ‖ .0688 ‖ 3 | 136 ‖ 4 | 182.844+ ‖ 5 | 6 hr. 3 m. $48\frac{1}{2}\frac{14}{17}$ sec.

311. ‖ 1 | 9.73295 ‖ 2 | 76, 1292 ‖ 3 | $69\frac{3}{7}$ from N. Haven

311. ‖ 4 | 4717 ‖ 5 | 137 ‖ 6 | $\frac{3}{8}$ ‖ 7 | $17\frac{7}{8}$ ‖ 8 | $\frac{3}{10}$

311. ‖ 9 | $4\frac{7}{24}$ ‖ 10 | 9.04 ‖ 11 | $4\frac{1}{6}$ ‖ 12 | 978 ‖ 13 | 51

311. ‖ 14 | 80.71 ‖ **312.** ‖ 15 | 7 mo. 15 da. 19hr. ‖ 16 | 26.25

312. ‖ 17 | 213.3125; 211.6875 ‖ 18 | $\frac{25}{42}$; $\frac{11}{42}$ ‖ 19 | 50

312. ‖ 20 | 879000 ‖ 21 | 792 ‖ 22 | 1 mi. 6 fur. 33 rd. $15\frac{1}{2}$ ft.

312. ‖ 23 | 3567 ‖ 24 | $108\frac{7}{75}$ ‖ 25 | 80 ‖ 26 | 120 ‖ 27 | 3800

312. ‖ 28 | 2400 ‖ 29 | 28; 60; 32 ‖ 30 | 2364; 1182;

313. ‖ 788; 394 ‖ 31 | $7\frac{1}{5}$ ‖ 32 | 3 ‖ 33 | $\frac{37}{240}$; $986.66\frac{2}{3}$

313. ‖ 34 | $1\frac{1}{4}$ ‖ 35 | 160; 120; 140 ‖ 36 | 2454; 3681;

313. ‖ 4294.50 ‖ 37 | 408 ‖ 38 | 62 ‖ 39 | 4 ‖ 40 | 288

313. ‖ 41 | $23\frac{1}{2}$ ‖ 42 | 2250 ‖ 314. ‖ 43 | 22500 ‖ 44 | 864

314. ‖ 45 | 300 ‖ 46 | 57 ‖ 47 | 52 yr. 11 mo. 20 da. $10\frac{1}{2}$ hr.

314. ‖ 48 | 111 ‖ 49 | $5\frac{15}{23}$ ‖ 50 | $32\frac{8}{11}$ m. ‖ 51 | 34782.60869
314. ‖ 52 | 3.6538+ ‖ 53 | $34\frac{4}{10}$ ‖ 54 | 7816.09195
314. ‖ 55 | 194.80$\frac{49}{77}$; 129.87$\frac{1}{77}$; 97.40$\frac{20}{77}$; 77.92$\frac{16}{77}$
314. ‖ 56 | 920.20 ; 2760.60 ; 5521.20 ‖ 315. ‖ 57 | 240
315. ‖ 58 | 3 hr. 20 min. ‖ 59 | 27.73084 ‖ 60 | 5 yr.
315. ‖ 61 | 196.83 ; 295.24 ‖ 62 | 96 ; 12 ; 12 ; 12
315. ‖ 63 | $16\frac{16}{17}$ ‖ 64 | 356.25 ‖ 65 | 8640 ‖ 66 | 4 yr
315. ‖ 11 mo. 27+ da. ‖ 67 | 1020.66 ‖ 316. ‖ 68 | 8925-
316. ‖ .544+ ‖ 69 | 1.20 ‖ 70 | 4 ‖ 71 | 4 ‖ 72 | 4 yd.
316. ‖ 73 | 423.36 ‖ 74 | 50 ‖ 75 | 780 ; 3120 ; 1560 ‖ 76 |
316. ‖ 35.8107+ ‖ 77 | 13, or A was 27 ‖ 78 | $8\frac{4}{7}$ ‖ 79 | 21; $8\frac{2}{5}$; 42

Fall Term, 1887
at "Temple Grove"
in Room 25 on long-hall
Third floor

9 7 7 0 3 : 9

11) 2770

2
44
304
304

$\frac{1}{11}$ [illegible] 21 $\frac{1}{7}$

21

[illegible] × 40 × 8 × 9

(Page 217. Ex. 5.)

$$\left.\begin{array}{rcr} 336 & : & 240 \\ 70 & : & 105 \\ 6 & : & 5 \\ 3 & : & 2 \\ [illegible] & : & 9 \end{array}\right\} :: 70 :$$

THE

NATIONAL SCHOOL BOOK SERIES,

COMPLETE FOR

SCHOOLS, ACADEMIES, AND COLLEGES OF ALL GRADES

IN ALL PARTS OF THE COUNTRY.

BARNES' NEW NATIONAL READERS. Five Numbers. Also Parker and Watson's National Readers and Spellers, and Watson's Independent Readers and Spellers.

BARNES' ELEMENTARY AND NATIONAL ARITHMETIC. Two Numbers. Also Davies' Arithmetics and Peck's Arithmetics.

BARNES' COMPLETE MATHEMATICAL SERIES. Including the Works of Charles Davies, LL.D., William G. Peck, LL.D., Joseph Ficklin, A. E. Church, [illegible] Bartlett, Courtenay and Hackley.

BARNES' BRIEF HISTORY SERIES. Embracing Outlines of General History, Ancient History, Modern History, France, Greece, and the United States.

BARNES' POPULAR DRAWING BOOK SERIES.

STEELE'S SCIENTIFIC COURSE. Seven Numbers.

MONTEITH'S GEOGRAPHICAL SERIES. [illegible] books for each grade. Also correct systems of map-drawing and wall maps.

MONTEITH'S POPULAR SCIENCE READER.

MONTEITH'S LESSONS IN POPULAR SCIENCE.

WORMAN'S MODERN LANGUAGE SERIES. [illegible] of French and Spanish, based upon the natural method of learning languages without the employment of English.

Also [illegible] Also Teachers' Library, [illegible] School Library and Stationery.

Teachers [illegible] educational information [illegible] illustrated [illegible] on application.

A. S. BARNES & CO., Publishers,

NEW YORK AND CHICAGO.